



ed3c



Université
de Paris



Eureka moments in the acquisition of mathematical concepts

Charlotte Barot, Louise Chevalier, Lucie Martin and Véronique Izard

13 janvier 2021

Introduction

How does it feel to understand a mathematical concept?

Famous scientists experience sudden “Eureka moments”, notably in mathematics Poincaré 1908, Hadamard 1959

Famous scientists experience sudden “Eureka moments”, notably in mathematics Poincaré 1908, Hadamard 1959

Sudden and unexpected understanding

Famous scientists experience sudden “Eureka moments”, notably in mathematics Poincaré 1908, Hadamard 1959

Sudden and unexpected understanding

Feeling of certainty

Famous scientists experience sudden “Eureka moments”, notably in mathematics Poincaré 1908, Hadamard 1959

Sudden and unexpected understanding

Feeling of certainty

Similar experiences are studied in the scope of problem solving where solutions come by “insight”

Famous scientists experience sudden “Eureka moments”, notably in mathematics Poincaré 1908, Hadamard 1959

Sudden and unexpected understanding

Feeling of certainty

Similar experiences are studied in the scope of problem solving where solutions come by “insight”

No awareness that it was about to come Metcalfe 1986, Metcalfe and Wiebe 1987, Bowden and Jung-Beeman 2003

Famous scientists experience sudden “Eureka moments”, notably in mathematics Poincaré 1908, Hadamard 1959

Sudden and unexpected understanding

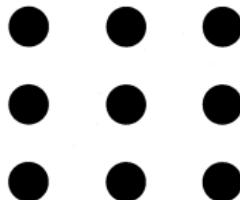
Feeling of certainty

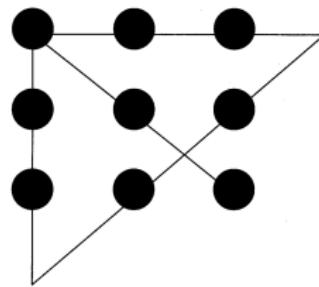
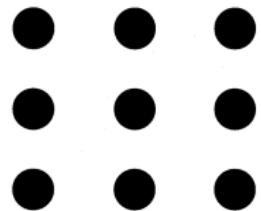
Similar experiences are studied in the scope of problem solving where solutions come by “insight”

No awareness that it was about to come Metcalfe 1986, Metcalfe and Wiebe 1987, Bowden and Jung-Beeman 2003

Immediately perceived as correct and relevant Kounios and Beeman 2014, Danek and Wiley 2017, Laukkonen et al. 2020

The nine dot problem, Lung and Dominowski 1985





So far, no evidence of insights while learning a new concept



Methodological difficulties : learning science concepts is difficult and protracted Carey 2009, Weber 2002, Asmuth and Rips 2006, Vosniadou 2019

Methodological difficulties : learning science concepts is difficult and protracted Carey 2009, Weber 2002, Asmuth and Rips 2006, Vosniadou 2019

Conducted in classrooms : cross sectional designs or longitudinal designs with long delays between sessions

Behr et al. Lesh 1984, Siegler 1995, Schauble 1996, Rittle-Johnson and Alibali 1999, Smith et al. 2005

Methodological difficulties : learning science concepts is difficult and protracted Carey 2009, Weber 2002, Asmuth and Rips 2006, Vosniadou 2019

Conducted in classrooms : cross sectional designs or longitudinal designs with long delays between sessions

Behr et al. Lesh 1984, Siegler 1995, Schauble 1996, Rittle-Johnson and Alibali 1999, Smith et al. 2005

Lab experiment with simple learning target for example inferring a rule for categorizing images Shepard 1961, Feldman 2000, Goodman et al. 2008, Ohlsson and Cosejo 2014, Marti et al. 2018

Goals of the study

Conceive a one session paradigm of conceptual learning

Conceive a one session paradigm of conceptual learning
See if participants report experiencing insights while learning
a new concept and if these insights are associated with a
better learning



Could concept learning rely on mechanisms that are not accessible to conscious reports? Schooler et al. 1993, Jameson et al. 1990, Miner and Reder 1994, Schwartz and Metcalfe, 1992

Could concept learning rely on mechanisms that are not accessible to conscious reports? Schooler et al. 1993, Jameson et al. 1990, Miner and Reder 1994, Schwartz and Metcalfe, 1992

Test the accessibility of concepts learning mechanisms

Could concept learning rely on mechanisms that are not accessible to conscious reports? Schooler et al. 1993, Jameson et al. 1990, Miner and Reder 1994, Schwartz and Metcalfe, 1992

Test the accessibility of concepts learning mechanisms

Collect ratings of confidence about participants' knowledge of the new concept and see if insight are related to some specific aspects of performance, even when we control for confidence ratings

Learning situation

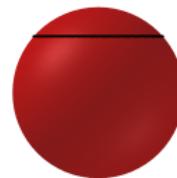
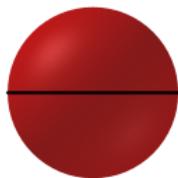
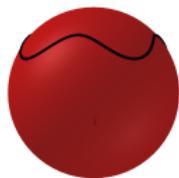
The new concept : geodesic

Geodesic is the generalization of straight line to curved surfaces

Starting from a given point on a surface, it is a path which has a constant direction and never turns

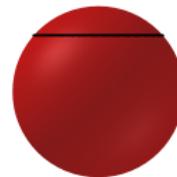
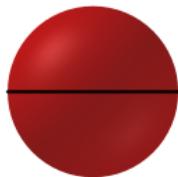
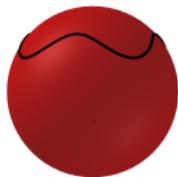
Learning situation : geodesic on the sphere

"Is it straight?"



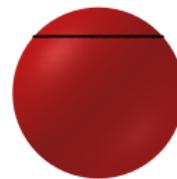
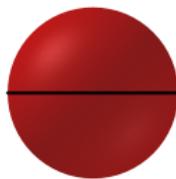
Learning situation : geodesic on the sphere

"Is it straight?"



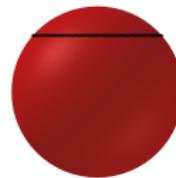
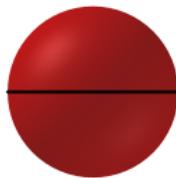
Learning situation : geodesic on the sphere

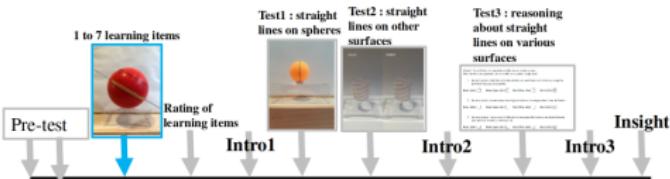
"Is it straight?"



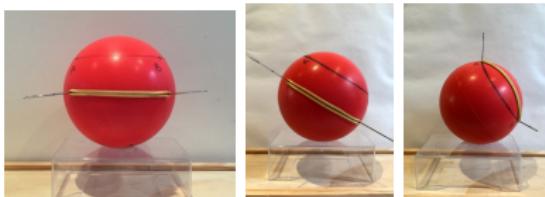
Learning situation : geodesic on the sphere

"Is it straight?"



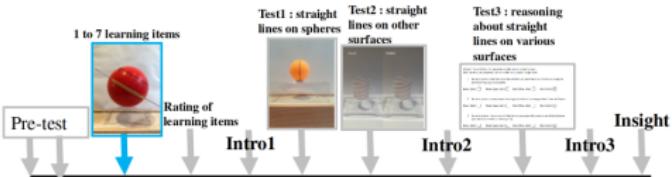


The rubber band lesson

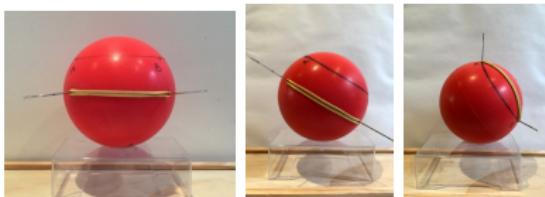


The elastic is straight and follows greater circles on a sphere, but when applied to a small circle, the elastic does not follow the line

Four conditions : 1, 3, 5 or 7 different lessons



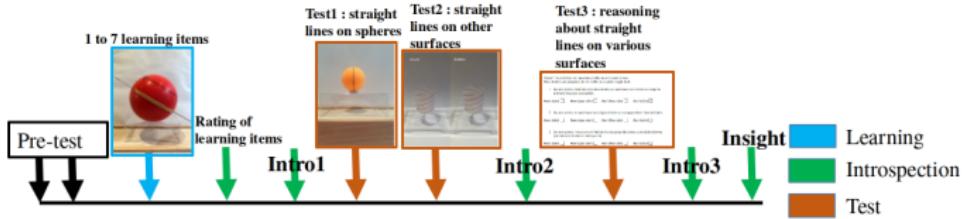
The rubber band lesson



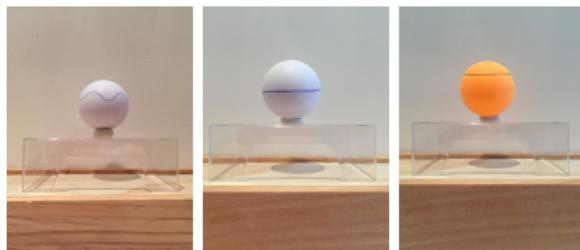
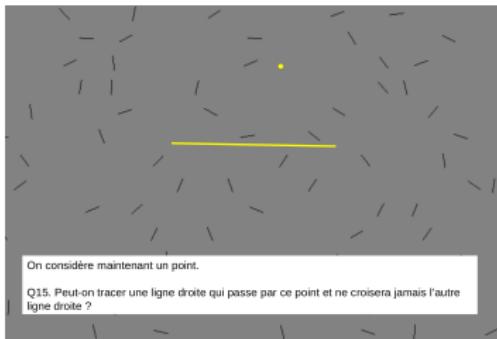
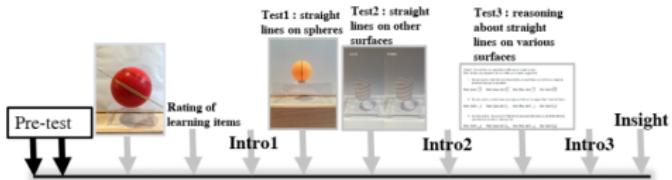
The elastic is straight and follows greater circles on a sphere, but when applied to a small circle, the elastic does not follow the line

Four conditions : 1, 3, 5 or 7 different lessons

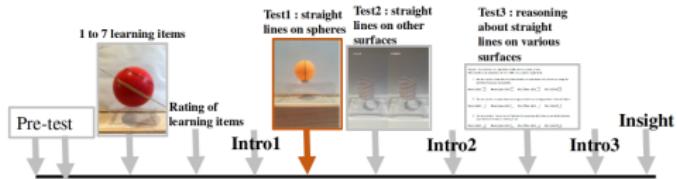
Goal : test whether participants' objective performance vary as a function of the number of lessons studied



N=56, 18-43 years ($M=25.5$), time $\approx 1h30$



Performance measures



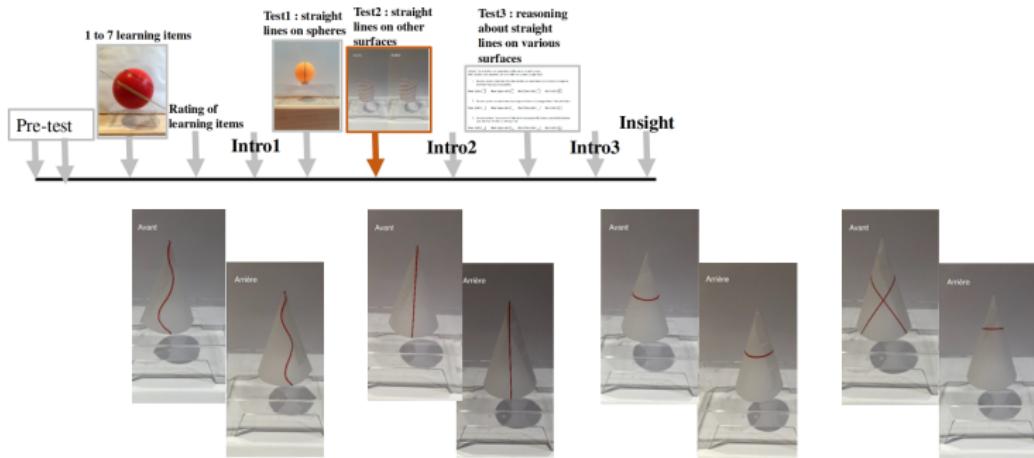
Not straight non planar lines

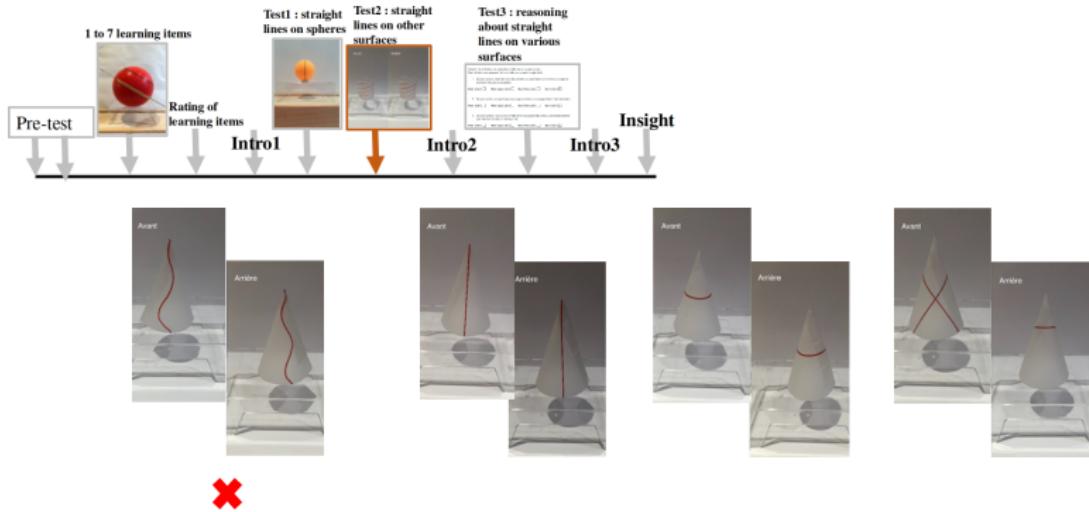


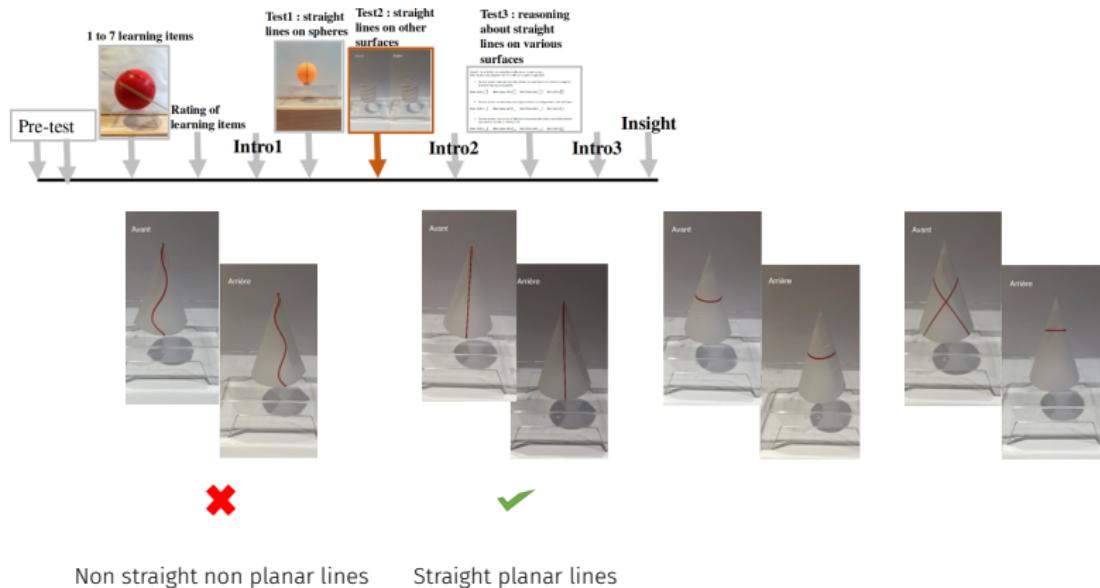
Straight planar lines

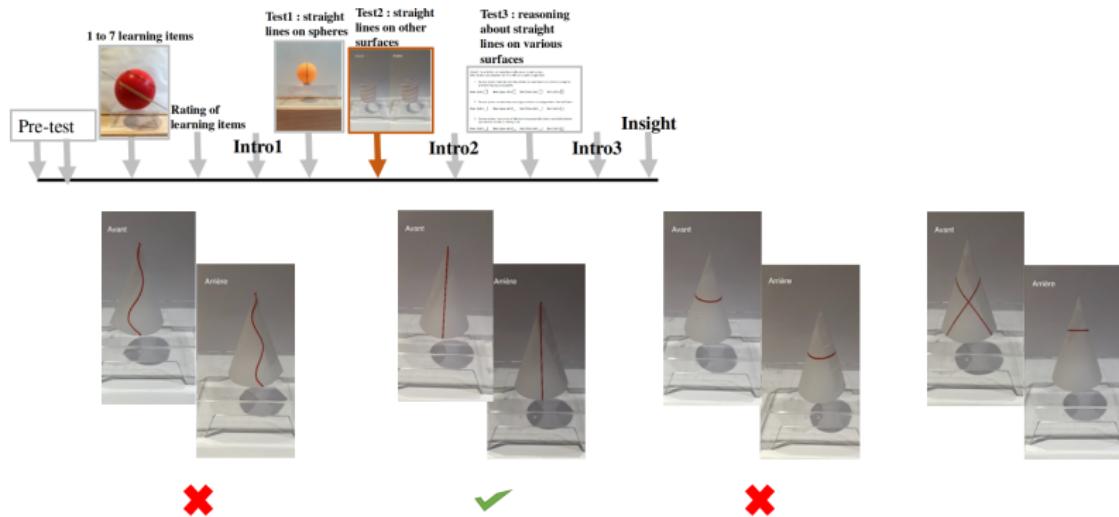


Not straight planar lines





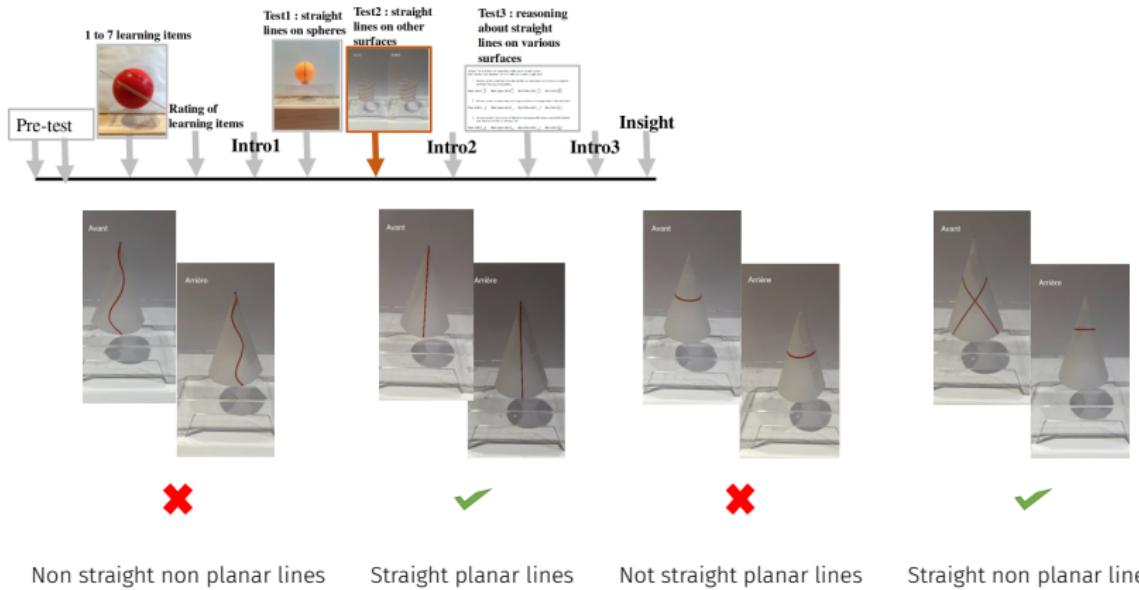


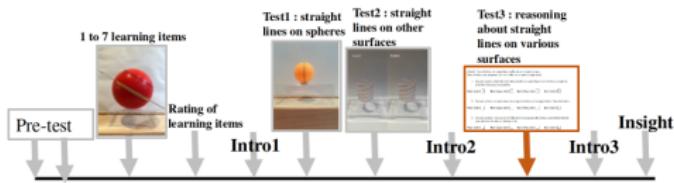


Non straight non planar lines

Straight planar lines

Not straight planar lines





Test condition Sphere

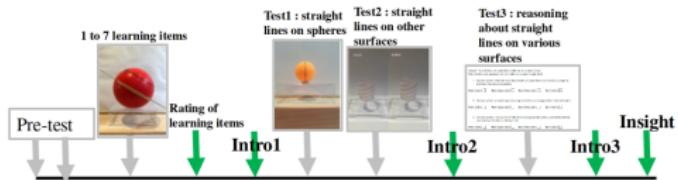
4. Sur une sphère, on peut tracer deux lignes droites qui ne se coupent jamais.

Non (sûr) **Non (pas sûr)** **Oui (Pas sûr)** **Oui (sûr)**

Test condition All surfaces

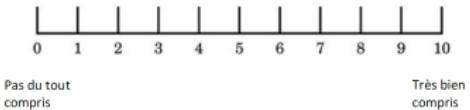
10. Il existe une surface sur laquelle par un point donné ne passe aucune ligne droite parallèle à une autre droite donnée.

Non (sûr) **Non (pas sûr)** **Oui (Pas sûr)** **Oui (sûr)**



Confidence rating (CR) from 0 to 10

Pensez-vous avoir compris la notion de ligne droite ?



Insight reports “Did you experienced an insight?”

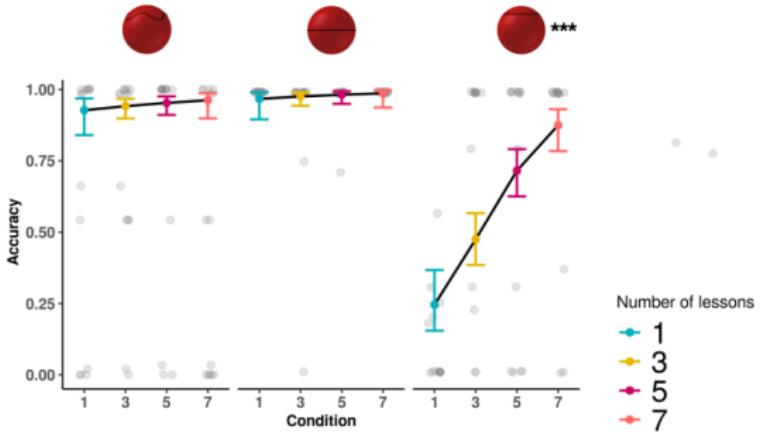
(description adapted from Danek and Wiley 2017)

Analyses

Does the paradigm allow participants to learn?

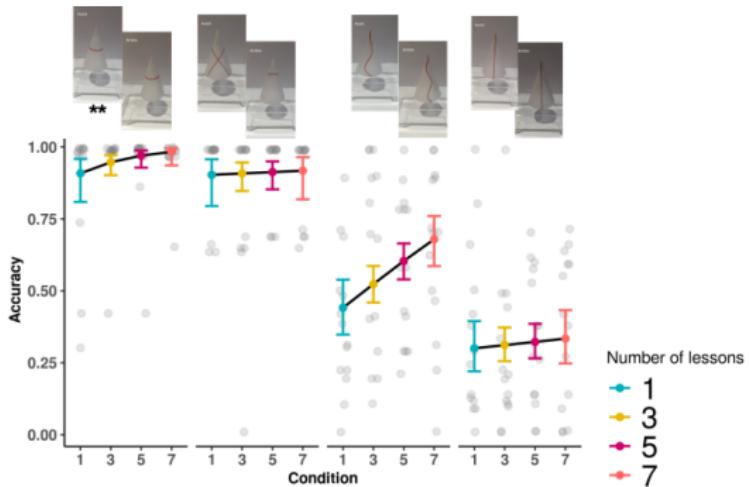
Does the paradigm allow participants to learn?

Do participants presented with more lessons perform better in the post-teaching tests related to the new concept?

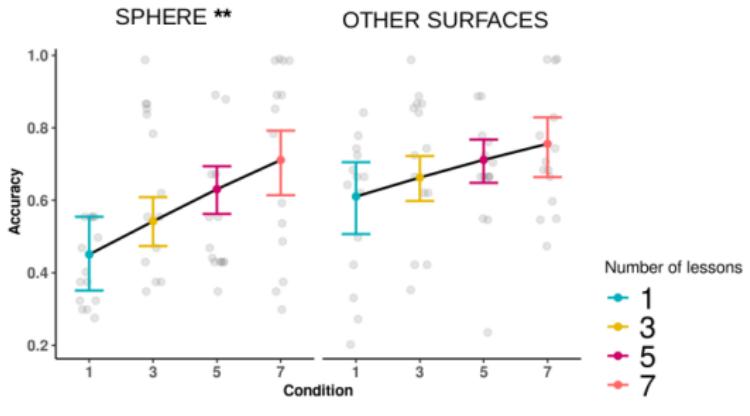


Predictions of the logistic mixed model by test condition and number of lessons, with individual participants' performance, corrected for years of mathematic education after 10th grade

N=56, 18-43 years ($M=25.5$)



N=56, 18-43 years ($M=25.5$)



N=56, 18-43 years ($M=25.5$)

Do participants report insights?

Do insights seem to be triggered by the learning process?

Do participants report insights?

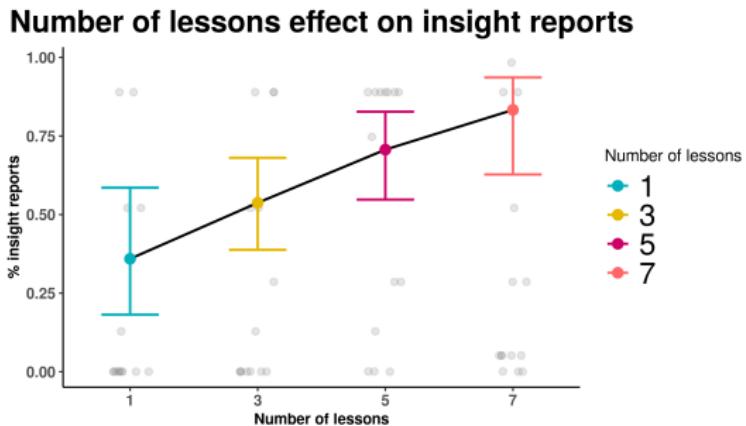
60 % of insight reports

Do insights seem to be triggered by the learning process?

Do insights seem to be triggered by the learning process?

If learning a new concept triggers insight, reports of insight experiences should be more frequent when participants receive more lessons to study

More insights reported in the conditions with more lessons (**)



Predictions of the binomial linear model by number of lessons, with individual participants' insight reports, corrected for years of mathematic education after 10th grade

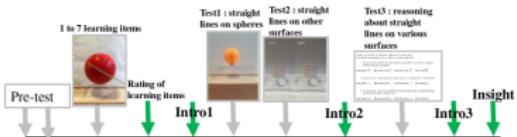
N=56, 18-43 years ($M=25.5$)

Are insights related to a good performance?

Are insights related to a good performance?

If insight experiences reflect key computations in the underlying learning processes, those participants who report insights should achieve better levels of objective understanding, even when controlling for global confidence about understanding the new concept

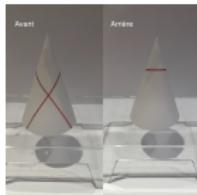
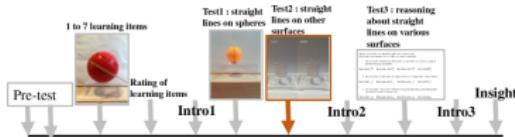
Are insights reports and confidence ratings dissociated?



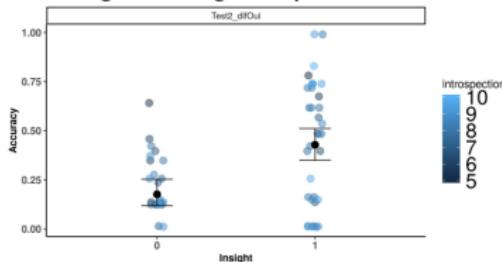
Introspection measure	CR 1	CR 2	CR 3	Insight
CR 1	X	df(54)=.6 (<.0001)	df(53)=.6 (<.0001)	df(54)=.27 (.13)
CR 2		X	df(53)=.85 (.<0001)	df(54)=.19 (.33)
CR 3	df(55)=.6 (<.0001)	df(55)=.85 (<.0001)	X	df(53)=.17 (.33)
Insight	df(56)=.15 (.81)	df(56)=.14 (.81)	df(55)=.15 (.81)	X

Spearman's rho coefficients and p values for pairwise correlation tests. Above diagonal : zero-order correlation, below : with number of lessons and education in mathematics as covariates. All p values were corrected for multiple comparisons using Holm's method

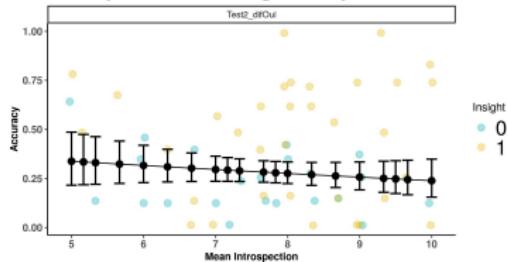
Are insight reports uniquely related to a better performance in specific aspects of learning?



Insight on Straight non planar lines **



Introspection on Straight non planar lines



Relation between performance and insight report or confidence ratings, in the condition that has no equivalent on the sphere (straight non-planar lines). Predicted performance for participants who did vs. did not report insight experiences (resp. different levels of confidence), and individual participants' performance corrected for years of mathematics education, number of lessons, and confidence level (resp. insight experiences)

Conclusion

Learning was effective and was function of the number of lessons

Learning was effective and was function of the number of lessons

Two characteristic signatures of conceptual learning :

Learning was effective and was function of the number of lessons

Two characteristic signatures of conceptual learning :

Learning was **difficult** : positive linear effects of the number of lessons on test performance

Learning was effective and was function of the number of lessons

Two characteristic signatures of conceptual learning :

Learning was **difficult** : positive linear effects of the number of lessons on test performance

Content learned was **inferentially rich** : participants were also able to draw inferences (better performance in generalization test conditions)

In the third test, teaching phase enabled participants (34%) to draw non trivial inferences : two straight lines drawn on a sphere can never be parallel, they necessarily cross

In the third test, teaching phase enabled participants (34%) to draw non trivial inferences : two straight lines drawn on a sphere can never be parallel, they necessarily cross

Counterintuitive for both Non/geometry-educated people Izard et al. 2011

In the third test, teaching phase enabled participants (34%) to draw non trivial inferences : two straight lines drawn on a sphere can never be parallel, they necessarily cross

Counterintuitive for both Non/geometry-educated people Izard et al. 2011

The experimental condition played a role in this understanding (linear trend of number of lessons for these two assertions, $\beta = 0.3$, $p = 0.001$, controlling for years of education in mathematics)

Learning a new concept gives rise to insight experiences (60%)

Learning a new concept gives rise to insight experiences (60%)

Extend the range of situations known to trigger insights Bowden 2005, Webb 2016, Danek and Wiley 2016, Laukkonen 2020, Danek and Wiley 2014, Tian et al. 2017, Canestrari et al. 2017, Laukkonen 2017

Insight experiences related to performance which was modulated across test conditions, even when controlling for the number of lessons, math education, and confidence

Insight experiences related to performance which was modulated across test conditions, even when controlling for the number of lessons, math education, and confidence

Exclude several deflationary explanations : confabulation, artificial effect of experimental condition, overconfidence

Insight experiences related to performance which was modulated across test conditions, even when controlling for the number of lessons, math education, and confidence

Exclude several deflationary explanations : confabulation, artificial effect of experimental condition, overconfidence

Reflect the functioning of learning processes as a term of a specific state, and the probability to reach this state increased when participants received more lessons to study

Insights predicted performance in one test of generalization even after factoring out variations in ratings of confidence

Insights predicted performance in one test of generalization even after factoring out variations in ratings of confidence

Learning abstract definitional properties of generalized straight lines involved processes that triggered insights (thought did not inform participants' introspective judgments of confidence)

Insights predicted performance in one test of generalization even after factoring out variations in ratings of confidence

Learning abstract definitional properties of generalized straight lines involved processes that triggered insights (thought did not inform participants' introspective judgments of confidence)

Dissociated introspection processes

Insights predicted performance in one test of generalization even after factoring out variations in ratings of confidence

Learning abstract definitional properties of generalized straight lines involved processes that triggered insights (thought did not inform participants' introspective judgments of confidence)

Dissociated introspection processes

Insights could be necessary steps in concept learning

Open questions

Are the insight experiences observed in the contexts of concept learning or problem solving qualitatively different, or do they reflect similar psychological processes?

Are the insight experiences observed in the contexts of concept learning or problem solving qualitatively different, or do they reflect similar psychological processes?

If all insight experiences turn out to indicate the termination of a search process, what is the nature of the search involved in conceptual learning?

Are the insight experiences observed in the contexts of concept learning or problem solving qualitatively different, or do they reflect similar psychological processes?

If all insight experiences turn out to indicate the termination of a search process, what is the nature of the search involved in conceptual learning?

Are insights experienced when learning other kinds of material, besides mathematical or science concepts?

Thanks for your attention!