

Clausal Form Exercises

Exercise 5.1

For each of the following sentences, say which set of clauses is the correct clausal form.

1. $p \wedge q \Rightarrow r \vee s$. $\{p, q, r, s\}$
 - . $\{\neg p, \neg q, r, s\}$ ✓
 - . $\{\neg p, \neg q, r\}, \{\neg p, \neg q, s\}$
 - . $\{\neg p, r, s\}, \{\neg q, r, s\}$
 - . $\{\neg p\}, \{\neg q\}, \{r\}, \{s\}$
2. $p \vee q \Rightarrow r \vee s$. $\{p, q, r, s\}$
 - . $\{\neg p, \neg q, r, s\}$
 - . $\{\neg p, \neg q, r\}, \{\neg p, \neg q, s\}$
 - . $\{\neg p, r, s\}, \{\neg q, r, s\}$ ✓
 - . $\{\neg p\}, \{\neg q\}, \{r\}, \{s\}$
3. $\neg(p \vee q \vee r)$
 - . $\{\neg p, \neg q, \neg r\}$
 - . $\{\neg p\}, \{\neg q\}, \{\neg r\}$ ✓
4. $\neg(p \wedge q \wedge r)$
 - . $\{\neg p, \neg q, \neg r\}$ ✓
 - . $\{\neg p\}, \{\neg q\}, \{\neg r\}$
5. $p \wedge q \Leftrightarrow r$
 - . $\{p, q\}, \{r\}$
 - . $\{\neg p, \neg q\}, \{r\}$
 - . $\{\neg p, \neg q, r\}, \{\neg r, p\}, \{\neg r, q\}$ ✓
 - . $\{\neg p, r\}, \{\neg q, r\}, \{p, q, \neg r\}$
 - . $\{p\}, \{q\}, \{r\}$

Exercise 5.2

In each of the following questions, say which of the answers best characterizes the result of applying resolution to the clauses shown.

1. $\{p, q, \neg r\}$ and $\{r, s\}$
 - . $\{p, q, s\}$ ✓
 - . $\{p, q, r, s\}$
 - . $\{p, q, \neg r, s\}$
 - . There are no resolvents.
2. $\{p, q, r\}$ and $\{r, \neg s, \neg t\}$
 - . $\{p, q, r, \neg s, \neg t\}$
 - . $\{p, q, \neg s, \neg t\}$
 - . There are no resolvents. ✓
3. $\{q, \neg q\}$ and $\{q, \neg q\}$
 - . $\{q, \neg q\}$ ✓

- . $\{q\}$
 - . $\{\neg q\}$
 - . $\{\}$
 - . There are no resolvents.
4. $\{\neg p, q, r\}$ and $\{p, \neg q, \neg r\}$
- . $\{\neg p, q, r, p, \neg q, \neg r\}$
 - . $\{q, r, \neg q, \neg r\}, \{\neg p, r, p, \neg r\}, \{\neg p, q, p, \neg q\}$ ✓
 - . $\{p, \neg p\}, \{q, \neg q\}, \{r, \neg r\}$
 - . $\{\}$
 - . There are no resolvents.

Exercise 5.3

Use Propositional Resolution to show that the clauses $\{p, q\}, \{\neg p, r\}, \{\neg p, \neg r\}, \{p, \neg q\}$ are not simultaneously satisfiable.

1	$\{p, q\}$	Premise
2	$\{\neg p, r\}$	Premise
3	$\{\neg p, \neg r\}$	Premise
4	$\{p, \neg q\}$	Premise
5	$\{p\}$	Resolution (1,4)
6	$\{\neg p\}$	Resolution (2,3)
7	$\{\}$	Resolution (5,6)

Exercise 5.4

Given the premises $(p \Rightarrow q)$ and $(r \Rightarrow s)$, use Propositional Resolution to prove the conclusion $(p \vee r \Rightarrow q \vee s)$.

Note: I already converted the premise to clausal form, I'll add the clausal form process later.

1	$\{\neg p, q\}$	Premise
2	$\{\neg r, s\}$	Premise
3	$\{p, r\}$	Premise
4	$\{\neg q\}$	Premise
5	$\{\neg s\}$	Premise
6	$\{\neg r\}$	Resolution (2,5)
7	$\{\neg p\}$	Resolution (1,4)
8	$\{p\}$	Resolution (3,6)
9	$\{\}$	Resolution (8,7)