Studying the aquisition of mathematical concepts through the generalization of a geometrical primitive from euclidean to non-euclidean geometry

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14 avril 2021

Introduction

How do we learn new mathematical concepts?

Studying the change of paradigm in geometry, from euclidean geometry to a more general geometry, using a fundamental concept : straight line

Why non euclidean geometry?

For almost 2000 years, euclidean geometry was considered as the natural geometry (Plato, Descartes, Kant)

Euclidean geometry is the most easy to conceive for humans (Spelke, Lee and Izard 2010; Izard, Pica and Dehaene 2011)

Non euclidean systems are late in the history of humanity

Emerged in a formal way, from a failed attempt to prove its contradiction

Some of its models contradicts basic postulates of euclidean geometry, notably the fact that there always exist a parallel line to another one

Euclidean geometry is historically considered as the most easy to conceive (Spelke, Lee and Izard 2010)

Is it the case for a fundamental concept such as straight line? Does it reflects the euclidean intuitions?

Straight line : a fundamental concept of euclidean geometry. Non primitive, used as a basic concept for more complex definitions, and hard to define without circularity

Euclid's definition : *A line is a length without width, and a straight line is a line equally placed between its points*

The generalization of straight line to curved surfaces is the geodesic

Starting from a given point on a surface, it is a path which has a constant direction and never turns

	Straight line	Geodesic
Constant direction	X	X
Shortest path	X	
Infinite	X	
Planar	X	

Straight line is a singular case of geodesic : a geodesic may intersect itself, be finite, and of non null curvature degree

Plan of the thesis

Intuition : does euclidean intuition shape the representation of straight line ? If yes, is it possible to change the initial intuition, or do humans just learn heuristics and methods to answer correctly to specific problems ? (Article in preparation, data in collection)

Dynamic of learning and metacognition : is there a specific metacognitive signature associated to a new learning situation ? (One article submitted, an online replication planned)

Novelty of concept : do participants build a new representation when learning or do they just use a new representation in addition, if any ? (Article in preparation, ongoing analyses)

Do spontaneous intuitions of straight lines on curved surfaces reflect euclidean bias ?

- Non mathematicians participants

Are the intuitions rigid to a formal learning ?

- Two groups of mathematicians, increasing degree of expertise : aggregés professors and experts in differential geometry

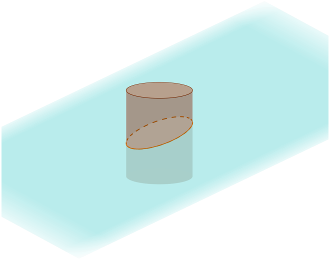
Does euclidean intuition shape the representation of straight line?

How do non mathematicians generalize spontaneously the concept of straight line ?

giving a definition which gives the essential property of geodesic which coincides with straight line, how do they manage to identify straigth lines on curved surfaces ?

Hypothesis : non mathematicians will tend to identify planar intersection lines as straight lines on curved surfaces

Planar intersections coïncide with straight directions in three dimensional space, but they rarely coïncide with actual straight lines (geodesics) on surfaces

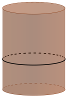
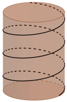
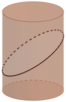
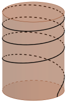


An ellipse resulting of a planar intersection on the cylinder

The experimental session

Definition : “A straight line is a line that always follows the same direction without ever turning, neither to the right nor to the left, and always goes straight ahead”

“Is this a straight line ?”

	Planar intersection	Non planar intersection
Straight		
Not straight		

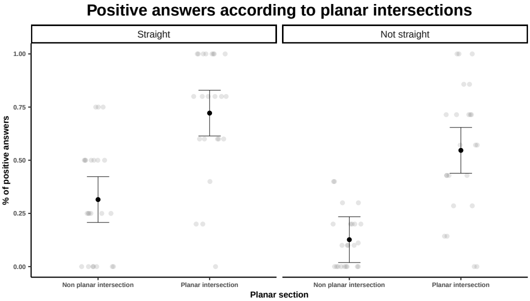
26 randomized trials, 4 surfaces (cone, cube, cylinder, sphere)

Hypotheses (preregistered) :

Is the factor of planarity a predictor of positive answers to the question “Is this a straight line ?” ?

Do participants tend to identify planar intersections as straight lines in a pool of pairs of non straight curves matched for length and curvature ?

Do the answers correlate to the answers of another group of participants asked to identify whether the lines were planar or not ?



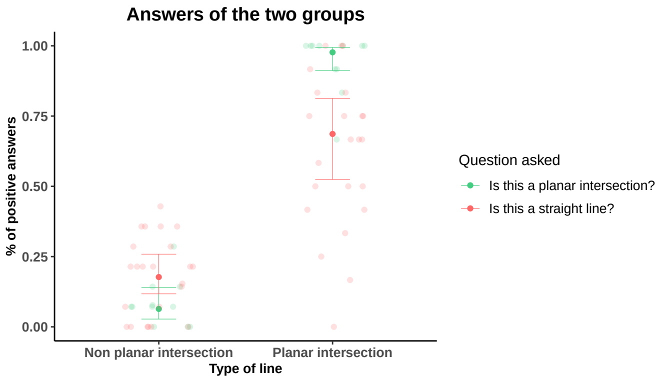
N=23

Factor planar intersection : $F(1,22)=53.32$, $p<.0001$

Factor straight : $F(1,22)=19.09$, $p=.0002$

No interaction

Visually identical effects when putting random effect for each object, to ensure this is not due to accidental properties of shapes or lines



group : $F(1,32)=2.28$ $p=.14$

planar intersection : $F(1,32)=213.60$, $p<.0001$

straight : $F(1,32)=8.45$, $p=.007$

group x straight $F(1,32)=6.94$, $p=.01$

Results synthesis

- Factor of planarity strongly predictive of participants' responses, transversaly to straightness
- Second, when comparing pairs of (non-straight) curves matched for length and curvature, participants were more likely to identify planar intersections as straight lines ($t(23)=5.14$, $p<.0001$).
- Third, participants' answers were strongly correlated to the answers of another group of participants ($N=11$) asked to identify whether the lines were planar or not ($r=0.81$, $p<.0001$), which suggests that the two concepts coïncide.

How do this euclidean intuition evolve with mathematical education ?

How do this euclidean intuition evolve with mathematical education ?

Two questions :

- When put in formal context, do participants reproduce this bias ? : Math educated participants (five years or more of formal education in maths, able to understand the exact definition of geodesic)
- Does the new concept collapses the former intuition or is there still traces of the planar heuristic ? Differential geometry experts (PhD or more in differential geometry or connex fields)

Thanks for your attention!