

## Proofs exercises

### Exercise 4.1

Given  $p$  and  $q$  and  $p \wedge q \Rightarrow r$ , use the Fitch system to prove  $r$ .

1	$p$	Premise
2	$q$	Presmise
3	$p \wedge q \Rightarrow r$	Presmise
4	$p \wedge q$	And Introduction (1,2)
5	$r$	Implication Elimination (3,4)

### Exercise 4.2

Given  $p \wedge q$ , use the Fitch system to prove  $q \vee r$

1	$p \wedge q$	Premise
2	$q$	And Elimination (1)
3	$q \vee r$	Or Introduction (2)

### Exercise 4.3

Given  $p \Rightarrow q$  and  $q \Leftrightarrow r$ , use the Fitch system to prove  $p \Rightarrow r$ .

1	$p \Rightarrow q$	Premise
2	$q \Leftrightarrow r$	Premise
3	$p$	Assumption
4	$q$	Implication Elimination (1,3)
5	$r$	Implication Elimination (5,4)
6	$p \Rightarrow r$	Implication Introduction (3,6)

### Exercise 4.4

Given  $p \Rightarrow q$  and  $m \Rightarrow p \wedge q$ , use the Fitch System to prove  $m \Rightarrow q$ .

1	$p \Rightarrow q$	Premise
2	$m \Rightarrow p \vee q$	Premise
3	$q$	Assumption
4	$q \Rightarrow q$	Implication Introduction (3,3)
5	$m$	Assumption
6	$p \vee q$	Implication Elimination (2,5)
7	$q$	Or Elimination (6,1,4)
8	$m \Rightarrow q$	Implication Introduction (5,7)

### Exercise 4.5

Given  $p \Rightarrow (q \Rightarrow r)$ , use the Fitch System to prove  $(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$

1	$p \Rightarrow (q \Rightarrow r)$	Premise
2	$p \Rightarrow q$	Assumption
3	$p$	Assumption
4	$q \Rightarrow r$	Implication Elimination (1,3)
5	$q$	Implication Elimination (2,3)
6	$r$	Implication Elimination (4,5)
7	$p \Rightarrow r$	Implication Introduction (3,6)
8	$(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$	Implication Introduction (2,7)

### Exercise 4.6

Use the Fitch System to prove  $p \Rightarrow (q \Rightarrow p)$

1	$p$	Assumption
2	$q$	Assumption
3	$p$	Reiteration (1)
4	$q \Rightarrow p$	Implication Introduction (2,3)
5	$p \Rightarrow (q \Rightarrow p)$	Implication Introduction (1,4)

### Exercise 4.7

Use the Fitch System to prove  $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$

1		$p \Rightarrow (q \Rightarrow r)$	Assumption
2		$p \Rightarrow q$	Assumption
3		$p$	Assumption
4		$q$	Implication Elimination (2,3)
5		$q \Rightarrow r$	Implication Elimination (1,3)
6		$r$	Implication Elimination (5,4)
7		$p \Rightarrow r$	Implication Introduction (3,6)
8		$(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$	Implication Introduction (2,7)
9		$(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$	Implication Introduction (1,8)

### Exercise 4.8

Use the Fitch System to prove  $(\neg p \Rightarrow q) \Rightarrow ((\neg p \Rightarrow \neg q) \Rightarrow p)$

1		$\neg p \Rightarrow q$	Assumption
2		$\neg p \Rightarrow \neg q$	Assumption
3		$\neg \neg p$	Negation Introduction (1,2)
4		$p$	Negation Elimination (3)
5		$(\neg p \Rightarrow \neg q) \Rightarrow p$	Implication Introduction (2,4)
6		$(\neg p \Rightarrow q) \Rightarrow ((\neg p \Rightarrow \neg q) \Rightarrow p)$	Implication Introduction (1,5)

### Exercise 4.9

Given  $p$ , use the Fitch System to prove  $\neg \neg p$

1		$p$	Premise
2		$\neg p$	Assumption
3		$p$	Reiteration (1)
4		$\neg p \Rightarrow p$	Implication Introduction (2,3)
5		$\neg p$	Assumption
6		$\neg p \Rightarrow \neg p$	Implication Introduction (5,5)
7		$\neg \neg p$	Negation Introduction (4,6)

### Exercise 4.10

Given  $p \Rightarrow q$ , use the Fitch System to prove  $\neg q \Rightarrow \neg p$ .

1	$p \Rightarrow q$	Premise
2	$\neg q$	Assumption
3	$p$	Assumption
4	$q$	Implication Elimination (1,3)
5	$p \Rightarrow q$	Implication Introduction (3,4)
6	$p$	Assumption
7	$\neg q$	Reiteration (2)
8	$p \Rightarrow \neg q$	Implication Introduction (6,7)
9	$\neg p$	Negation Introduction (5,8)
10	$\neg q \Rightarrow \neg p$	Implication Introduction (2,9)

### Exercise 4.11

Given  $p \Rightarrow q$ , use the Fitch System to prove  $\neg p \vee q$

1	$p \Rightarrow q$	Premise
2	$\neg(\neg p \vee q)$	Assumption
3	$\neg p$	Assumption
4	$\neg p \vee q$	Or Introduction (3)
5	$\neg p \Rightarrow \neg p \vee q$	Implication Introduction (3,4)
6	$\neg p$	Assumption
7	$\neg(\neg p \vee q)$	Reiteration (2)
8	$\neg p \Rightarrow \neg(\neg p \vee q)$	Implication Introduction (6,7)
9	$\neg\neg p$	Negation Introduction (5,8)
10	$p$	Negation Elimination (9)
11	$\neg(\neg p \vee q) \Rightarrow p$	Implication Introduction (2,10)
12	$\neg(\neg p \vee q)$	Assumption
13	$\neg(\neg p \vee q) \Rightarrow \neg(\neg p \vee q)$	Implication Introduction (12,12)
14	$\neg(\neg p \vee q)$	Assumption
15	$p$	Implication Elimination (11,14)
16	$q$	Implication Elimination (1,15)
17	$\neg p \vee q$	Or Introduction (16)
18	$\neg(\neg p \vee q) \Rightarrow \neg p \vee q$	Implication Introduction (14,17)
19	$\neg\neg(\neg p \vee q)$	Negation Introduction (18,13)
20	$\neg p \vee q$	Negation Elimination (19)

## Exercise 4.12

Use the Fitch System to prove  $((p \Rightarrow q) \Rightarrow p) \Rightarrow p$ .

1		$(p \Rightarrow q) \Rightarrow p$	Assumption
2		$\neg p$	Assumption
3		$p$	Assumption
4		$\neg q$	Assumption
5		$p$	Reiteration (3)
6		$\neg q \Rightarrow p$	Implication Introduction (4,5)
7		$\neg q$	Assumption
8		$\neg p$	Reiteration (2)
9		$\neg q \Rightarrow \neg p$	Implication Introduction (7,8)
10		$\neg \neg q$	Negation Introduction (6,9)
11		$q$	Negation Elimination (10)
12		$p \Rightarrow q$	Implication Introduction (3,11)
13		$p$	Implication Elimination (1,12)
14		$\neg p \Rightarrow p$	Implication Introduction (2,13)
15		$\neg p$	Assumption
16		$\neg p \Rightarrow \neg p$	Implication Introduction (15,15)
17		$\neg \neg p$	Negation Introduction (14,16)
18		$p$	Negation Elimination (17)
19		$((p \Rightarrow q) \Rightarrow p) \Rightarrow p$	Implication Introduction (1,18)

## Exercise 4.13

Given  $\neg(p \vee q)$ , use the Fitch system to prove  $(\neg p \wedge \neg q)$ .

1	$\neg(p \vee q)$	Premise
2	$p$	Assumption
3	$p \vee q$	Or Introduction (2)
4	$p \Rightarrow p \vee q$	Implication Introduction (2,3)
5	$p$	Assumption
6	$\neg(p \vee q)$	Reiteration (1)
7	$p \Rightarrow \neg(p \vee q)$	Implication Introduction (5,6)
8	$\neg p$	Negation Introduction (4,7)
9	$q$	Assumption
10	$p \vee q$	Or Introduction (9)
11	$q \Rightarrow p \vee q$	Implication Introduction (9,10)
12	$q$	Assumption
13	$\neg(p \vee q)$	Reiteration (1)
14	$q \Rightarrow \neg(p \vee q)$	Implication Introduction (12,13)
15	$\neg q$	Negation Introduction (11,14)
16	$\neg p \wedge \neg q$	And Introduction (8,15)

## Exercise 4.14

Use the Fitch system to prove the tautology  $(p \vee \neg p)$

1	$\neg(p \vee \neg p)$	Assumption
2	$p$	Assumption
3	$p \vee \neg p$	Or Introduction (2)
4	$p \Rightarrow p \vee \neg p$	Implication Introduction (2,3)
5	$p$	Assumption
6	$\neg(p \vee \neg p)$	Reiteration (1)
7	$p \Rightarrow \neg(p \vee \neg p)$	Implication Introduction (5,6)
8	$\neg p$	Negation Introduction (4,7)
9	$\neg(p \vee \neg p) \Rightarrow \neg p$	Implication Introduction (1,8)
10	$\neg(p \vee \neg p)$	Assumption
11	$\neg p$	Assumption
12	$p \vee \neg p$	Or Introduction (11)
13	$\neg p \Rightarrow p \vee \neg p$	Implication Introduction (11,12)
14	$\neg p$	Assumption
15	$\neg(p \vee \neg p)$	Reiteration (10)
16	$\neg p \Rightarrow \neg(p \vee \neg p)$	Implication Introduction (14,15)
17	$\neg\neg p$	Negation Introduction (13,16)
18	$p$	Negation Elimination (17)
19	$\neg(p \vee \neg p) \Rightarrow p$	Implication Introduction (10,18)
20	$\neg\neg(p \vee \neg p)$	Negation Introduction (19,9)
21	$p \vee \neg p$	Negation Elimination (20)