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Convolutional Neural Networks

Summary

1. Convolution
    a. Kernel convolution
    b. Convolutional layer
2. Pooling
    a. Concept
    b. Exemple

Convolution

Kernel convolution

Intuition

Original image

0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0
0	0	1	0	0	1	0	0
0	0	1	0	0	1	0	0
0	0	0	0	0	0	0	0
0	1	0	0	0	0	1	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0

Pixel values

0	0	0
0	0	1
0	0	1

0	0	0
0	1	0
0	1	0

...

0	0	1
0	0	1
0	0	1

...

Subpart of the image multiplied by the kernel

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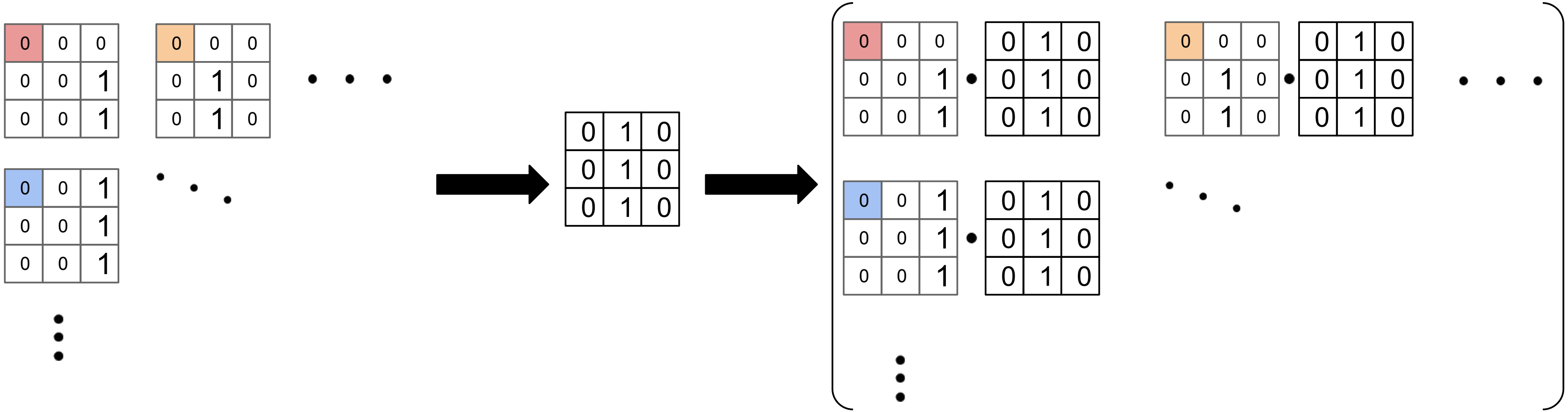
Kernel convolution

Intuition



Subpart of the image

Kernel

Dot product each subpart of the image with the kernel (also called filter).
Each product gives a scalar, all those scalars give use a new matrix which will be the input of the next layer (after applying an activation function)

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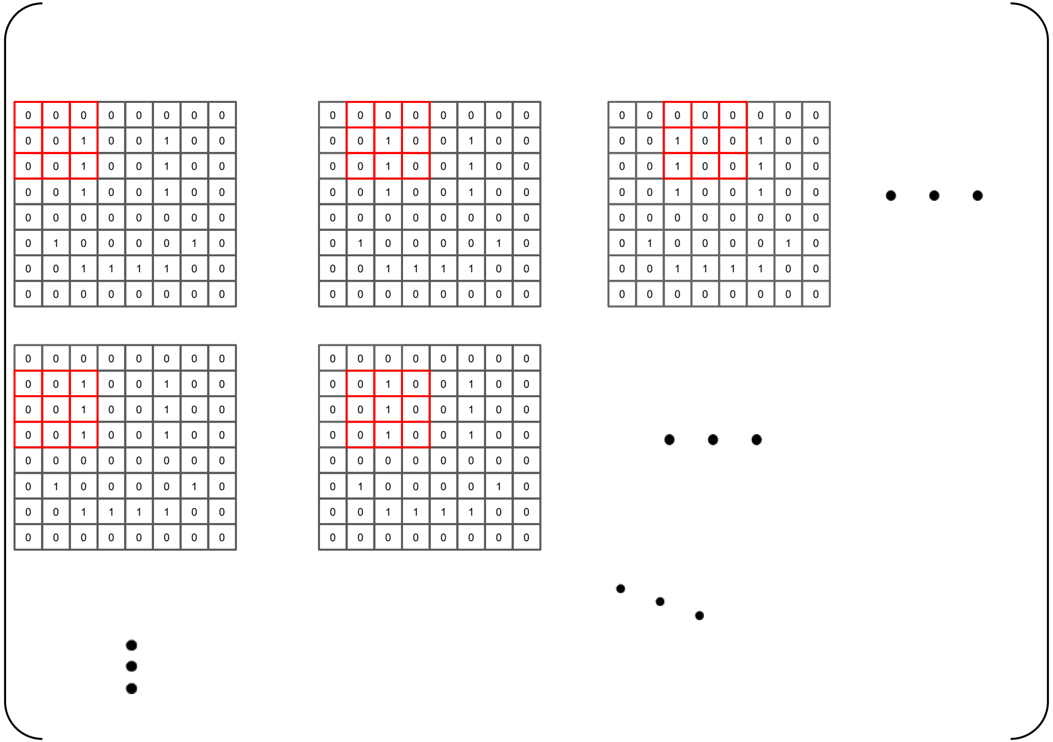
Kernel convolution

Intuition

$$\begin{bmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&1&0&0 \\ 0&0&1&0&0&1&0&0 \\ 0&0&1&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&1&0 \\ 0&0&1&1&1&1&0&0 \\ 0&0&0&0&0&0&0&0 \end{bmatrix} \bullet \begin{bmatrix} 0&1&0 \\ 0&1&0 \\ 0&1&0 \end{bmatrix} \Rightarrow$$



Each red highlighted submatrix represents the part that is being multiplied by the kernel, the result being a matrix.

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Kernel convolution

Mathematical expression

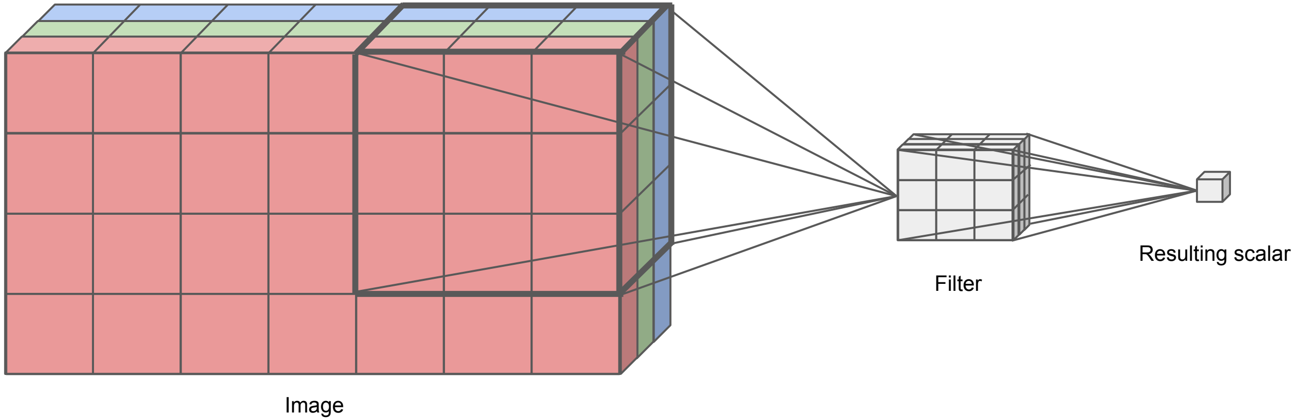
Let P be the matrix containing the image pixels and K the kernel matrix.
We can express the value (i,j) in the matrix resulting of the convolution of P by K thanks to the following formula:

$$Conv_{i,j}(P, K) = \langle P[i, i+k; j, j+k], K \rangle_F$$

Where $\langle \rangle_F$ represents the *Frobenius inner product* and P[a,b;c,d] represents the submatrix of P composed of the intersections of the rows in [a,b] and the columns in [c,d].

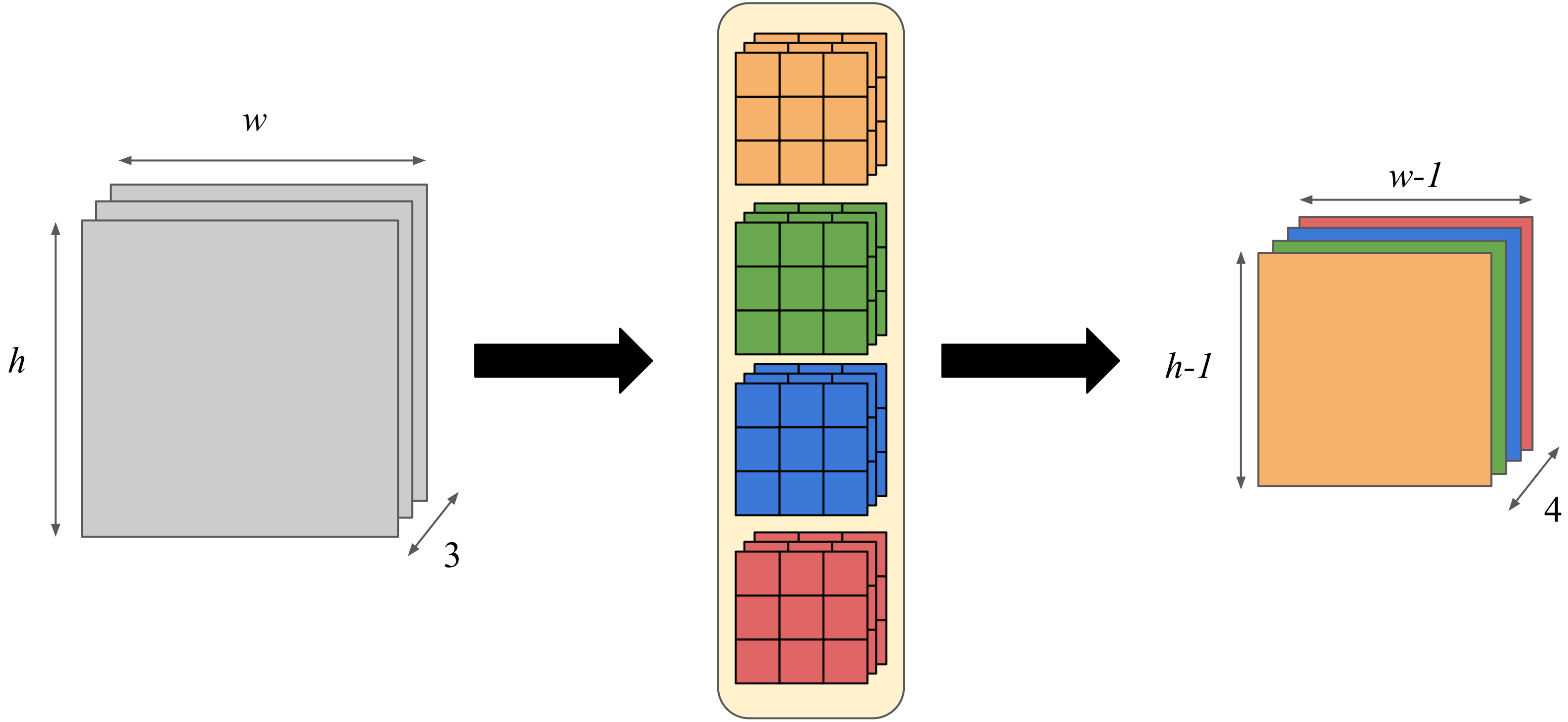
Convolutional layer

A convolutional layer is a set of kernels. Such a layer takes as input a 3-dimensional array (width, height, depth) and output another 3-dimensional array resulting of the convolution of the input with all the kernel.



Result of applying a kernel of dimension 3x3 (extended to 3x3x3 for the dot product) on a subpart of the image. This operation is done for all the image resulting in a 2-dimensional array, and for all the filters resulting in a 3-dimensional array

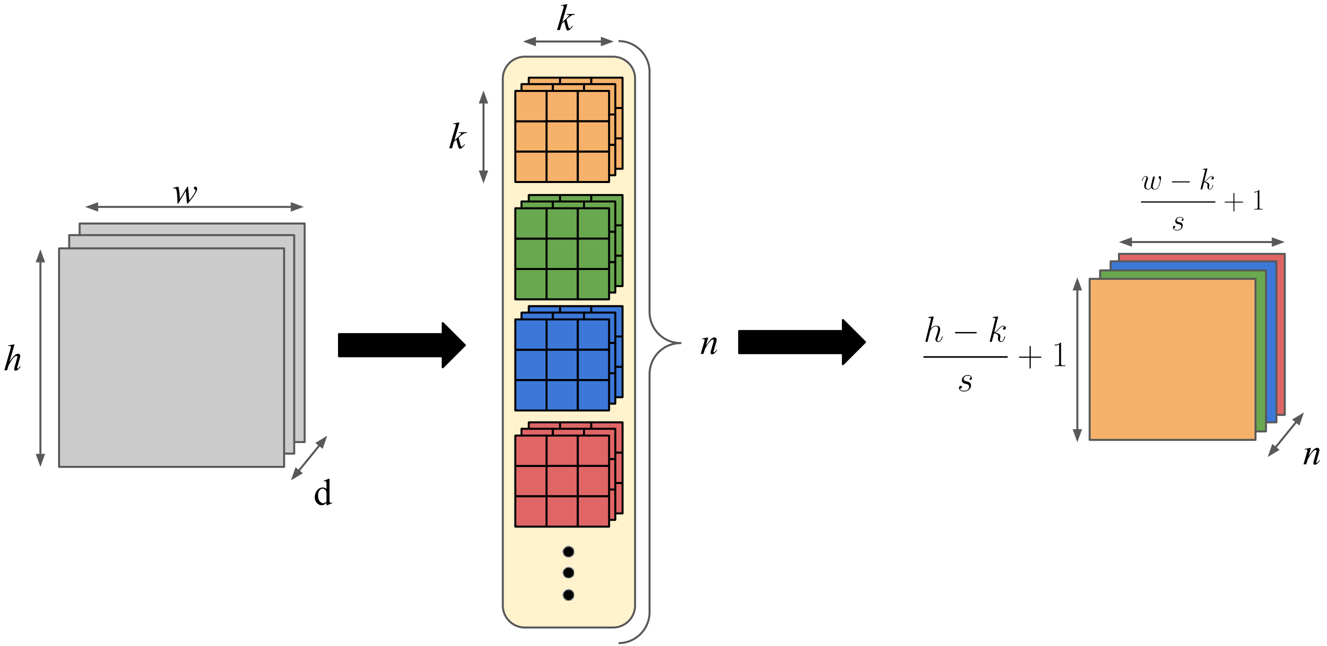
Convolutional layer



Example of convolutional layer with a stride of 1

Convolutional layer

Let P be a matrix of dimension w x h x d, C be a set of n filter each with a dimension of k x k and s be the stride of those filters.
Then the dimension of the output is $$\left(\frac{w-k}{s}+1, \frac{h-k}{s}+1, n\right)$$



Pooling

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Concepts

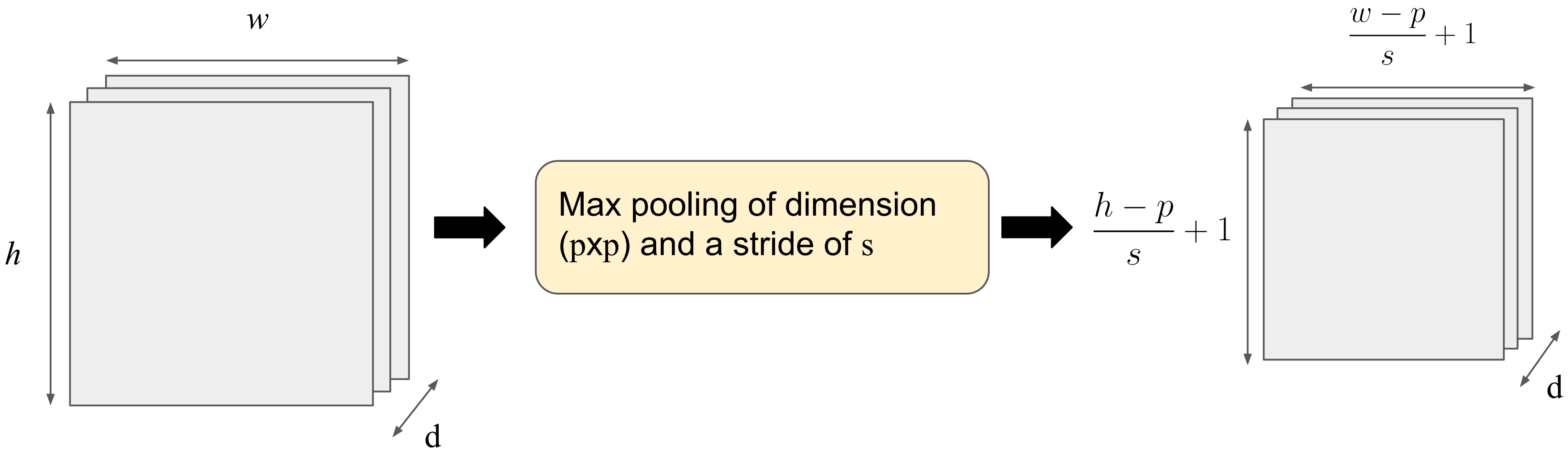
A pooling layer that extract information about a submatrix. For a exemple a max pooling layer will take the maximum value of all the designated submatrix and build a new matrix from it. This pooling is applied on each channel of the 3-dimensional array. So a pooling layer takes as input a 3-dimensional array and output another 3-dimensional with a (possible) different width and height but same depth.

The dimension of the output is: $(\frac{w-p}{s}+1, \frac{h-p}{s}+1, d)$

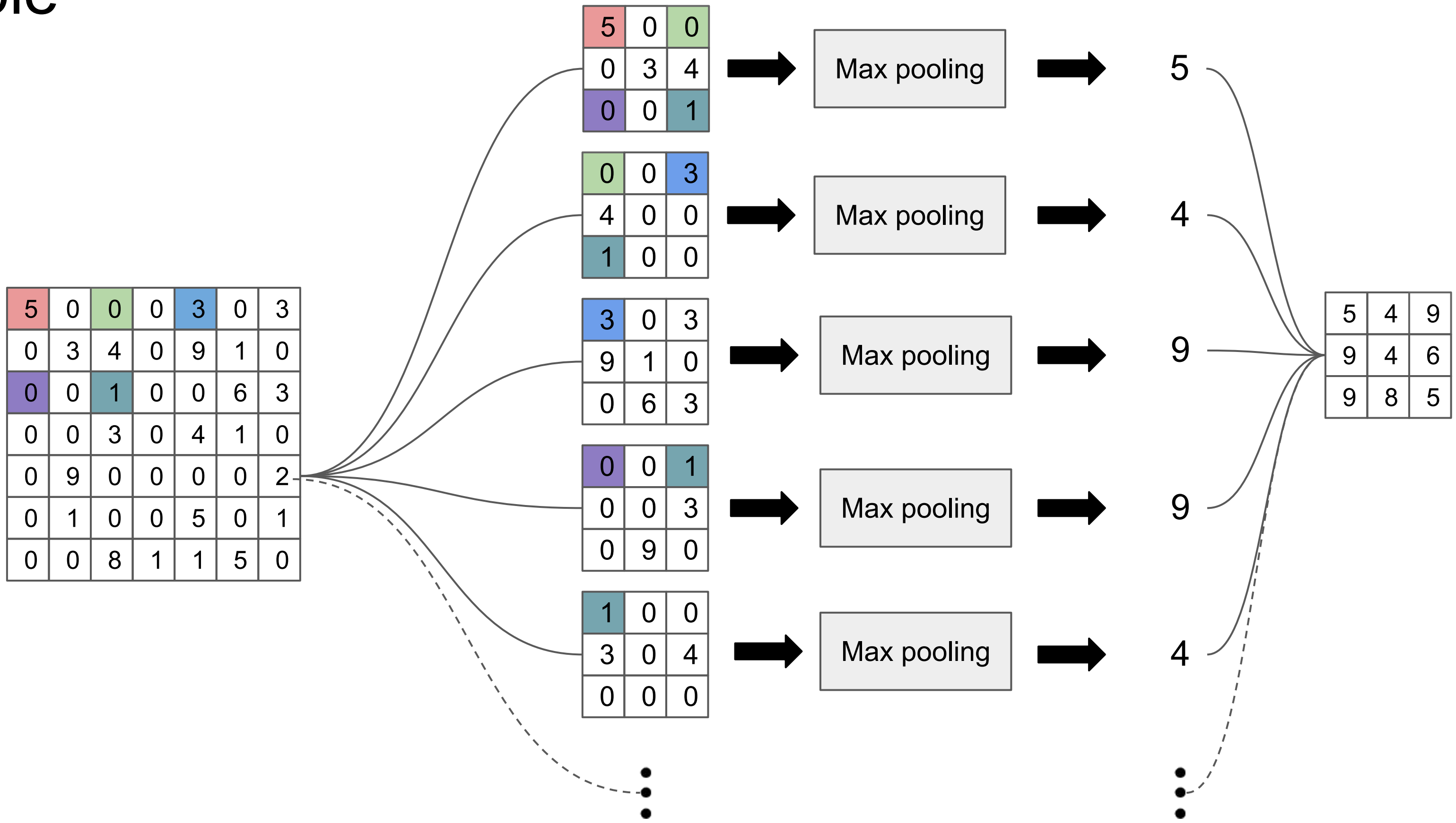
with:

$$w \rightarrow width\ of\ the\ input$$
$$h \rightarrow height\ of\ the\ input$$
$$(p \times p) \rightarrow dimensions\ of\ the\ pooling$$
$$s \rightarrow stride\ of\ the\ pooling$$

Concepts


w
h
d
Max pooling of dimension (pxp) and a stride of s
$\frac{h-p}{s}+1$
$\frac{w-p}{s}+1$
d

Example


5 0 0 0 3 0 3
0 3 4 0 9 1 0
0 0 1 0 0 6 3
0 0 3 0 4 1 0
0 9 0 0 0 0 2
0 1 0 0 5 0 1
0 0 8 1 1 5 0
5 0 0
0 3 4
0 0 1
0 0 3
4 0 0
1 0 0
3 0 3
9 1 0
0 6 3
0 0 1
0 0 3
0 9 0
1 0 0
3 0 4
0 0 0
Max pooling
Max pooling
Max pooling
Max pooling
Max pooling
5
4
9
9
4
5 4 9
9 4 6
9 8 5