Propositional Proof

Rule of inferences

"A rule of inference is a pattern of reasoning consisting of some schemas, called premises, and one or more additional schemas, called conclusions."

Fitch rules of inferences

And Introduction (AI)

$$\frac{\begin{array}{l}\varphi_1\\ \cdots\\ \varphi_n\end{array}}{\varphi_1 \land \cdots \land \varphi_2}$$

And Elimination (AE)

$$\frac{\varphi_1 \land \cdots \land \varphi_i \land \cdots \land \varphi_n}{\varphi_i}$$

Or Introduction (OI)

$$\frac{\varphi_i}{\varphi_1 \lor \cdots \lor \varphi_i \lor \cdots \lor \varphi_n}$$

Or Elimination (OE)

$$\frac{\begin{array}{l}\varphi \Rightarrow \chi\\ \psi \Rightarrow \chi\\ \varphi \lor \psi\end{array}}{\chi}$$

Negation Introduction (NI)

$$\frac{\begin{array}{l}\varphi \Rightarrow \psi\\ \varphi \Rightarrow \neg\psi\end{array}}{\neg\varphi}$$

Negation Elimination (NE)

$$\frac{\neg\neg\varphi}{\varphi}$$

Implication Introduction (II)

$$\frac{\varphi \vdash \psi}{\varphi \Rightarrow \psi}$$

Implication Elimination (IE)

$$\frac{\begin{array}{l}\varphi \Rightarrow \psi \\ \varphi\end{array}}{\psi}$$

Biconditional Introduction

$$\frac{\begin{array}{l}\varphi \Rightarrow \psi \\ \psi \Rightarrow \varphi\end{array}}{\varphi \Leftrightarrow \psi}$$

Biconditional Elimination

$$\frac{\varphi \Leftrightarrow \psi}{\begin{array}{l}\varphi \Rightarrow \psi \\ \psi \Rightarrow \varphi\end{array}}$$

Proof tips

$\varphi \Rightarrow \psi$

1. Assume φ and prove ψ
2. Use Implication Introduction to derive $\varphi \Rightarrow \psi$

$\varphi \wedge \psi$

1. Prove φ
2. Prove ψ
3. Use And Introduction to derive $\varphi \wedge \psi$

$\varphi \vee \psi$

1. Prove either φ or ψ
2. Use Or Introduction to derive $\varphi \vee \psi$

$\neg\varphi$

1. Assume φ and prove a contradiction ($\varphi \Rightarrow \psi$ and $\varphi \Rightarrow \neg\psi$)
2. Use Negation Introduction to derive $\neg\varphi$

φ

1. Assume $\neg\varphi$ and prove a contradiction ($\neg\varphi \Rightarrow \psi$ and $\neg\varphi \Rightarrow \neg\psi$)
2. Use Negation Introduction to derive $\neg\neg\varphi$
3. Use Negation Elimination to derive φ

ψ when we have $\varphi \Rightarrow \psi$ as a premise

1. Prove φ
2. Use Implication Elimination to derive ψ

$\varkappa$ when we have $\varphi \vee \psi$ as a premise

1. Prove $\varphi \Rightarrow \varkappa$
2. Prove $\psi \Rightarrow \varkappa$
3. Use Or Elimination to derive $\varkappa$