

Exercise 2 - BRDF Sampling

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QUESTION 2.2.3

Answer. Let X be the random variable accounting for the number of loop required to generate a sample. Then

$$\mathbb{E}[X] = \frac{12}{\pi}$$

Proof. At each loop, $\mathbb{P}(w_i \in \text{hemisphere}) = \text{hemisphere volume} / \text{cube volume} = \frac{1}{2} \frac{4\pi}{3} (\frac{1}{2})^3 = \frac{\pi}{12}$.

Hence for $n \geq 1$, $\mathbb{P}(X = n) = \frac{\pi}{12} (1 - \frac{\pi}{12})^{n-1}$.

X has a geometric distribution of parameter $p = \frac{\pi}{12}$.

Thus $\mathbb{E}[X] = p^{-1} = \frac{12}{\pi} \simeq 4$. □

QUESTION 2.3.2A

TODO

QUESTION 2.3.2B

TODO