# Computing the efficient frontier for the 0/1 biobjective uncapacitated facility location problem

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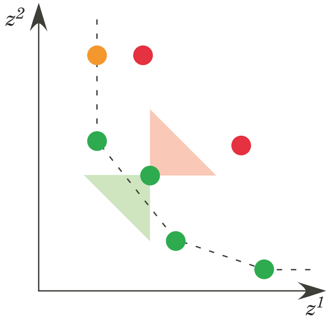
- Multi-Objective Combinatorial Optimization (MOCO) problems

$$\min \{ Cx : Ax \geqq b, x \in \mathbb{Z}^n \}$$

- Objectives optimized simultaneously
- Efficient solutions $X_E$, Non dominated points $Y_N$ (efficient frontier)
- Preferences of the decision-maker introduced a posteriori



# 1. Context

## 0/1 biobjective uncapacitated facility location problem (bi-0/1UFLP)

$$\left[\begin{array}{lllr} v\text{–}\min & \left\{ z^k = \sum_{i\in I}\sum_{j\in J} c_{ij}^k x_{ij} + \sum_{j\in J} r_j^k s_j \quad k=1,2 \right\} & & (0) \\ s/c & \sum_{j\in J} x_{ij} = 1 & \forall i \in I & (1) \\ & x_{ij} \le s_j & \forall i\in I,\ \forall j \in J & (2) \\ & x_{ij},\ s_j \in \{0,1\} & \forall i\in I,\ \forall j \in J & (3) \end{array}\right]$$

- single objective version
  - NP-hard [Krarup and Pruzan, 1983]
  - practical situations (example in telecom [Gourdin and al. 2002])
- multi-objective version
  - exact and approximated algorithms for small size bi (multi) objective instances [Fernandez and Puerto 2003]
  - data and numerical instances available [Harris and al. 2009; 2011]

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An **exact algorithm** in two steps

- **paving** $Y_N$ :
  - Set of boxes giving a rigorous description of areas of $Y$ which include potentially non-dominated points.
  - A box is defined by **an unique** combination of opened facilities
  - Initial paving obtained with **a branch and bound** algorithm
  - Decomposition of the paving by dichotomy and filtering on Pareto dominance relation
  - Recomposition of the paving for the sets issued from the same origin
- **generating** $Y_N$ :
  - Compute the efficient solutions $X_E$ for each box
  - Non-dominated points are computed with **a label setting algorithm** on boxes.
  - Return a complete set of efficient solutions.

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# 2. Algorithm

## Definitions and notations

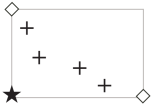
Let $J_1 \subseteq J$, set of indexes for facilities opened

**Paving** : set of boxes characterizing rigorously the existence of potential efficient solutions

**Box** : Sub-set of dimension 2 in $Y$ bounded by (at most) two feasible solutions corresponding to lexicographic optimal solutions on both objectives for $J_1$

Remarkable points related to a box $\mathcal{B}$ :

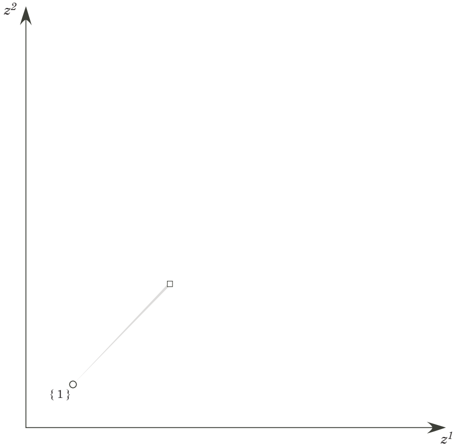
$\diamond$ : $z_{lex^1}(\mathcal{B}), z_{lex^2}(\mathcal{B})$, lex-optimal points
$\star$ : $z_I(\mathcal{B})$, ideal point
$+$ : $Y_{ND}(\mathcal{B})$, non dominated points



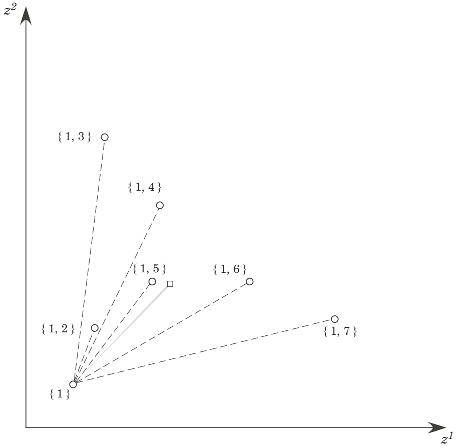
when $\#J_1 = 1 \Rightarrow$ remarkable points correspond to a same single point

Example with 7 potential facilities 1...7 .

- ○ : point corresponding to the opening of one facility $\{1\}$
- □ : point corresponding to the assignment of all customers to the opened facility. It corresponds to the performances of a feasible solution.

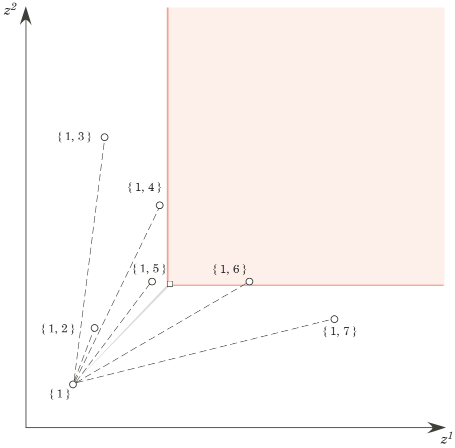


Others points (symbol $\circ$) corresponding to the opening of two facilities $\{1, 2\}$ à $\{1, 7\}$.

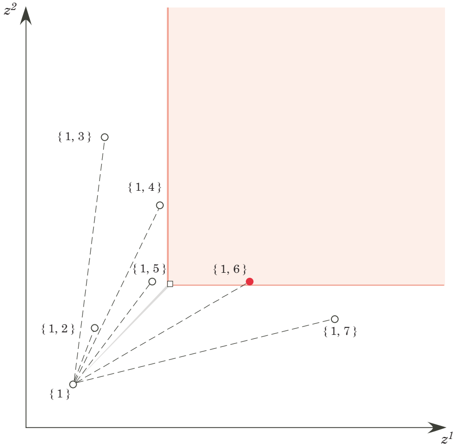


Examination of the dominance cone with origin □

Test: ∃? an origin point dominated by the performance of at least one feasible solution

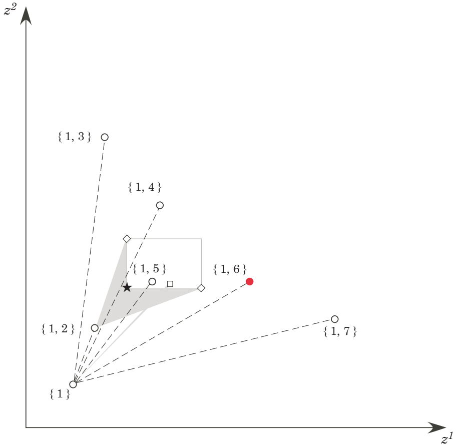


The point corresponding to opened facities $\{1, 6\}$ is dominated. It is pruned (domination by origin).



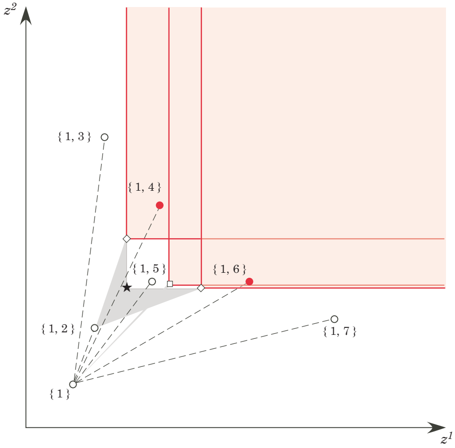
Expansion of domain for the opened facilities $\{1, 2\}$ giving a box.

The two feasible solutions ($\diamond$) for the box are the two lexicographic optimal solutions.



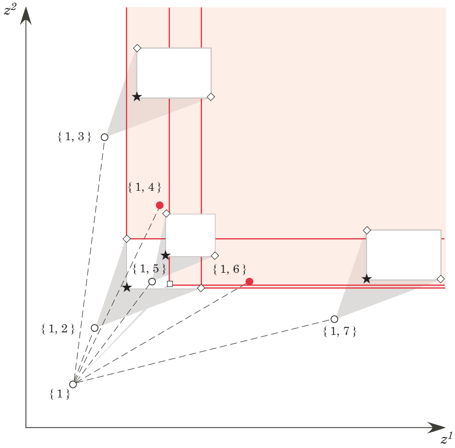
Examination of the dominance cone with origin $\diamond$

The point corresponding to opened facities $\{1, 4\}$ is dominated. It is pruned (domination by origin).



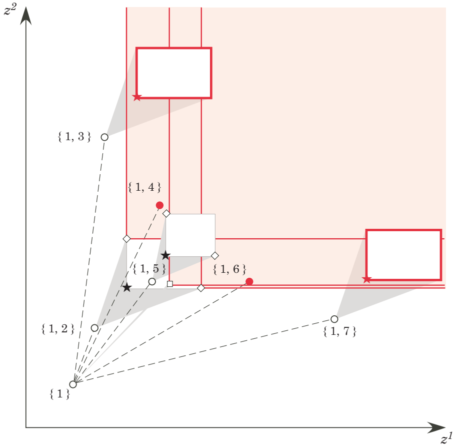
Expansion of domain for the remaining facilities giving a set of boxes.

Test: ∃? an ideal point of a box dominated by the performance of at least one feasible solution

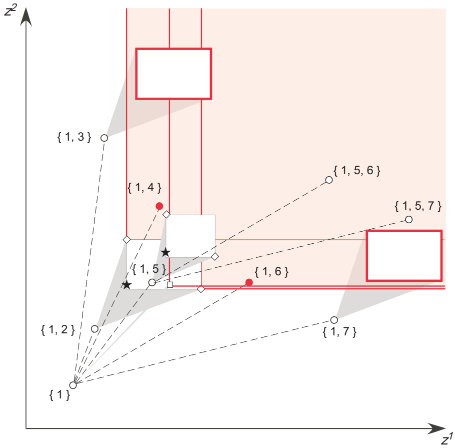


Examination of the dominance cone with origin $\diamond$ and $\square$

Expansions with origin $\{1,3\}$ and $\{1,7\}$ are dominated. They are pruned (domination by expansion).

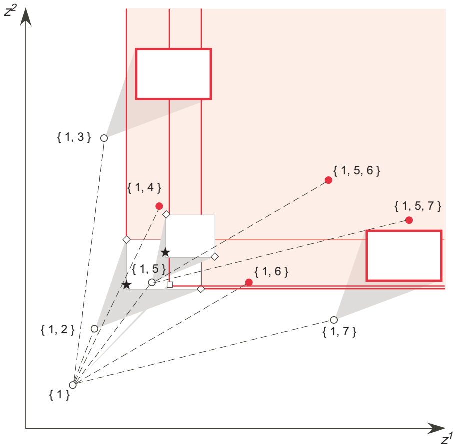


Others points (symbol $\circ$) corresponding to the opening of three facilities $\{1, 5, 6\}$ et $\{1, 5, 7\}$.



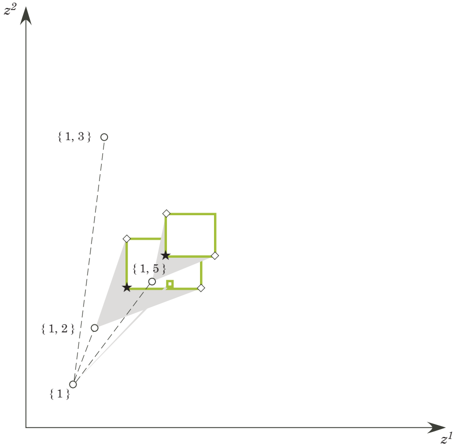
Examination of the dominance cone with origin $\diamond$

The points corresponding to opened facilities $\{1, 5, 6\}$ et $\{1, 5, 7\}$ are dominated. They are pruned (domination by origin).



Paving partial resulting.

$\{1, 2\}$ and $\{1, 3\}$ are pending.



- **Search strategy:** breadth first search
- **Branching rule:** variables $s_j$
- **Pruning strategy:** 3 dominance tests
- **Output :** non dominated expansed nodes

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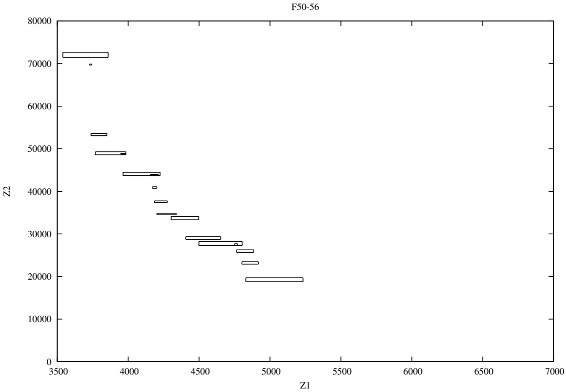
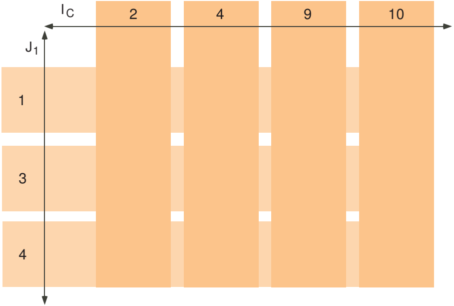


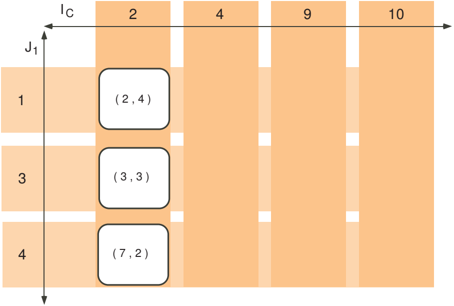

Instance with 30 facilities and 90 customers

- A box is defined by the opened facilities: e.g. $J_1 = \{1, 3, 4\}$
- Set of indexes for the non trivial assignments: e.g. $I_c = \{2, 4, 9, 10\}$



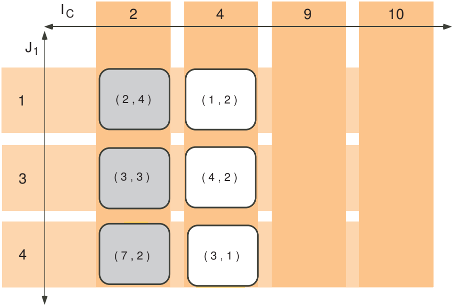
Customer 2 has three potential assignments:

- [2, 4]
- [3, 3]
- [7, 2]



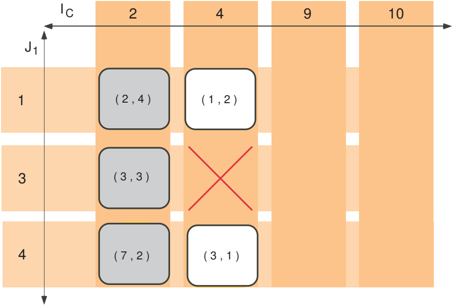
Assignment of customer 4 to facility 3 is dominated. $\Longrightarrow$ pruned

- [2, 4]
- [3, 3]
- [7, 2]



For each solution already computed, the possible assignments for customer 4 are added.

- [2, 4]
- [3, 3]
- [7, 2]

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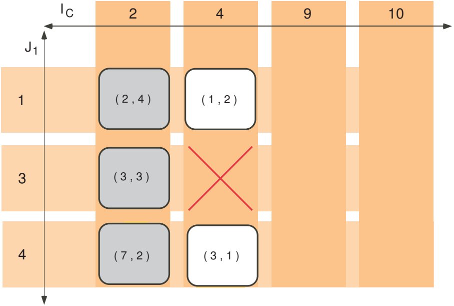
For each solution already computed, the possible assignments for customer 4 are added.

- [3, 6]
- [4, 5]
- [8, 4]
- [5, 5]
- [7, 4]
- [10, 3]



For each solution already computed, the possible assignments for customer 4 are added.

- [3, 6]
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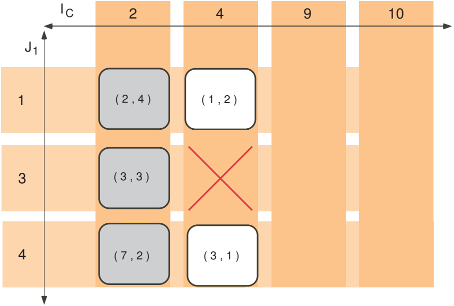
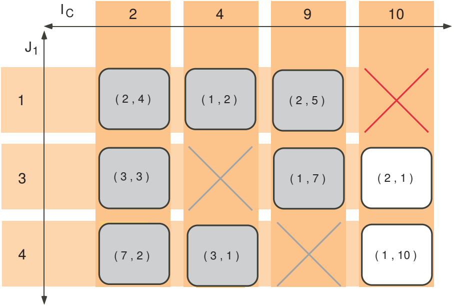
- ~~[8, 4]~~
- ~~[5, 5]~~
- [7, 4]
- [10, 3]



In fine: 6 labels (solutions) not dominated.

- [7, 12]
- [8, 11]
- [11, 10]
- [14, 9]
- [6, 14]
- [5, 23]

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- dataset F [Fernandez and Puerto 2003]
  - 28 instances
  - source: collection obtained in pasting two mono-obj UFLP
  - size : n=30; m=90
  - correlation of objectives : $\approx 0.0$
  - ranges : $c_{ij}^k \in [0, 100]$; $r_j^k \in [200, 28000]$
  - http://www-eio.upc.es/%7Eelena/sscplp/
- dataset H [Harris et al 2011]
  - 5 instances
  - source : green logistic (obj1 : cost; obj2 : $CO_2$)
  - size : n=10; m=2000...10000
  - correlation of objectives : $\approx 0.99$
  - ranges : $c_{ij}^k \in [0, 43000]$; $r_j^k$ unique $\forall j \in J$ per instance
  - http://users.cs.cf.ac.uk/C.L.Mumford/Research%20Topics/FLP/papers/data/
- Soon available on the MCDMLib, section MOCOlib :
  - http:// http://mcdmsociety.org/MCDMlib.html

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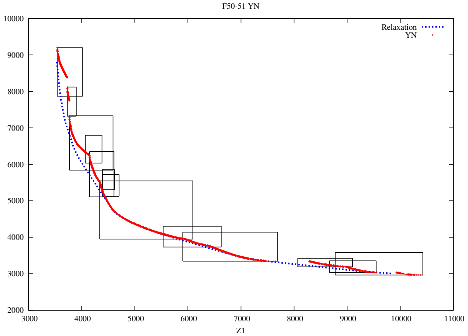
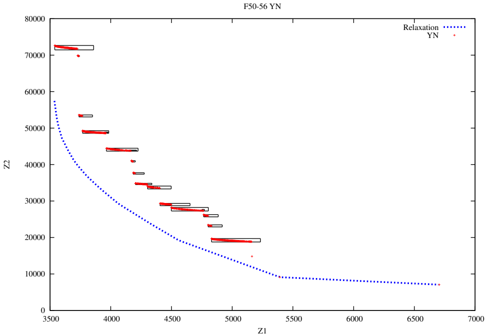



Instance F50-51




Instance F50-56

| | Paving | | | Generation | |
|---|---|---|---|---|---|
| H | (1) | (2) | CPUt ms | (3) | CPUt ms |
| min | 1 023 | 11 | 411 | 13 412 | 104 |
| avg | 1 023 | 12 | 1 200 | 295 868 | 35 142 |
| max | 1 023 | 13 | 1 994 | 731 385 | 122 691 |
| F | | | | | |
| min | 280 | 3 | 1 | 3 | $<1$ |
| avg | 8 885 | 18 | 53 | 421 | 6 |
| max | 48 741 | 152 | 399 | 1 229 | 29 |

(1) : expansed boxes
(2) : non-dominated expansed boxes
(3) : non-dominated points

| H | (1) | (2) ms | (3) ms |
|---|---|---|---|
| min | 15h39min | - | 515 |
| avg | N.A. | - | 36 342 |
| max | N.A. | - | 124 285 |
| tot | N.A. | - | 181 713 |
| F | ms | ms | ms |
| min | 1 000 | 59 000* | 1 |
| avg | 94 428 | 97 200* | 59 |
| max | 300 000 | 162 000* | 428 |
| tot | 2 644 000 | - | 1 669 |

(1) : $\epsilon$-constraint with Cplex
(2) : algorithm of Fernandez and Puerto [2003]
(3) : algorithm proposed (paving + generation)

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# 4. Summary

## Conclusion and future investigations

- introduced the principle of rigorous paving by boxes in MOCO
- paving is useful for a decision-maker aiming to navigate on $Y_N$
- proposed algorithm is efficient for the benchmarks known
- observed some $Y_N$ with previously unpublished characteristics
- ...

- reinforce the algorithm which is somewhere brute-force
- identify the computational limits of the proposed algorithm
- quid of this algorithm for situations with more than 2 objectives
- version of this algorithm for situations with capacities
- ...

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