Ric Ergenbright/CORBIS

CHAPTER 26

Statistical Process Control

Organizations are (or ought to be) concerned about the quality of the products and services they offer. A key to maintaining and improving quality is systematic use of *data* in place of intuition or anecdotes. In the words of Stan Sigman, former CEO of Cingular Wireless, "What gets measured gets managed."[1]

Because using data is a key to improving quality, statistical methods have much to contribute. Simple tools are often the most effective. A scatterplot and perhaps a regression line can show how the time to answer telephone calls to a corporate call center influences the percent of callers who hang up before their calls are answered. The design of a new product as simple as a multivitamin tablet may involve interviewing samples of consumers to learn what vitamins and minerals they want included and using randomized comparative experiments in designing the manufacturing process. An experiment might discover, for example, what combination of moisture level in the raw vitamin powder and pressure in the tablet-forming press produces the right tablet hardness.

Quality is a vague idea. You may feel that a restaurant serving filet mignon is a higher-quality establishment than a fast-food outlet that serves hamburgers. For statistical purposes we need a narrower concept: *consistently meeting standards appropriate for a specific product or service*. By this definition of quality, the expensive restaurant may serve low-quality filet mignon while the fast-food outlet serves

IN THIS CHAPTER WE COVER...

high-quality hamburgers. The hamburgers are freshly grilled, are served at the right temperature, and are the same every time you visit. Statistically minded management can assess quality by sampling hamburgers and measuring the time from order to being served, the temperature of the burgers, and their tenderness.

This chapter focuses on just one aspect of statistics for improving quality: *statistical process control*. The techniques are simple and are based on sampling distributions (Chapter 11), but the underlying ideas are important and a bit subtle.

Processes

In thinking about statistical inference, we distinguish between the *sample* data we have in hand and the wider *population* that the data represent. We hope to use the sample to draw conclusions about the population. In thinking about quality improvement, it is often more natural to speak of *processes* rather than populations. This is because work is organized in processes. Some examples are

- processing an application for admission to a university and deciding whether or not to admit the student;
- reviewing an employee's expense report for a business trip and issuing a reimbursement check;
- hot forging to shape a billet of titanium into a blank that, after machining, will become part of a medical implant for hip, knee, or shoulder replacement.

Each of these processes is made up of several successive operations that eventually produce the output—an admission decision, reimbursement check, or metal component.

PROCESS

A **process** is a chain of activities that turns inputs into outputs.

We can accommodate processes in our sample-versus-population framework: think of the population as containing all the outputs that would be produced by the process if it ran forever in its present state. The outputs produced today or this week are a sample from this population. Because the population doesn't actually exist now, it is simpler to speak of a process and of recent output as a sample from the process in its present state.

Describing processes

flowchart
cause-and-effect diagram

The first step in improving a process is to understand it. Process understanding is often presented graphically using two simple tools: flowcharts and cause-and-effect diagrams. A **flowchart** is a picture of the stages of a process. A **cause-and-effect diagram** organizes the logical relationships between the inputs

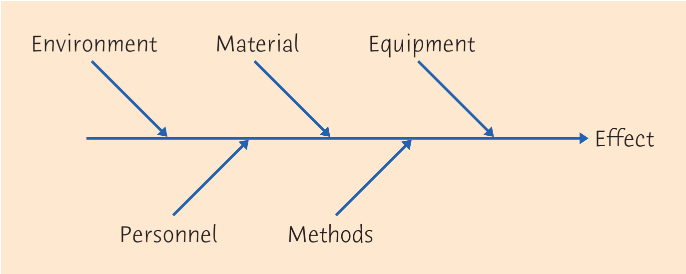

FIGURE 26.1
An outline for a cause-and-effect diagram. To complete the diagram, group causes under these main headings in the form of branches.

and stages of a process and an output. Sometimes the output is successful completion of the process task; sometimes it is a quality problem that we hope to solve. A good starting outline for a cause-and-effect diagram appears in Figure 26.1. The main branches organize the causes and serve as a skeleton for detailed entries. You can see why these are sometimes called "fishbone diagrams." An example will illustrate the use of these graphs.[2]

EXAMPLE 26.1 Hot forging

Hot forging involves heating metal to a plastic state and then shaping it by applying thousands of pounds of pressure to force the metal into a die (a kind of mold). Figure 26.2 is a flowchart of a typical hot-forging process.[3]

A process improvement team, after making and discussing this flowchart, came to several conclusions:

- Inspecting the billets of metal received from the supplier adds no value. We should insist that the supplier be responsible for the quality of the material. The supplier should put in place good statistical process control. We can then eliminate the inspection step.
- Can we buy the metal billets already cut to rough length and ground smooth by the supplier, thus eliminating the cost of preparing the raw material ourselves?
- Heating the metal billet and forging (pressing the hot metal into the die) are the heart of the process. We should concentrate our attention here.

The team then prepared a cause-and-effect diagram (Figure 26.3) for the heating and forging part of the process. The team members shared their specialist knowledge of the causes in their areas, resulting in a more complete picture than any one person could produce. Figure 26.3 (page 26-5) is a simplified version of the actual diagram. We have given some added detail for the "Hammer stroke" branch under "Equipment" to illustrate the next level of branches. Even this requires some knowledge of hot forging to understand. Based on detailed discussion of the diagram, the team decided what variables to measure and at what stages of the process to measure them. Producing well-chosen data is the key to improving the process. ■

We will apply statistical methods to a series of measurements made on a process. Deciding what specific variables to measure is an important step in quality

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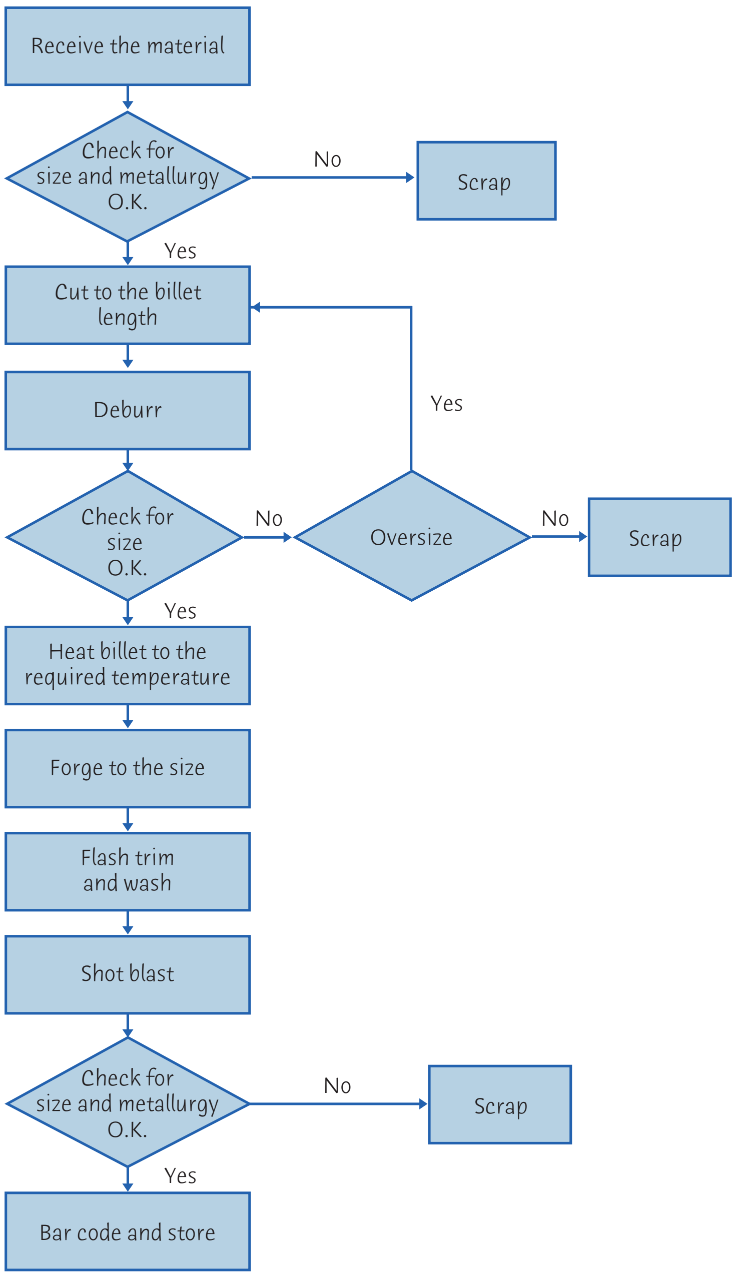


FIGURE 26.2
Flowchart of the hot-forging process in Example 26.1. Use this as a model for flowcharts: decision points appear as diamonds, and other steps in the process appear as rectangles. Arrows represent flow from step to step.

improvement. Often we use a "performance measure" that describes an output of a process. A company's financial office might record the percent of errors that outside auditors find in expense account reports or the number of data entry errors per week. The personnel department may measure the time to process employee insurance claims or the percent of job offers that are accepted. In the case of complex processes, it is wise to measure key steps within the process rather than just final outputs. The process team in Example 26.1 might recommend that the temperature of the die and of the billet be measured just before forging.

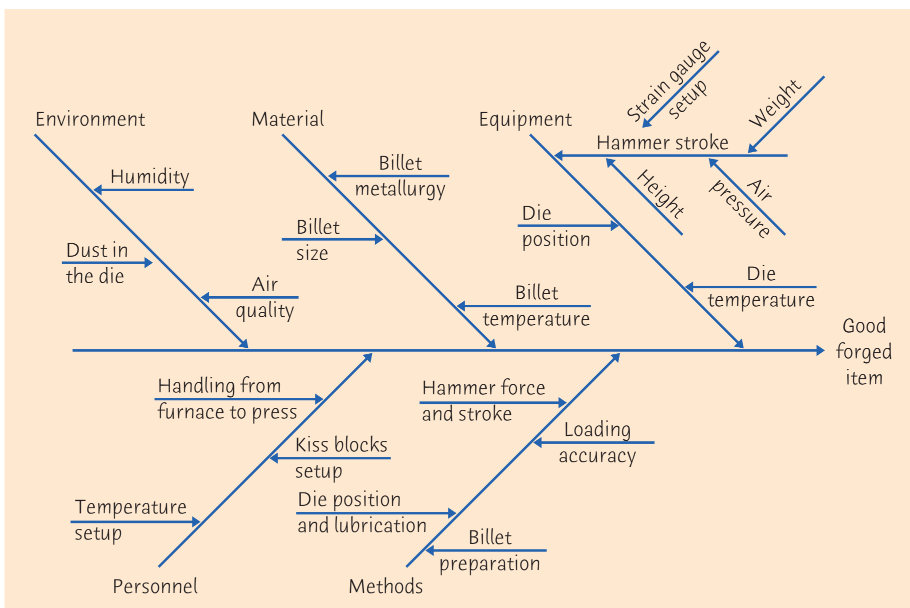


FIGURE 26.3
Simplified cause-and-effect diagram of the hot-forging process in Example 26.1. Good cause-and-effect diagrams require detailed knowledge of the specific process.

APPLY YOUR KNOWLEDGE

26.1 Describe a process. Choose a process that you know well. If you lack experience with actual business or manufacturing processes, choose a personal process such as cooking scrambled eggs or balancing your checkbook. Make a flowchart of the process. Make a cause-and-effect diagram that presents the factors that lead to successful completion of the process.

Simon Watson/Ford Pix/Getty Images

26.2 Describe a process. Each weekday morning, you must get to work or to your first class on time. Make a flowchart of your daily process for doing this, starting when you wake. Be sure to include the time at which you plan to start each step.

26.3 Process measurement. Based on your description of the process in Exercise 26.1, suggest specific variables that you might measure in order to

(a) assess the overall quality of the process.

(b) gather information on a key step within the process.

Pareto charts

26.4 Pareto charts. Pareto charts are bar graphs with the bars ordered by height. They are often used to isolate the "vital few" categories on which we should focus our attention. Here is an example. A large medical center, financially pressed by restrictions on reimbursement by insurers and the government, looked at losses broken down by diagnosis. Government standards place cases into Diagnostic Related Groups (DRGs). For example, major joint replacements (mostly hip and knee) are DRG 209.[4] Here is what the hospital found:

DRG	Percent of losses
104	5.2
107	10.1
109	7.7
116	13.7
148	6.8
209	15.2
403	5.6
430	6.8
462	9.4

What percent of total losses do these 9 DRGs account for? Make a Pareto chart of losses by DRG. Which DRGs should the hospital study first when attempting to reduce its losses?

26.5 **Pareto charts.** Continue the study of the process of getting to work or class on time from Exercise 26.2. If you kept good records, you could make a Pareto chart of the reasons (special causes) for late arrivals at work or class. Make a Pareto chart that you think roughly describes your own reasons for lateness. That is, list the reasons from your experience and chart your estimates of the percent of late arrivals each reason explains.

The idea of statistical process control

The goal of statistical process control is to make a process stable over time and then keep it stable unless planned changes are made. You might want, for example, to keep your weight constant over time. A manufacturer of machine parts wants the critical dimensions to be the same for all parts. "Constant over time" and "the same for all" are not realistic requirements. They ignore the fact that *all processes have variation*. Your weight fluctuates from day to day; the critical dimension of a machined part varies a bit from item to item; the time to process a college admission application is not the same for all applications. Variation occurs in even the most precisely made product due to small changes in the raw material, the adjustment of the machine, the behavior of the operator, and even the temperature in the plant. Because variation is always present, we can't expect to hold a variable exactly constant over time. The statistical description of stability over time requires that the *pattern of variation* remain stable, not that there be no variation in the variable measured.

STATISTICAL CONTROL

A variable that continues to be described by the same distribution when observed over time is said to be in statistical control, or simply **in control.**

Control charts are statistical tools that monitor a process and alert us when the process has been disturbed so that it is now **out of control.** This is a signal to find and correct the cause of the disturbance.

In the language of statistical quality control, a process that is in control has only **common cause** variation. Common cause variation is the inherent variability of the system, due to many small causes that are always present. When the normal functioning of the process is disturbed by some unpredictable event, **special cause** variation is added to the common cause variation. We hope to be able to discover what lies behind special cause variation and eliminate that cause to restore the stable functioning of the process.

common cause

special cause

EXAMPLE 26.2 Common cause, special cause

Imagine yourself doing the same task repeatedly, say folding an advertising flyer, stuffing it into an envelope, and sealing the envelope. The time to complete the task will vary a bit, and it is hard to point to any one reason for the variation. Your completion time shows only common cause variation.

Now the telephone rings. You answer, and though you continue folding and stuffing while talking, your completion time rises beyond the level expected from common causes alone. Answering the telephone adds special cause variation to the common cause variation that is always present. The process has been disturbed and is no longer in its normal and stable state.

If you are paying temporary employees to fold and stuff advertising flyers, you avoid this special cause by not having telephones present and by asking the employees to turn off their cell phones while they are working. ■

Control charts work by distinguishing the always-present common cause variation in a process from the additional variation that suggests that the process has been disturbed by a special cause. A control chart sounds an alarm when it sees too much variation. The most common application of control charts is to monitor the performance of industrial and business processes. The same methods, however, can be used to check the stability of quantities as varied as the ratings of a television show, the level of ozone in the atmosphere, and the gas mileage of your car. Control charts combine graphical and numerical descriptions of data with use of sampling distributions.

APPLY YOUR KNOWLEDGE

26.6 Special causes. Jeannine participates in bicycle road races. She regularly rides 25 kilometers over the same course in training. Her time varies a bit from day to day but is generally stable. Give several examples of special causes that might raise Jeannine's time on a particular day.

26.7 Common causes, special causes. In Exercise 26.1, you described a process that you know well. What are some sources of common cause variation in this process? What are some special causes that might at times drive the process out of control?

26.8 Common causes, special causes. Each weekday morning, you must get to work or to your first class on time. The time at which you reach work or class varies from day to day, and your planning must allow for this variation. List several common causes of variation in your arrival time. Then list several special causes that might result in unusual variation leading to either early or (more likely) late arrival.

$\bar{x}$ charts for process monitoring

When you first apply control charts to a process, the process may not be in control. Even if it is in control, you don't yet understand its behavior. You will have to collect data from the process, establish control by uncovering and removing special causes, and then set up control charts to maintain control. We call this the **chart setup** stage. Later, when the process has been operating in control for some time, you understand its usual behavior and have a long run of data from the process. You keep control charts to monitor the process because a special cause could erupt at any time. We will call this **process monitoring.**[5]

chart setup

process monitoring

Although in practice chart setup precedes process monitoring, the big ideas of control charts are more easily understood in the process-monitoring setting. We will start there, then discuss the more complex chart setup setting.

Choose a quantitative variable x that is an important measure of quality. The variable might be the diameter of a part, the number of envelopes stuffed in an hour, or the time to respond to a customer call. Here are the conditions for process monitoring.

PROCESS-MONITORING CONDITIONS

Measure a quantitative variable x that has a **Normal distribution.** The process has been operating in control for a long period, so that we know the **process mean** μ and the **process standard deviation** σ that describe the distribution of x as long as the process remains in control.

In practice, we must of course estimate the process mean and standard deviation from past data on the process. Under the process-monitoring conditions, we have very many observations and the process has remained in control. The law of large numbers tells us that estimates from past data will be very close to the truth about the process. That is, at the process-monitoring stage we can act as if we know the true values of μ and σ. *Note carefully that μ and σ describe the center and spread of the variable x only as long as the process remains in control.* A special cause may at any time disturb the process and change the mean, the standard deviation, or both.

CAUTION

To make control charts, begin by taking small samples from the process at regular intervals. For example, we might measure 4 or 5 consecutive parts or time the responses to 4 or 5 consecutive customer calls. There is an important idea here: *the observations in a sample are so close together that we can assume that the process is stable during this short period of time.* Variation within the same sample gives us a benchmark for the common cause variation in the process. *The process standard deviation σ refers to the standard deviation within the time period spanned by one sample.* If the process remains in control, the same σ describes the standard deviation of observations across any time period. Control charts help us decide whether this is the case.

$\bar{x}$ chart

We start with the $\bar{x}$ **chart** based on plotting the means of the successive samples. Here is the outline:

1. **Take samples** of size n from the process at regular intervals. **Plot the means** $\bar{x}$ of these samples against the order in which the samples were taken.
2. We know that the sampling distribution of $\bar{x}$ under the process-monitoring conditions is Normal with mean μ and standard deviation $\sigma/\sqrt{n}$ (see text page 300). Draw a solid **center line** on the chart at height μ.
3. The 99.7 part of the 68–95–99.7 rule for Normal distributions (text page 74) says that, as long as the process remains in control, 99.7% of the values of $\bar{x}$ will fall between $\mu - 3\sigma/\sqrt{n}$ and $\mu + 3\sigma/\sqrt{n}$. Draw dashed **control limits** on the chart at these heights. The control limits mark off the range of variation in sample means that we expect to see when the process remains in control.

center line

control limits

If the process remains in control and the process mean and standard deviation do not change, we will rarely observe an $\bar{x}$ outside the control limits. Such an $\bar{x}$ is therefore a signal that the process has been disturbed.

EXAMPLE 26.3 Manufacturing computer monitors

A manufacturer of computer monitors must control the tension on the mesh of fine vertical wires that lies behind the surface of the viewing screen. Too much tension will tear the mesh, and too little will allow wrinkles. Tension is measured by an electrical device with output readings in millivolts (mV). The manufacturing process has been stable with mean tension $\mu = 275$ mV and process standard deviation $\sigma = 43$ mV.

The mean 275 mV and the common cause variation measured by the standard deviation 43 mV describe the stable state of the process. If these values are not satisfactory—for example, if there is too much variation among the monitors—the manufacturer must make some fundamental change in the process. This might involve buying new equipment or changing the alloy used in the wires of the mesh. In fact, the common cause variation in mesh tension does not affect the performance of the monitors. We want to watch the process and maintain its current condition.

The operator measures the tension on a sample of 4 monitors each hour. Table 26.1 gives the last 20 samples. The table also gives the mean $\bar{x}$ and the standard deviation s for each sample. The operator did not have to calculate these—modern measuring equipment often comes equipped with software that automatically records $\bar{x}$ and s and even produces control charts. ■

Figure 26.4 is an $\bar{x}$ control chart for the 20 mesh tension samples in Table 26.1. We have plotted each sample mean from the table against its sample number. For example, the mean of the first sample is 253.4 mV, and this is the value plotted for Sample 1. The center line is at $\mu = 275$ mV. The upper and lower control limits are

$$\mu + 3\frac{\sigma}{\sqrt{n}} = 275 + 3\frac{43}{\sqrt{4}} = 275 + 64.5 = 339.5 \text{ mV} \qquad \text{(UCL)}$$

$$\mu - 3\frac{\sigma}{\sqrt{n}} = 275 - 3\frac{43}{\sqrt{4}} = 275 - 64.5 = 210.5 \text{ mV} \qquad \text{(LCL)}$$

TABLE 26.1 Twenty control chart samples of mesh tension (in millivolts)

SAMPLE	TENSION MEASUREMENTS				SAMPLE MEAN	STANDARD DEVIATION
1	234.5	272.3	234.5	272.3	253.4	21.8
2	311.1	305.8	238.5	286.2	285.4	33.0
3	247.1	205.3	252.6	316.1	255.3	45.7
4	215.4	296.8	274.2	256.8	260.8	34.4
5	327.9	247.2	283.3	232.6	272.7	42.5
6	304.3	236.3	201.8	238.5	245.2	42.8
7	268.9	276.2	275.6	240.2	265.2	17.0
8	282.1	247.7	259.8	272.8	265.6	15.0
9	260.8	259.9	247.9	345.3	278.5	44.9
10	329.3	231.8	307.2	273.4	285.4	42.5
11	266.4	249.7	231.5	265.2	253.2	16.3
12	168.8	330.9	333.6	318.3	287.9	79.7
13	349.9	334.2	292.3	301.5	319.5	27.1
14	235.2	283.1	245.9	263.1	256.8	21.0
15	257.3	218.4	296.2	275.2	261.8	33.0
16	235.1	252.7	300.6	297.6	271.5	32.7
17	286.3	293.8	236.2	275.3	272.9	25.6
18	328.1	272.6	329.7	260.1	297.6	36.5
19	316.4	287.4	373.0	286.0	315.7	40.7
20	296.8	350.5	280.6	259.8	296.9	38.8

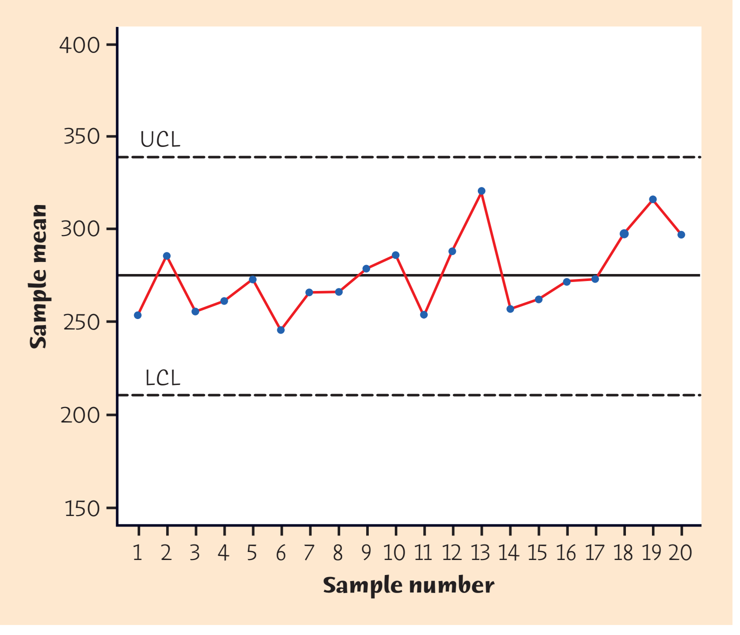


FIGURE 26.4
$\bar{x}$ chart for the mesh tension data of Table 26.1. No points lie outside the control limits.

As is common, we have labeled the control limits UCL for upper control limit and LCL for lower control limit.

EXAMPLE 26.4 Interpreting $\bar{x}$ charts

Figure 26.4 is a typical $\bar{x}$ chart for a process in control. The means of the 20 samples do vary, but all lie within the range of variation marked out by the control limits. We are seeing the common cause variation of a stable process.

Figures 26.5 and 26.6 illustrate two ways in which the process can go out of control. In Figure 26.5, the process was disturbed by a special cause sometime between Sample 12 and Sample 13. As a result, the mean tension for Sample 13 falls above the upper control limit. It is common practice to mark all out-of-control points with an "x" to call attention to them. A search for the cause begins as soon as we see a point out of control. Investigation finds that the mounting of the tension-measuring device has slipped, resulting in readings that are too high. When the problem is corrected, Samples 14 to 20 are again in control.

Figure 26.6 shows the effect of a steady upward drift in the process center, starting at Sample 11. You see that some time elapses before the $\bar{x}$ for Sample 18 is out of control. Process drift results from gradual changes such as the wearing of a cutting tool or overheating. The one-point-out signal works better for detecting sudden large disturbances than for detecting slow drifts in a process. ■

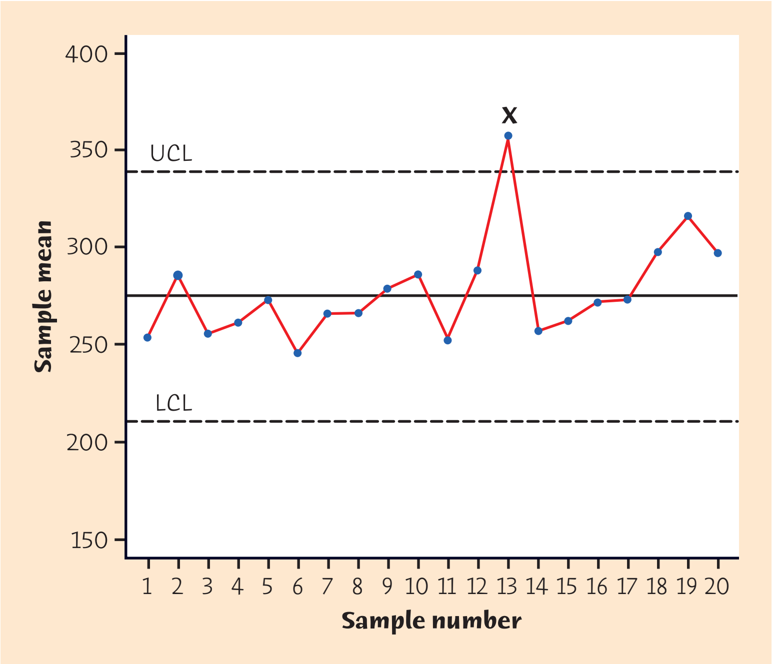
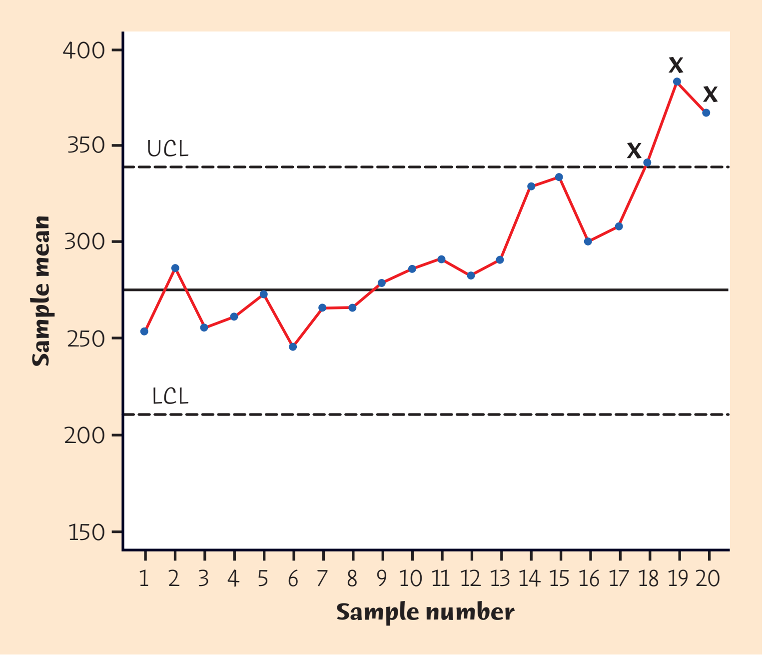


FIGURE 26.5

This $\bar{x}$ chart is identical to that in Figure 26.4 except that a special cause has driven $\bar{x}$ for Sample 13 above the upper control limit. The out-of-control point is marked with an x.

FIGURE 26.6
The first 10 points on this $\bar{x}$ chart are as in Figure 26.4. The process mean drifts upward after Sample 10, and the sample means $\bar{x}$ reflect this drift. The points for Samples 18, 19, and 20 are out of control.



APPLY YOUR KNOWLEDGE

26.9 Auto thermostats. A maker of auto air conditioners checks a sample of 4 thermostatic controls from each hour's production. The thermostats are set at 75°F and then placed in a chamber where the temperature is raised gradually. The temperature at which the thermostat turns on the air conditioner is recorded. The process mean should be $\mu = 75$°F. Past experience indicates that the response temperature of properly adjusted thermostats varies with $\sigma = 0.5$°F. The mean response temperature $\bar{x}$ for each hour's sample is plotted on an $\bar{x}$ control chart. Calculate the center line and control limits for this chart.

26.10 Tablet hardness. A pharmaceutical manufacturer forms tablets by compressing a granular material that contains the active ingredient and various fillers. The hardness of a sample from each lot of tablets is measured in order to control the compression process. The process has been operating in control with mean at the target value $\mu = 11.5$ kilograms (kg) and estimated standard deviation $\sigma = 0.2$ kg. Table 26.2 gives three sets of data, each representing $\bar{x}$ for 20 successive samples of $n = 4$ tablets. One set remains in control at the target value. In a second set, the process mean μ shifts suddenly to a new value. In a third, the process mean drifts gradually.

(a) What are the center line and control limits for an $\bar{x}$ chart for this process?

(b) Draw a separate $\bar{x}$ chart for each of the three data sets. Mark any points that are beyond the control limits.

(c) Based on your work in (b) and the appearance of the control charts, which set of data comes from a process that is in control? In which case

TABLE 26.2 Three sets of $\bar{x}$'s from 20 samples of size 4

SAMPLE	DATA SET A	DATA SET B	DATA SET C
1	11.602	11.627	11.495
2	11.547	11.613	11.475
3	11.312	11.493	11.465
4	11.449	11.602	11.497
5	11.401	11.360	11.573
6	11.608	11.374	11.563
7	11.471	11.592	11.321
8	11.453	11.458	11.533
9	11.446	11.552	11.486
10	11.522	11.463	11.502
11	11.664	11.383	11.534
12	11.823	11.715	11.624
13	11.629	11.485	11.629
14	11.602	11.509	11.575
15	11.756	11.429	11.730
16	11.707	11.477	11.680
17	11.612	11.570	11.729
18	11.628	11.623	11.704
19	11.603	11.472	12.052
20	11.816	11.531	11.905

does the process mean shift suddenly and at about which sample do you think that the mean changed? Finally, in which case does the mean drift gradually?

s charts for process monitoring

The $\bar{x}$ charts in Figures 26.4, 26.5, and 26.6 were easy to interpret because the process standard deviation remained fixed at 43 mV. The effects of moving the process mean away from its in-control value (275 mV) are then clear to see. We know that even the simplest description of a distribution should give both a measure of center and a measure of spread. So it is with control charts. We must monitor both the process center, using an $\bar{x}$ chart, and the process spread, using a control chart for the sample standard deviation s.

The standard deviation s does not have a Normal distribution, even approximately. Under the process-monitoring conditions, the sampling distribution of s is skewed to the right. Nonetheless, control charts for any statistic are based on the "plus or minus three standard deviations" idea motivated by the 68–95–99.7 rule for Normal distributions. Control charts are intended to be practical tools that are easy to use. Standard practice in process control therefore ignores such details as the effect of non-Normal sampling distributions. Here is the general control chart setup for a sample statistic Q (short for "quality characteristic").

THREE-SIGMA CONTROL CHARTS

To make a **three-sigma (3σ) control chart** for any statistic Q:

1. Take samples from the process at regular intervals and plot the values of the statistic Q against the order in which the samples were taken.
2. Draw a **center line** on the chart at height μ_Q, the mean of the statistic when the process is in control.
3. Draw upper and lower **control limits** on the chart three standard deviations of Q above and below the mean. That is,

$$\text{UCL} = \mu_Q + 3\sigma_Q$$
$$\text{LCL} = \mu_Q - 3\sigma_Q$$

   Here σ_Q is the standard deviation of the sampling distribution of the statistic Q when the process is in control.
4. The chart produces an **out-of-control signal** when a plotted point lies outside the control limits.

We have applied this general idea to $\bar{x}$ charts. If μ and σ are the process mean and standard deviation, the statistic $\bar{x}$ has mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. The center line and control limits for $\bar{x}$ charts follow from these facts.

What are the corresponding facts for the sample standard deviation s? Study of the sampling distribution of s for samples from a Normally distributed process characteristic gives these facts:

1. The *mean* of s is a constant times the process standard deviation σ, $\mu_s = c_4\sigma$.
2. The *standard deviation* of s is also a constant times the process standard deviation, $\sigma_s = c_5\sigma$.

The constants are called c_4 and c_5 for historical reasons. Their values depend on the size of the samples. For large samples, c_4 is close to 1. That is, the sample standard deviation s has little bias as an estimator of the process standard deviation σ. Because statistical process control often uses small samples, we pay attention to the value of c_4. Following the general pattern for three-sigma control charts:

1. The *center line* of an s chart is at $c_4\sigma$.
2. The *control limits* for an s chart are at

$$\text{UCL} = \mu_s + 3\sigma_s = c_4\sigma + 3c_5\sigma = (c_4 + 3c_5)\sigma$$
$$\text{LCL} = \mu_s - 3\sigma_s = c_4\sigma - 3c_5\sigma = (c_4 - 3c_5)\sigma$$

   That is, the control limits UCL and LCL are also constants times the process standard deviation. These constants are called (again for historical reasons) B_6

and B_5. We don't need to remember that $B_6 = c_4 + 3c_5$ and $B_5 = c_4 - 3c_5$, because tables give us the numerical values of B_6 and B_5.

$\bar{x}$ AND s CONTROL CHARTS FOR PROCESS MONITORING[6]

Take regular samples of size n from a process that has been in control with process mean μ and process standard deviation σ. The center line and control limits for an **$\bar{x}$ chart** are

$$\text{UCL} = \mu + 3\frac{\sigma}{\sqrt{n}}$$
$$\text{CL} = \mu$$
$$\text{LCL} = \mu - 3\frac{\sigma}{\sqrt{n}}$$

The center line and control limits for an **s chart** are

$$\text{UCL} = B_6\sigma$$
$$\text{CL} = c_4\sigma$$
$$\text{LCL} = B_5\sigma$$

The **control chart constants** c_4, B_5, and B_6 depend on the sample size n.

Table 26.3 gives the values of the control chart constants c_4, c_5, B_5, and B_6 for samples of sizes 2 to 10. This table makes it easy to draw s charts. The table has no B_5 entries for samples of size smaller than $n = 6$. The lower control limit for an s chart is zero for samples of sizes 2 to 5. This is a consequence of the fact that s has a right-skewed distribution and takes only values greater than zero. Three standard deviations above the mean (UCL) lies on the long right side of the distribution. Three standard deviations below the mean (LCL) on the short left side is below zero, so we say that LCL = 0.

TABLE 26.3 Control chart constants

SAMPLE SIZE n	c_4	c_5	B_5	B_6
2	0.7979	0.6028		2.606
3	0.8862	0.4633		2.276
4	0.9213	0.3889		2.088
5	0.9400	0.3412		1.964
6	0.9515	0.3076	0.029	1.874
7	0.9594	0.2820	0.113	1.806
8	0.9650	0.2622	0.179	1.751
9	0.9693	0.2459	0.232	1.707
10	0.9727	0.2321	0.276	1.669

EXAMPLE 26.5 $\bar{x}$ and s charts for mesh tension

Figure 26.7 is the s chart for the computer monitor mesh tension data in Table 26.1. The samples are of size $n = 4$ and the process standard deviation in control is $\sigma =$ 43 mV. The center line is therefore

$$\text{CL} = c_4\sigma = (0.9213)(43) = 39.6 \text{ mV}$$

The control limits are

$$\text{UCL} = B_6\sigma = (2.088)(43) = 89.8$$

$$\text{LCL} = B_5\sigma = (0)(43) = 0$$

Figures 26.4 and 26.7 go together: they are $\bar{x}$ and s charts for monitoring the mesh-tensioning process. Both charts are in control, showing only common cause variation within the bounds set by the control limits.

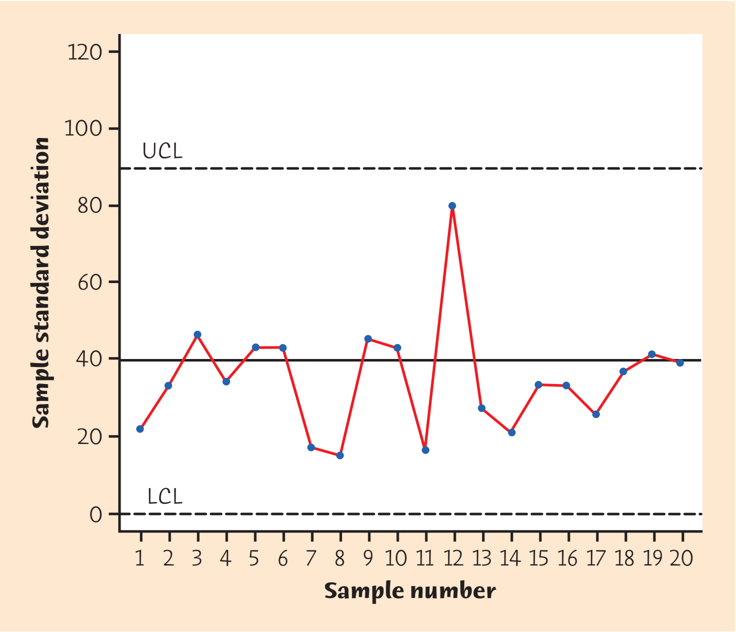


FIGURE 26.7
s chart for the mesh tension data of Table 26.1. Both the s chart and the $\bar{x}$ chart (Figure 26.4) are in control.

Figures 26.8 and 26.9 are $\bar{x}$ and s charts for the mesh-tensioning process when a new and poorly trained operator takes over between Samples 10 and 11. The new operator introduces added variation into the process, increasing the process standard deviation from its in-control value of 43 mV to 60 mV. The $\bar{x}$ chart in Figure 26.8 shows one point out of control. Only on closer inspection do we see that the spread of the $\bar{x}$'s increases after Sample 10. In fact, the process mean has remained unchanged at 275 mV. The apparent lack of control in the $\bar{x}$ chart is entirely due to the larger process variation. There is a lesson here: *it is difficult to interpret an $\bar{x}$ chart unless s is in control. When you look at $\bar{x}$ and s charts, always start with the s chart.*

CAUTION

The s chart in Figure 26.9 shows lack of control starting at Sample 11. As usual, we mark the out-of-control points by an "x." The points for Samples 13 and 15 also lie above the UCL, and the overall spread of the sample points is much greater than for the first 10 samples. In practice, the s chart would call for action after Sample 11. We would ignore the $\bar{x}$ chart until the special cause (the new operator) for the lack of control in the s chart has been found and removed by training the operator. ■

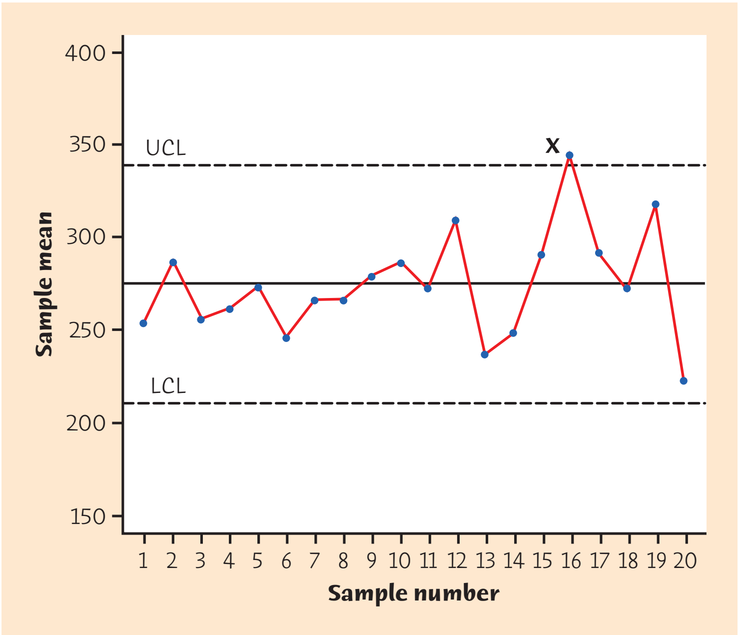


FIGURE 26.8

$\bar{x}$ chart for mesh tension when the process variability increases after Sample 10. The $\bar{x}$ chart does show the increased variability, but the s chart is clearer and should be read first.

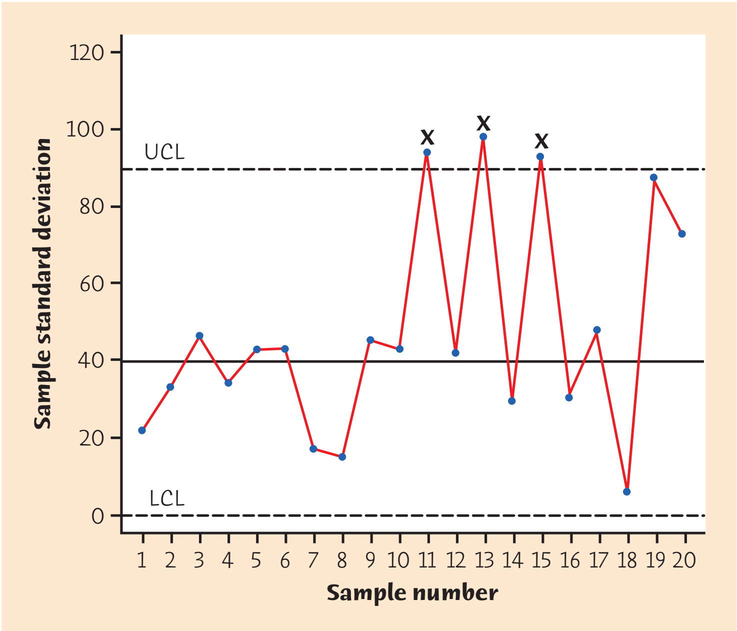


FIGURE 26.9

s chart for mesh tension when the process variability increases after Sample 10. Increased within-sample variability is clearly visible. Find and remove the s-type special cause before reading the $\bar{x}$ chart.

Example 26.5 suggests a strategy for using $\bar{x}$ and s charts in practice. First examine the s chart. Lack of control on an s chart is due to special causes that affect the observations *within a sample* differently. New and nonuniform raw material, a new and poorly trained operator, and mixing results from several machines or several operators are typical "s-type" special causes.

Once the s chart is in control, the stable value of the process standard deviation σ means that the variation within samples serves as a benchmark for detecting variation in the level of the process over the longer time periods between samples. The $\bar{x}$ chart, with control limits that depend on σ, does this. The $\bar{x}$ chart, as we saw in Example 26.5, responds to s-type causes as well as to longer-range changes in the process, so it is important to eliminate s-type special causes first. Then the $\bar{x}$ chart will alert us to, for example, a change in process level caused by new raw material that differs from that used in the past or a gradual drift in the process level caused by wear in a cutting tool.

EXAMPLE 26.6 s-type and $\bar{x}$-type special causes

A large health maintenance organization (HMO) uses control charts to monitor the process of directing patient calls to the proper department or doctor's receptionist. Each day at a random time, 5 consecutive calls are recorded electronically. The first call today is handled quickly by an experienced operator, but the next goes to a newly hired operator who must ask a supervisor for help. The sample has a large s, and lack of control signals the need to train new hires more thoroughly.

The same HMO monitors the time required to receive orders from its main supplier of pharmaceutical products. After a long period in control, the $\bar{x}$ chart shows a systematic shift downward in the mean time because the supplier has changed to a more efficient delivery service. This is a desirable special cause, but it is nonetheless a systematic change in the process. The HMO will have to establish new control limits that describe the new state of the process, with smaller process mean μ. ■

The second setting in Example 26.6 reminds us that a major change in the process returns us to the chart setup stage. In the absence of deliberate changes in the process, process monitoring uses the same values of μ and σ for long periods of time. There is one important exception: careful monitoring and removal of special causes as they occur can permanently reduce the process σ. If the points on the s chart remain near the center line for a long period, it is wise to update the value of σ to the new, smaller value and compute new values of UCL and LCL for both $\bar{x}$ and s charts.

APPLY YOUR KNOWLEDGE

26.11 Responding to applicants. The personnel department of a large company records a number of performance measures. Among them is the time required to respond to an application for employment, measured from the time the application arrives. Suggest some plausible examples of each of the following.

(a) Reasons for common cause variation in response time.

(b) s-type special causes.

(c) $\overline{x}$-type special causes.

26.12 Auto thermostats. In Exercise 26.9 you gave the center line and control limits for an $\overline{x}$ chart. What are the center line and control limits for an s chart for this process?

26.13 Tablet hardness. Exercise 26.10 concerns process control data on the hardness of tablets (measured in kilograms) for a pharmaceutical product. Table 26.4 gives data for 20 new samples of size 4, with the $\overline{x}$ and s for each sample. The process has been in control with mean at the target value $\mu = 11.5$ kg and standard deviation $\sigma = 0.2$ kg.

TABLE 26.4 Twenty samples of size 4, with $\overline{x}$ and s

SAMPLE	HARDNESS (KILOGRAMS)				$\overline{x}$	s
1	11.432	11.350	11.582	11.184	11.387	0.1660
2	11.791	11.323	11.734	11.512	11.590	0.2149
3	11.373	11.807	11.651	11.651	11.620	0.1806
4	11.787	11.585	11.386	11.245	11.501	0.2364
5	11.633	11.212	11.568	11.469	11.470	0.1851
6	11.648	11.653	11.618	11.314	11.558	0.1636
7	11.456	11.270	11.817	11.402	11.486	0.2339
8	11.394	11.754	11.867	11.003	11.504	0.3905
9	11.349	11.764	11.402	12.085	11.650	0.3437
10	11.478	11.761	11.907	12.091	11.809	0.2588
11	11.657	12.524	11.468	10.946	11.649	0.6564
12	11.820	11.872	11.829	11.344	11.716	0.2492
13	12.187	11.647	11.751	12.026	11.903	0.2479
14	11.478	11.222	11.609	11.271	11.395	0.1807
15	11.750	11.520	11.389	11.803	11.616	0.1947
16	12.137	12.056	11.255	11.497	11.736	0.4288
17	12.055	11.730	11.856	11.357	11.750	0.2939
18	12.107	11.624	11.727	12.207	11.916	0.2841
19	11.933	10.658	11.708	11.278	11.394	0.5610
20	12.512	12.315	11.671	11.296	11.948	0.5641

(a) Make both $\overline{x}$ and s charts for these data based on the information given about the process.

(b) At some point, the within-sample process variation increased from $\sigma = 0.2$ kg to $\sigma = 0.4$ kg. About where in the 20 samples did this happen? What is the effect on the s chart? On the $\overline{x}$ chart?

(c) At that same point, the process mean changed from $\mu = 11.5$ kg to $\mu = 11.7$ kg. What is the effect of this change on the s chart? On the $\overline{x}$ chart?

26.14 Dyeing yarn. The unique colors of the cashmere sweaters your firm makes result from heating undyed yarn in a kettle with a dye liquor. The pH (acidity) of the liquor is critical for regulating dye uptake and hence the final color. There are 5 kettles, all of which receive dye liquor from a common source. Twice each day, the

Ric Ergenbright/CORBIS

pH of the liquor in each kettle is measured, giving samples of size 5. The process has been operating in control with $\mu = 4.22$ and $\sigma = 0.127$.

(a) Give the center line and control limits for the s chart.

(b) Give the center line and control limits for the $\bar{x}$ chart.

26.15 Mounting-hole distances. Figure 26.10 reproduces a data sheet from the floor of a factory that makes electrical meters.[7] The sheet shows measurements on the distance between two mounting holes for 18 samples of size 5. The heading informs us that the measurements are in multiples of 0.0001 inch above 0.6000 inch. That is, the first measurement, 44, stands for 0.6044 inch. All the measurements end in 4. Although we don't know why this is true, it is clear that in effect the measurements were made to the nearest 0.001 inch, not to the nearest 0.0001 inch.

Calculate $\bar{x}$ and s for the first two samples. The data file *ex26-15.dat* contains $\bar{x}$ and s for all 18 samples. Based on long experience with this process, you are keeping control charts based on $\mu = 43$ and $\sigma = 12.74$. Make s and $\bar{x}$ charts for the data in Figure 26.10 and describe the state of the process.

VARIABLES CONTROL CHART ($\bar{X}$ & R)

Part No.	Chart No.
32506	1

Part name (project)	Operation (process)	Specification limits
Metal frame	Distance between mounting holes	0.6054" ± 0.0010"

Operator	Machine	Gage	Unit of measure	Zero equals
	R-5		0.0001"	0.6000"

Date		3/7				3/8						3/9							
Time		8:30	10:30	11:45	1:30	8:15	10:15	11:45	2:00	3:00	4:00	8:30	10:00	11:45	1:30	2:30	3:30	4:30	5:30
Sample measurements	1	44	64	34	44	34	34	54	64	24	34	34	54	44	24	54	54	54	54
	2	44	44	44	54	14	64	64	34	54	44	44	44	24	24	24	34	34	24
	3	44	34	54	54	84	34	34	54	44	44	34	24	34	54	54	24	74	64
	4	44	34	44	34	54	44	44	44	34	34	64	54	34	44	44	44	44	34
	5	64	54	54	44	44	44	34	44	34	34	34	24	44	44	44	54	54	44
Average, $\bar{X}$																			
Range, R		20	30	20	20	70	30	30	30	30	10	30	30	20	30	40	30	40	40

FIGURE 26.10

A process control record sheet kept by operators, for Exercise 26.15. This is typical of records kept by hand when measurements are not automated. We will see in the next section why such records mention $\bar{x}$ and R control charts rather than $\bar{x}$ and s charts.

26.16 Dyeing yarn: special causes. The process described in Exercise 26.14 goes out of control. Investigation finds that a new type of yarn was recently introduced. The pH in the kettles is influenced by both the dye liquor and the yarn. Moreover, on a few occasions a faulty valve on one of the kettles had allowed water to enter that kettle; as a result, the yarn in that kettle had to be discarded. Which of these special causes appears on the s chart and which on the $\bar{x}$ chart? Explain your answer.

Using control charts

We are now familiar with the ideas that undergird all control charts and also with the details of making $\bar{x}$ and s charts. This section discusses two topics related to using control charts in practice.

$\bar{x}$ and R charts We have seen that it is essential to monitor both the center and the spread of a process. Control charts were originally intended to be used by factory workers with limited knowledge of statistics in the era before even calculators, let alone software, were common. In that environment, it takes too long to calculate standard deviations. The $\bar{x}$ chart for center was therefore combined with a control chart for spread based on the **sample range** rather than the sample standard deviation. The range R of a sample is just the difference between the largest and smallest observations. It is easy to find R without a calculator. Using R rather than s to measure the spread of samples replaces the s chart with an **R chart.** It also changes the $\bar{x}$ chart because the control limits for $\bar{x}$ use the estimated process spread. So $\bar{x}$ and R charts differ in the details of both charts from $\bar{x}$ and s charts.

sample range

R chart

Because the range R uses only the largest and smallest observations in a sample, it is less informative than the standard deviation s calculated from all the observations. For this reason, $\bar{x}$ and s charts are now preferred to $\bar{x}$ and R charts. R charts remain common because tradition dies hard and also because it is easier for workers to understand R than s. In this short introduction, we concentrate on the principles of control charts, so we won't give the details of constructing $\bar{x}$ and R charts. These details appear in any text on quality control.[8] If you meet a set of $\bar{x}$ and R charts, remember that the interpretation of these charts is just like the interpretation of $\bar{x}$ and s charts.

Additional out-of-control signals So far, we have used only the basic "one point beyond the control limits" criterion to signal that a process may have gone out of control. We would like a quick signal when the process moves out of control, but we also want to avoid "false alarms," signals that occur just by chance when the process is really in control. The standard 3σ control limits are chosen to prevent too many false alarms, because an out-of-control signal calls for an effort to find and remove a special cause. As a result, *$\bar{x}$ charts are often slow to respond to a gradual drift in the process center that continues for some time before finally forcing a reading outside the control limits*. We can speed the response of a control chart to lack of control—at the cost of also enduring more false alarms—by adding patterns other than "one-point-out" as signals. The most common step in this direction is to add a *runs signal* to the $\bar{x}$ chart.

CAUTION

OUT-OF-CONTROL SIGNALS

$\bar{x}$ and s or $\bar{x}$ and R control charts produce an out-of-control signal if:

- **One point out:** A single point lies outside the 3σ control limits of either chart.
- **Run:** The $\bar{x}$ chart shows 9 consecutive points above the center line or 9 consecutive points below the center line. The signal occurs when we see the 9th point of the run.

EXAMPLE 26.7 Using the runs signal

Figure 26.11 reproduces the $\overline{x}$ chart from Figure 26.6. The process center began a gradual upward drift at Sample 11. The chart shows the effect of the drift—the sample means plotted on the chart move gradually upward, with some random variation. The one-point-out signal does not call for action until Sample 18 finally produces an $\overline{x}$ above the UCL. The runs signal reacts more quickly: Sample 17 is the 9th consecutive point above the center line. ■

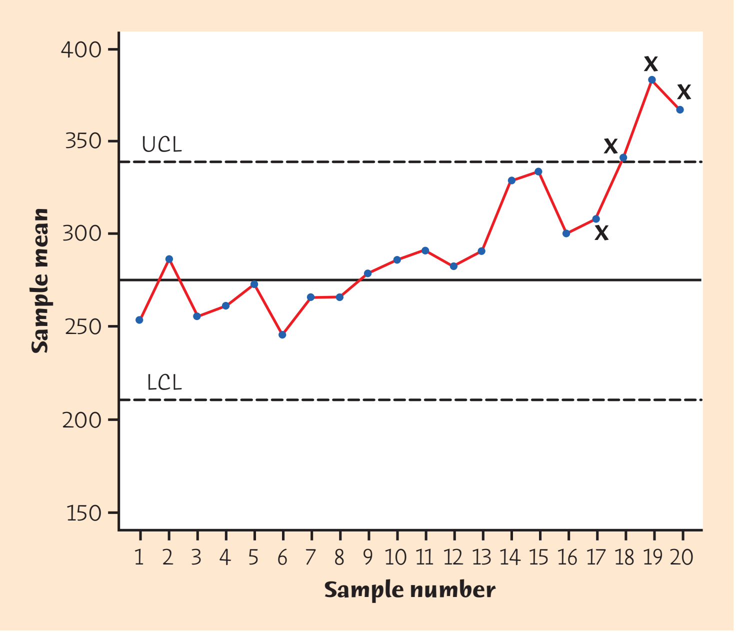


FIGURE 26.11
$\overline{x}$ chart for mesh tension data when the process center drifts upward. The "run of 9" signal gives an out-of-control warning at Sample 17.

It is a mathematical fact that the runs signal responds to a gradual drift more quickly (on the average) than the one-point-out signal does. The motivation for a runs signal is that when a process is in control, the probability of a false alarm is about the same for the runs signal as for the one-point-out signal. There are many other signals that can be added to the rules for responding to $\overline{x}$ and s or $\overline{x}$ and R charts. *In our enthusiasm to detect various special kinds of loss of control, it is easy to forget that adding signals always increases the frequency of false alarms.* Frequent false alarms are so annoying that the people responsible for responding soon begin to ignore out-of-control signals. It is better to use only a few signals and to reserve signals other than one-point-out and runs for processes that are known to be prone to specific special causes for which there is a tailor-made signal.[9]

CAUTION

APPLY YOUR KNOWLEDGE

26.17 Special causes. Is each of the following examples of a special cause most likely to first result in (i) one-point-out on the s or R chart, (ii) one-point-out on the $\overline{x}$ chart, or (iii) a run on the $\overline{x}$ chart? In each case, briefly explain your reasoning.

(a) An etching solution deteriorates as more items are etched.

(b) Buildup of dirt reduces the precision with which parts are placed for machining.

(c) A new customer service representative for a Spanish-language help line is not a native speaker and has difficulty understanding customers.

(d) A data entry employee grows less attentive as her shift continues.

26.18 Mixtures. Here is an artificial situation that illustrates an unusual control chart pattern. Invoices are processed and paid by two clerks, one very experienced and the other newly hired. The experienced clerk processes invoices quickly. The new hire must often refer to a handbook and is much slower. Both are quite consistent, so that their times vary little from invoice to invoice. It happens that each sample of invoices comes from one of the clerks, so that some samples are from one and some from the other clerk. Sketch the $\bar{x}$ chart pattern that will result.

Setting up control charts

When you first approach a process that has not been carefully studied, it is quite likely that the process is not in control. Your first goal is to discover and remove special causes and so bring the process into control. Control charts are an important tool. Control charts for *process monitoring* follow the process forward in time to keep it in control. Control charts at the *chart setup* stage, on the other hand, look back in an attempt to discover the present state of the process. An example will illustrate the method.

EXAMPLE 26.8 Viscosity of an elastomer

The viscosity of a material is its resistance to flow when under stress. Viscosity is a critical characteristic of rubber and rubber-like compounds called elastomers, which have many uses in consumer products. Viscosity is measured by placing specimens of the material above and below a slowly rotating roller, squeezing the assembly, and recording the drag on the roller. Measurements are in "Mooney units," named after the inventor of the instrument.

A specialty chemical company is beginning production of an elastomer that is supposed to have viscosity 45 ± 5 Mooneys. Each lot of the elastomer is produced by "cooking" raw material with catalysts in a reactor vessel. Table 26.5 records $\bar{x}$ and s from samples of size $n = 4$ lots from the first 24 shifts as production begins.[10] An s chart therefore monitors variation among lots produced during the same shift. If the s chart is in control, an $\bar{x}$ chart looks for shift-to-shift variation. ■

Estimating μ We do not know the process mean μ and standard deviation σ. What shall we do? Sometimes we can easily adjust the center of a process by setting some control, such as the depth of a cutting tool in a machining operation or the temperature of a reactor vessel in a pharmaceutical plant. In such cases it is usual to simply take the process mean μ to be the target value, the depth or temperature

TABLE 26.5 $\bar{x}$ and s for 24 samples of elastomer viscosity

SAMPLE	$\bar{x}$	s	SAMPLE	$\bar{x}$	s
1	49.750	2.684	13	47.875	1.118
2	49.375	0.895	14	48.250	0.895
3	50.250	0.895	15	47.625	0.671
4	49.875	1.118	16	47.375	0.671
5	47.250	0.671	17	50.250	1.566
6	45.000	2.684	18	47.000	0.895
7	48.375	0.671	19	47.000	0.447
8	48.500	0.447	20	49.625	1.118
9	48.500	0.447	21	49.875	0.447
10	46.250	1.566	22	47.625	1.118
11	49.000	0.895	23	49.750	0.671
12	48.125	0.671	24	48.625	0.895

that the design of the process specifies as correct. The $\bar{x}$ chart then helps us keep the process mean at this target value.

There is less likely to be a "correct value" for the process mean μ if we are monitoring response times to customer calls or data entry errors. In Example 26.8, we have the target value 45 Mooneys, but there is no simple way to set viscosity at the desired level. In such cases, we want the μ we use in our $\bar{x}$ chart to describe the center of the process as it has actually been operating. To do this, just take the mean of all the individual measurements in the past samples. Because the samples are all the same size, this is just the mean of the sample $\bar{x}$'s. The overall "mean of the sample means" is therefore usually called $\bar{\bar{x}}$. For the 24 samples in Table 26.5,

$$\begin{aligned}\bar{\bar{x}} &= \frac{1}{24}(49.750 + 49.375 + \cdots + 48.625)\\ &= \frac{1161.125}{24} = 48.380\end{aligned}$$

Estimating σ It is almost never safe to use a "target value" for the process standard deviation σ because it is almost never possible to directly adjust process variation. We must estimate σ from past data. We want to combine the sample standard deviations s from past samples rather than use the standard deviation of all the individual observations in those samples. That is, in Example 26.8, we want to combine the 24 sample standard deviations in Table 26.5 rather than calculate the standard deviation of the 96 observations in these samples. The reason is that it is the *within-sample* variation that is the benchmark against which we compare the longer-term process variation. Even if the process has been in control, we want only the variation over the short time period of a single sample to influence our value for σ.

There are several ways to estimate σ from the sample standard deviations. In practice, software may use a somewhat sophisticated method and then calculate the control limits for you. We use a simple method that is traditional in quality control because it goes back to the era before software. If we are basing chart setup on k past samples, we have k sample standard deviations $s_1, s_2, \ldots, s_k$. Just average these to get

$$\bar{s} = \frac{1}{k}(s_1 + s_2 + \cdots + s_k)$$

For the viscosity example, we average the s-values for the 24 samples in Table 26.5:

$$\begin{aligned}\bar{s} &= \frac{1}{24}(2.684 + 0.895 + \cdots + 0.895)\\ &= \frac{24.156}{24} = 1.0065\end{aligned}$$

Combining the sample s-values to estimate σ introduces a complication: the samples used in process control are often small (size $n = 4$ in the viscosity example), so s has some bias as an estimator of σ. Recall that $\mu_s = c_4\sigma$. The mean $\bar{s}$ inherits this bias: its mean is also not σ but $c_4\sigma$. The proper estimate of σ corrects this bias. It is

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

We get control limits from past data by using the estimates $\bar{\bar{x}}$ and $\hat{\sigma}$ in place of the μ and σ used in charts at the process-monitoring stage. Here are the results.[11]

$\bar{x}$ AND s CONTROL CHARTS USING PAST DATA

Take regular samples of size n from a process. Estimate the process mean μ and the process standard deviation σ from past samples by

$$\hat{\mu} = \bar{\bar{x}} \qquad \text{(or use a target value)}$$

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

The center line and control limits for an **$\bar{x}$ chart** are

$$\begin{aligned}\text{UCL} &= \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}}\\ \text{CL} &= \hat{\mu}\\ \text{LCL} &= \hat{\mu} - 3\frac{\hat{\sigma}}{\sqrt{n}}\end{aligned}$$

The center line and control limits for an **s chart** are

$$\begin{aligned}\text{UCL} &= B_6\hat{\sigma}\\ \text{CL} &= c_4\hat{\sigma} = \bar{s}\\ \text{LCL} &= B_5\hat{\sigma}\end{aligned}$$

If the process was not in control when the samples were taken, these should be regarded as trial control limits.

We are now ready to outline the chart setup procedure for elastomer viscosity.

Step 1. As usual, we look first at an s chart. For chart setup, control limits are based on the same past data that we will plot on the chart. Calculate from Table 26.5 that

$$\bar{s} = 1.0065$$

$$\hat{\sigma} = \frac{\bar{s}}{c_4} = \frac{1.0065}{0.9213} = 1.0925$$

The center line and control limits for an s chart based on past data are

$$\text{UCL} = B_6\hat{\sigma} = (2.088)(1.0925) = 2.281$$
$$\text{CL} = \bar{s} = 1.0065$$
$$\text{LCL} = B_5\hat{\sigma} = (0)(1.0925) = 0$$

Figure 26.12 is the s chart. The points for Shifts 1 and 6 lie above the UCL. Both are near the beginning of production. Investigation finds that the reactor operator made an error on one lot in each of these samples. The error changed the viscosity of that lot and increased s for that one sample. The error will not be repeated now that the operators have gained experience. That is, this special cause has already been removed.

Step 2. Remove the two values of s that were out of control. This is proper because the special cause responsible for these readings is no longer present. Recalculate from the remaining 22 shifts that $\bar{s} = 0.854$ and $\hat{\sigma} = 0.854/0.9213 = 0.927$.

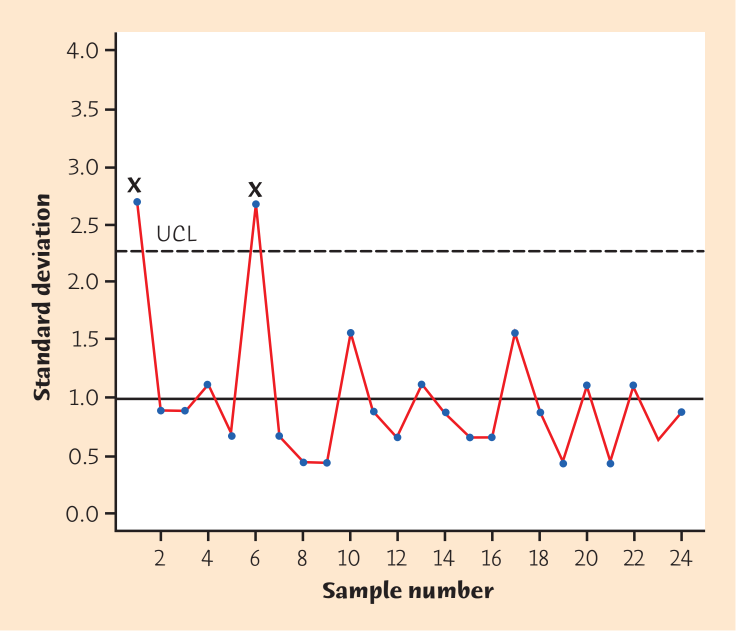


FIGURE 26.12
s chart based on past data for the viscosity data of Table 26.5. The control limits are based on the same s-values that are plotted on the chart. Points 1 and 6 are out of control.

Make a new s chart with

$$\text{UCL} = B_6\hat{\sigma} = (2.088)(0.927) = 1.936$$
$$\text{CL} = \bar{s} = 0.854$$
$$\text{LCL} = B_5\hat{\sigma} = (0)(0.927) = 0$$

We don't show the chart, but you can see from Table 26.5 that none of the remaining s-values lies above the new, lower, UCL; the largest remaining s is 1.566. If additional points were now out of control, we would repeat the process of finding and eliminating s-type causes until the s chart for the remaining shifts was in control. In practice, of course, this is often a challenging task.

Step 3. Once s-type causes have been eliminated, make an $\bar{x}$ chart *using only the samples that remain* after dropping those that had out-of-control s-values. For the 22 remaining samples, we know that $\hat{\sigma} = 0.927$ and we calculate that $\bar{\bar{x}} = 48.4716$. The center line and control limits for the $\bar{x}$ chart are

$$\text{UCL} = \bar{\bar{x}} + 3\frac{\hat{\sigma}}{\sqrt{n}} = 48.4716 + 3\frac{0.927}{\sqrt{4}} = 49.862$$

$$\text{CL} = \bar{\bar{x}} = 48.4716$$

$$\text{LCL} = \bar{\bar{x}} - 3\frac{\hat{\sigma}}{\sqrt{n}} = 48.4716 - 3\frac{0.927}{\sqrt{4}} = 47.081$$

Figure 26.13 is the $\bar{x}$ chart. Shifts 1 and 6 have been dropped. Seven of the 22 points are beyond the 3σ limits, four high and three low. Although within-shift variation is now stable, there is excessive variation from shift to shift. To find the

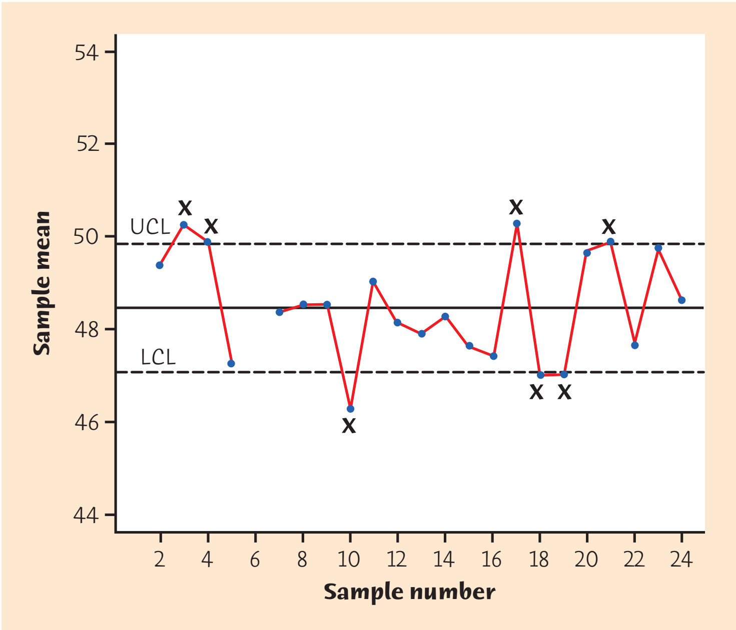


FIGURE 26.13
$\bar{x}$ chart based on past data for the viscosity data of Table 26.5. The samples for Shifts 1 and 6 have been removed because s-type special causes active in those samples are no longer active. The $\bar{x}$ chart shows poor control.

cause, we must understand the details of the process, but knowing that the special cause or causes operate between shifts is a big help. If the reactor is set up anew at the beginning of each shift, that's one place to look more closely.

Step 4. Once the $\overline{x}$ and s charts are both in control (looking backward), use the estimates $\hat{\mu}$ and $\hat{\sigma}$ from the points in control to set tentative control limits to monitor the process going forward. If it remains in control, we can update the charts and move to the process-monitoring stage.

APPLY YOUR KNOWLEDGE

26.19 From setup to monitoring. Suppose that when the chart setup project of Example 26.8 is complete, the points remaining after removing special causes have $\overline{\overline{x}} = 48.7$ and $\overline{s} = 0.92$. What are the center line and control limits for the $\overline{x}$ and s charts you would use to monitor the process going forward?

26.20 Estimating process parameters. The $\overline{x}$ and s control charts for the mesh-tensioning example (Figures 26.4 and 26.7) were based on $\mu = 275$ mV and $\sigma = 43$ mV. Table 26.1 gives the 20 most recent samples from this process.

(a) Estimate the process μ and σ based on these 20 samples.

(b) Your calculations suggest that the process σ may now be less than 43 mV. Explain why the s chart in Figure 26.7 (page 26-16) suggests the same conclusion. (If this pattern continues, we would eventually update the value of σ used for control limits.)

26.21 Hospital losses. Table 26.6 gives data on the losses (in dollars) incurred by a hospital in treating major joint replacement (DRG 209) patients.[12] The hospital

TABLE 26.6 Hospital losses for 15 samples of DRG 209 patients

SAMPLE	LOSS (DOLLARS)								SAMPLE MEAN	STANDARD DEVIATION
1	6835	5843	6019	6731	6362	5696	7193	6206	6360.6	521.7
2	6452	6764	7083	7352	5239	6911	7479	5549	6603.6	817.1
3	7205	6374	6198	6170	6482	4763	7125	6241	6319.8	749.1
4	6021	6347	7210	6384	6807	5711	7952	6023	6556.9	736.5
5	7000	6495	6893	6127	7417	7044	6159	6091	6653.2	503.7
6	7783	6224	5051	7288	6584	7521	6146	5129	6465.8	1034.3
7	8794	6279	6877	5807	6076	6392	7429	5220	6609.2	1104.0
8	4727	8117	6586	6225	6150	7386	5674	6740	6450.6	1033.0
9	5408	7452	6686	6428	6425	7380	5789	6264	6479.0	704.7
10	5598	7489	6186	5837	6769	5471	5658	6393	6175.1	690.5
11	6559	5855	4928	5897	7532	5663	4746	7879	6132.4	1128.6
12	6824	7320	5331	6204	6027	5987	6033	6177	6237.9	596.6
13	6503	8213	5417	6360	6711	6907	6625	7888	6828.0	879.8
14	5622	6321	6325	6634	5075	6209	4832	6386	5925.5	667.8
15	6269	6756	7653	6065	5835	7337	6615	8181	6838.9	819.5

has taken from its records a random sample of 8 such patients each month for 15 months.

(a) Make an s control chart using center lines and limits calculated from these past data. There are no points out of control.

(b) Because the s chart is in control, base the $\bar{x}$ chart on all 15 samples. Make this chart. Is it also in control?

26.22 A cutting operation. A machine tool in your plant is cutting an outside diameter. A sample of 4 pieces is taken near the end of each hour of production. Table 26.7 gives $\bar{x}$ and s for the first 21 samples, coded in units of 0.0001 inch from the center of the specifications. The specifications allow a range of ± 0.0002 inch about the center (a range of -2 to $+2$ as coded).

TABLE 26.7 $\bar{x}$ and s for 21 samples of outside diameter

SAMPLE	$\bar{x}$	s	SAMPLE	$\bar{x}$	s
1	−0.14	0.48	12	0.55	0.10
2	0.09	0.26	13	0.50	0.25
3	0.17	0.24	14	0.37	0.45
4	0.08	0.38	15	0.69	0.21
5	−0.17	0.50	16	0.47	0.34
6	0.36	0.26	17	0.56	0.42
7	0.30	0.39	18	0.78	0.08
8	0.19	0.31	19	0.75	0.32
9	0.48	0.13	20	0.49	0.23
10	0.29	0.13	21	0.79	0.12
11	0.48	0.25			

(a) Make an s chart based on past data and comment on control of short-term process variation.

(b) Because the data are coded about the center of the specs, we have a given target $\mu = 0$ (as coded) for the process mean. Make an $\bar{x}$ chart and comment on control of long-term process variation. What special $\bar{x}$-type cause probably explains the lack of control of $\bar{x}$?

26.23 The Boston Marathon. The Boston Marathon has been run each year since 1897. Winning times were highly variable in the early years, but control improved as the best runners became more professional. A clear downward trend continued until the 1980s. Rick plans to make a control chart for the winning times from 1950 to the present. The first few times are 153, 148, 152, 139, 141, and 138 minutes. Calculation from the winning times from 1950 to 2007 gives

$$\bar{x} = 134.552 \text{ minutes} \qquad \text{and} \qquad s = 6.375 \text{ minutes}$$

Rick draws a center line at $\bar{x}$ and control limits at $\bar{x} \pm 3s$ for a plot of individual winning times. Explain carefully why these control limits are too wide to effectively signal unusually fast or slow times.

Comments on statistical control

Having seen how $\bar{x}$ and s (or $\bar{x}$ and R) charts work, we can turn to some important comments and cautions about statistical control in practice.

Focus on the process rather than on the products This is a fundamental idea in statistical process control. We might attempt to attain high quality by careful inspection of the finished product, measuring every completed forging and reviewing every outgoing invoice and expense account payment. Inspection of finished products can ensure good quality, but it is expensive. Perhaps more important, final inspection comes too late: when something goes wrong early in a process, much bad product may be produced before final inspection discovers the problem. This adds to the expense, because the bad product must then be scrapped or reworked.

The small samples that are the basis of control charts are intended to monitor the process at key points, not to ensure the quality of the particular items in the samples. If the process is kept in control, we know what to expect in the finished product. We want to do it right the first time, not inspect and fix finished product.

Rational subgroups The interpretation of control charts depends on the distinction between $\bar{x}$-type special causes and s-type special causes. This distinction in turn depends on how we choose the samples from which we calculate s (or R). We want the variation *within* a sample to reflect only the item-to-item chance variation that (when in control) results from many small common causes. Walter Shewhart, the founder of statistical process control, used the term **rational subgroup** to emphasize that we should think about the process when deciding how to choose samples.

rational subgroup

EXAMPLE 26.9 Random sampling versus rational subgroups

A pharmaceutical manufacturer forms tablets by compressing a granular material that contains the active ingredient and various fillers. To monitor the compression process, we will measure the hardness of a sample from each 10 minutes' production of tablets. Should we choose a random sample of tablets from the several thousand produced in a 10-minute period?

A random sample would contain tablets spread across the entire 10 minutes. It fairly represents the 10-minute period, but that isn't what we want for process control. If the setting of the press drifts or a new lot of filler arrives during the 10 minutes, the spread of the sample will be increased. That is, a random sample contains both the short-term variation among tablets produced in quick succession and the longer-term variation among tablets produced minutes apart. We prefer to measure a rational subgroup of 5 consecutive tablets every 10 minutes. We expect the process to be stable during this very short time period, so that variation within the subgroups is a benchmark against which we can see special cause variation. ■

Samples of consecutive items are rational subgroups when we are monitoring the output of a single activity that does the same thing over and over again. Several consecutive items is the most common type of sample for process control. There is no formula for choosing samples that are rational subgroups. You must think about causes of variation in your process and decide which you are willing to think of as common causes that you will not try to eliminate. Rational subgroups are samples chosen to express variation due to these causes and no others. Because the choice requires detailed process knowledge, we will usually accept samples of consecutive items as being rational subgroups.

Why statistical control is desirable To repeat, if the process is kept in control, we know what to expect in the finished product. The process mean μ and standard deviation σ remain stable over time, so (assuming Normal variation) the 99.7 part of the 68–95–99.7 rule tells us that almost all measurements on individual products will lie in the range $\mu \pm 3\sigma$. These are sometimes called the **natural tolerances** for the product. *Be careful to distinguish $\mu \pm 3\sigma$, the range we expect for individual measurements, from the $\bar{x}$ chart control limits $\mu \pm 3\sigma/\sqrt{n}$, which mark off the expected range of sample means.*

natural tolerances

CAUTION

EXAMPLE 26.10 Natural tolerances for mesh tension

The process of setting the mesh tension on computer monitors has been operating in control. The $\bar{x}$ and s charts were based on $\mu = 275$ mV and $\sigma = 43$ mV. The s chart in Figure 26.7 and your calculation in Exercise 26.20 suggest that the process σ is now less than 43 mV. We may prefer to calculate the natural tolerances from the recent data on 20 samples (80 monitors) in Table 26.1 (page 26-10). The estimate of the mean is $\bar{\bar{x}} = 275.065$, very close to the target value.

Now a subtle point arises. The estimate $\hat{\sigma} = \bar{s}/c_4$ used for past-data control charts is based entirely on variation *within the samples*. That's what we want for control charts, because within-sample variation is likely to be "pure common cause" variation. Even when the process is in control, there is some additional variation from sample to sample, just by chance. So the variation in the process output will be greater than the variation within samples. *To estimate the natural tolerances, we should estimate σ from all 80 individual monitors rather than by averaging the 20 within-sample standard deviations.* The standard deviation for all 80 mesh tensions is

$$s = 38.38$$

(For a sample of size 80, c_4 is very close to 1, so we can ignore it.)

We are therefore confident that almost all individual monitors will have mesh tension

$$\bar{\bar{x}} \pm 3s = 275.065 \pm (3)(38.38) \doteq 275 \pm 115$$

We expect mesh tension measurements to vary between 160 and 390 mV. You see that the spread of individual measurements is wider than the spread of sample means used for the control limits of the $\bar{x}$ chart. ■

The natural tolerances in Example 26.10 depend on the fact that the mesh tensions of individual monitors follow a Normal distribution. We know that the process was in control when the 80 measurements in Table 26.1 were made, so we can graph them to assess Normality.

APPLY YOUR KNOWLEDGE

26.24 No incoming inspection. The computer makers who buy monitors require that the monitor manufacturer practice statistical process control and submit control charts for verification. This allows the computer makers to eliminate inspection of monitors as they arrive, a considerable cost saving. Explain carefully why incoming inspection can safely be eliminated.

26.25 Natural tolerances. Table 26.6 (page 26-28) gives data on hospital losses for samples of DRG 209 patients. The distribution of losses has been stable over time. What are the natural tolerances within which you expect losses on nearly all such patients to fall?

26.26 Normality? Do the losses on the 120 individual patients in Table 26.6 appear to come from a single Normal distribution? Make a graph and discuss what it shows. Are the natural tolerances you found in the previous exercise trustworthy?

Don't confuse control with capability!

CAUTION

A process in control is stable over time. We know how much variation the finished product will show. Control charts are, so to speak, the voice of the process telling us what state it is in. *There is no guarantee that a process in control produces products of satisfactory quality.* "Satisfactory quality" is measured by comparing the product to some standard outside the process, set by technical specifications, customer expectations, or the goals of the organization. These external standards are unrelated to the internal state of the process, which is all that statistical control pays attention to.

CAPABILITY

Capability refers to the ability of a process to meet or exceed the requirements placed on it.

Capability has nothing to do with control—except for the very important point that if a process is not in control, it is hard to tell if it is capable or not.

EXAMPLE 26.11 Capability

The primary customer for our monitors is a large maker of computers. The customer informed us that adequate image quality requires that the mesh tension lie between 100 and 400 mV. Because the mesh-tensioning process is in control, we know (Example 26.10) that almost all monitors will have mesh tension between 160 and 390 mV. The process is capable of meeting the customer's requirement.

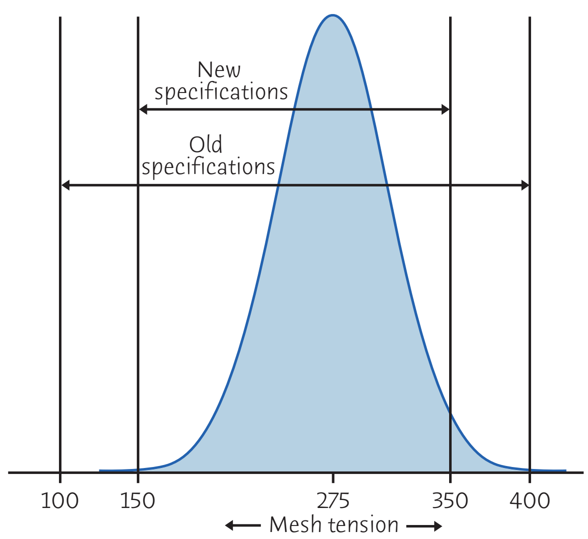


FIGURE 26.14
Comparison of the distribution of mesh tension (Normal curve) with original and tightened specifications. The process in its current state is not capable of meeting the new specifications.

Figure 26.14 compares the distribution of mesh tension for individual monitors with the customer's specifications. The distribution of tension is approximately Normal, and we estimate its mean to be very close to 275 mV and the standard deviation to be about 38.4 mV. The distribution is safely within the specifications.

Times change, however. As computer buyers demand better screen quality, the computer maker restudies the effect of mesh tension and decides to require that tension lie between 150 and 350 mV. These new specification limits also appear in Figure 26.14. The process is not capable of meeting the new requirements. The process remains in control. The change in its capability is entirely due to a change in external requirements. ■

Because the mesh-tensioning process is in control, we know that it is not capable of meeting the new specifications. That's an advantage of control, but the fact remains that control does not guarantee capability. *If a process that is in control does not have adequate capability, fundamental changes in the process are needed.* The process is doing as well as it can and displays only the chance variation that is natural to its present state. Better training for workers, new equipment, or more uniform material may improve capability, depending on the findings of a careful investigation.

CAUTION

APPLY YOUR KNOWLEDGE

26.27 Describing capability. If the mesh tension of individual monitors follows a Normal distribution, we can describe capability by giving the percent of monitors that meet specifications. The old specifications for mesh tension are 100 to 400 mV. The new specifications are 150 to 350 mV. Because the process is in control, we can estimate that tension has mean 275 mV and standard deviation 38.4 mV.

(a) What percent of monitors meet the old specifications?

(b) What percent meet the new specifications?

26.28 Improving capability. The center of the specifications for mesh tension in the previous exercise is 250 mV, but the center of our process is 275 mV. We can improve capability by adjusting the process to have center 250 mV. This is an easy adjustment that does not change the process variation. What percent of monitors now meet the new specifications?

26.29 Mounting-hole distances. Figure 26.10 (page 26-20) displays a record sheet for 18 samples of distances between mounting holes in an electrical meter. The data file *ex26-15.dat* adds $\bar{x}$ and s for each sample. In Exercise 26.15, you found that Sample 5 was out of control on the process-monitoring s chart. The special cause responsible was found and removed. Based on the 17 samples that were in control, what are the natural tolerances for the distance between the holes?

26.30 Mounting-hole distances, continued. The record sheet in Figure 26.10 gives the specifications as 0.6054 ± 0.0010 inch. That's 54 ± 10 as the data are coded on the record sheet. Assuming that the distance varies Normally from meter to meter, about what percent of meters meet the specifications?

Control charts for sample proportions

We have considered control charts for just one kind of data: measurements of a quantitative variable in some meaningful scale of units. We describe the distribution of measurements by its center and spread and use $\bar{x}$ and s or $\bar{x}$ and R charts for process control. There are control charts for other statistics that are appropriate for other kinds of data. The most common of these is the p chart for use when the data are proportions.

***p* CHART**

A ***p* chart** is a control chart based on plotting sample proportions $\hat{p}$ from regular samples from a process against the order in which the samples were taken.

EXAMPLE 26.12 *p* chart settings

Here are two examples of the usefulness of p charts:

- Measure two dimensions of a manufactured part and also grade its surface finish by eye. The part conforms if both dimensions lie within their specifications and the finish is judged acceptable. Otherwise, it is nonconforming. Plot the proportion of nonconforming parts in samples of parts from each shift.
- An urban school system records the percent of its eighth-grade students who are absent three or more days each month. Because students with high absenteeism in eighth grade often fail to complete high school, the school system has launched programs to reduce absenteeism. These programs include calls to parents of absent students, public-service messages to change community expectations, and measures to ensure that the schools are safe and attractive. A p chart will show if the programs are having an effect. ■

The manufacturing example illustrates an advantage of p charts: they can combine several specifications in a single chart. Nonetheless, p charts have been rendered outdated in many manufacturing applications by improvements in typical levels of quality. For example, Delphi, the largest North American auto electronics manufacturer, says that it reduced its proportion of problem parts from 200 per million in 1997 to 20 per million in 2001.[13] At either of these levels, even large samples of parts will rarely contain any bad parts. The sample proportions will almost all be 0, so that plotting them is uninformative. It is better to choose important measured characteristics—voltage at a critical circuit point, for example—and keep $\overline{x}$ and s charts. Even if the voltage is satisfactory, quality can be improved by moving it yet closer to the exact voltage specified in the design of the part.

The school absenteeism example is a management application of p charts. More than 20% of all American eighth-graders miss three or more days of school per month, and this proportion is higher in large cities. A p chart will be useful. Proportions of "things going wrong" are often higher in business processes than in manufacturing, so that p charts are an important tool in business.

Control limits for p charts

We studied the sampling distribution of a sample proportion $\hat{p}$ in Chapter 19. The center line and control limits for a 3σ control chart follow directly from the facts stated there, in the box on text page 502. We ought to call such charts "$\hat{p}$ charts" because they plot sample proportions. Unfortunately, they have always been called p charts in quality control circles. We will keep the traditional name but also keep our usual notation: p is a *process* proportion and $\hat{p}$ is a *sample* proportion.

p CHART USING PAST DATA

Take regular samples from a process that has been in control. Estimate the process proportion p of "successes" by

$$\overline{p} = \frac{\text{total number of successes in past samples}}{\text{total number of individuals in these samples}}$$

The center line and control limits for a **p chart** for future samples of size n are

$$\text{UCL} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

$$\text{CL} = \overline{p}$$

$$\text{LCL} = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

Common **out-of-control signals** are one sample proportion $\hat{p}$ outside the control limits or a run of 9 sample proportions on the same side of the center line.

If we have k past samples of the *same* size n, then $\overline{p}$ is just the average of the k sample proportions. In some settings, you may meet samples of unequal size—differing numbers of students enrolled in a month or differing numbers of parts inspected in a shift. The average $\overline{p}$ estimates the process proportion p even when the sample sizes vary. Note that the control limits use the actual size n of a sample.

EXAMPLE 26.13 Reducing absenteeism

Unscheduled absences by clerical and production workers are an important cost in many companies. You have been asked to improve absenteeism in a production facility where 12% of the workers are now absent on a typical day.

Start with data: the Pareto chart in Figure 26.15 shows that there are major differences among supervisors in the absenteeism rate of their workers. You retrain all the supervisors in human relations skills, using B, E, and H as discussion leaders. In addition, a trainer works individually with supervisors I and D. You also improve lighting and other work conditions.

Are your actions effective? You hope to see a reduction in absenteeism. To view progress (or lack of progress), you will keep a p chart of the proportion of absentees. The plant has 987 production workers. For simplicity, you just record the number who are absent from work each day. Only unscheduled absences count, not planned time off such as vacations. Each day you will plot

$$\hat{p} = \frac{\text{number of workers absent}}{987}$$

You first look back at data for the past three months. There were 64 workdays in these months. The total workdays available for the workers was

$$(64)(987) = 63{,}168 \text{ person-days}$$

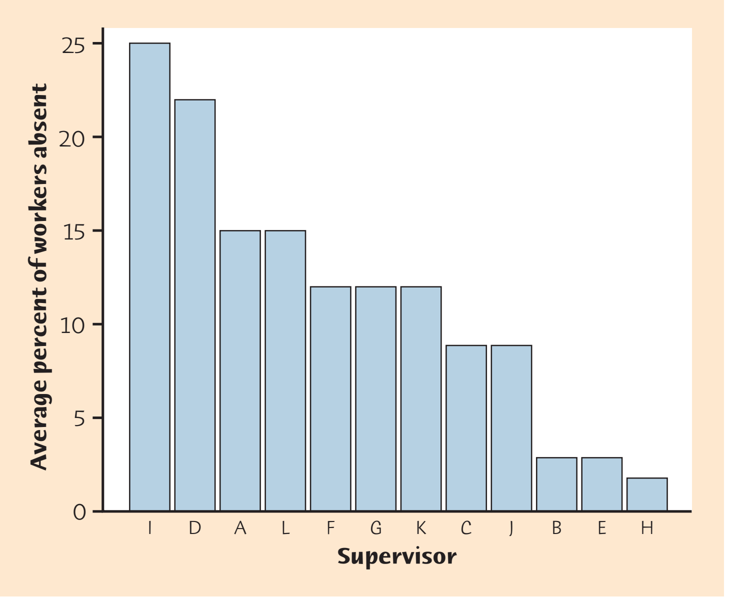


FIGURE 26.15
Pareto chart of the average absenteeism rate for workers reporting to each of 12 supervisors.

Absences among all workers totaled 7580 person-days. The average daily proportion absent was therefore

$$\overline{p} = \frac{\text{total days absent}}{\text{total days available for work}}$$

$$= \frac{7580}{63{,}168} = 0.120$$

The daily rate has been in control at this level.

These past data allow you to set up a p chart to monitor future proportions absent:

$$\text{UCL} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.120 + 3\sqrt{\frac{(0.120)(0.880)}{987}}$$

$$= 0.120 + 0.031 = 0.151$$

$$\text{CL} = \overline{p} = 0.120$$

$$\text{LCL} = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.120 - 3\sqrt{\frac{(0.120)(0.880)}{987}}$$

$$= 0.120 - 0.031 = 0.089$$

Table 26.8 gives the data for the next four weeks. Figure 26.16 is the p chart. ■

TABLE 26.8 Proportions of workers absent during four weeks

	M	T	W	Th	F	M	T	W	Th	F
Workers absent	129	121	117	109	122	119	103	103	89	105
Proportion $\hat{p}$	0.131	0.123	0.119	0.110	0.124	0.121	0.104	0.104	0.090	0.106

	M	T	W	Th	F	M	T	W	Th	F
Workers absent	99	92	83	92	92	115	101	106	83	98
Proportion $\hat{p}$	0.100	0.093	0.084	0.093	0.093	0.117	0.102	0.107	0.084	0.099

Figure 26.16 shows a clear downward trend in the daily proportion of workers who are absent. Days 13 and 19 lie below LCL, and a run of 9 days below the center line is achieved at Day 15 and continues. The points marked "x" are therefore all out of control. It appears that a special cause (the various actions you took) has reduced the absenteeism rate from around 12% to around 10%. The last two weeks' data suggest that the rate has stabilized at this level. You will update the chart based on the new data. If the rate does not decline further (or even rises again as the effect of your actions wears off), you will consider further changes.

Example 26.13 is a bit oversimplified. The number of workers available did not remain fixed at 987 each day. Hirings, resignations, and planned vacations change the number a bit from day to day. The control limits for a day's $\hat{p}$ depend on n, the number of workers that day. If n varies, the control limits will move in and out

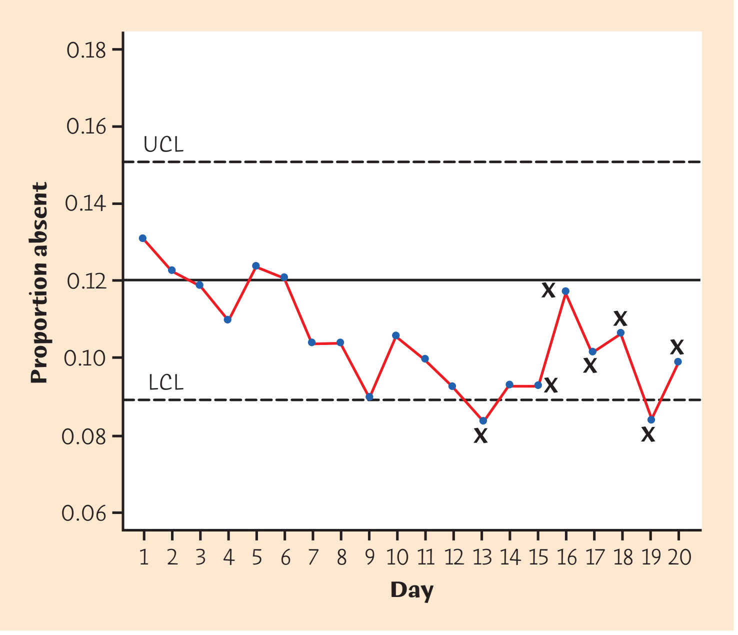


FIGURE 26.16

p chart for daily proportion of workers absent over a four-week period. The lack of control shows an improvement (decrease) in absenteeism. Update the chart to continue monitoring the process.

from day to day. Software will do the extra arithmetic needed for a different n each day, but as long as the count of workers remains close to 987 the greater detail will not change your conclusion.

A single p chart for all workers is not the only, or even the best, choice in this setting. Because of the important role of supervisors in absenteeism, it would be wise to also keep separate p charts for the workers under each supervisor. These charts may show that you must reassign some supervisors.

APPLY YOUR KNOWLEDGE

26.31 Setting up a *p* chart. After inspecting Figure 26.16, you decide to monitor the next four weeks' absenteeism rates using a center line and control limits calculated from the second two weeks' data recorded in Table 26.8. Find $\overline{p}$ for these 10 days and give the new values of CL, LCL, and UCL. (Until you have more data, these are trial control limits. As long as you are taking steps to improve absenteeism, you have not reached the process-monitoring stage.)

26.32 Unpaid invoices. The controller's office of a corporation is concerned that invoices that remain unpaid after 30 days are damaging relations with vendors. To assess the magnitude of the problem, a manager searches payment records for invoices that arrived in the past 10 months. The average number of invoices is 2875 per month, with relatively little month-to-month variation. Of all these invoices, 960 remained unpaid after 30 days.

(a) What is the total number of invoices studied? What is $\overline{p}$?

(b) Give the center line and control limits for a p chart on which to plot the future monthly proportions of unpaid invoices.

26.33 Lost baggage. The Department of Transportation reports that about 1 of every 200 passengers on domestic flights of the 10 largest U.S. airlines files a report of mishandled baggage. Starting with this information, you plan to sample records for 1000 passengers per day at a large airport to monitor the effects of efforts to reduce mishandled baggage. What are the initial center line and control limits for a chart of the daily proportion of mishandled-baggage reports? (You will find that LCL < 0. Because proportions $\hat{p}$ are always 0 or positive, take LCL $= 0$.)

26.34 Aircraft rivets. After completion of an aircraft wing assembly, inspectors count the number of missing or deformed rivets. There are hundreds of rivets in each wing, but the total number varies depending on the aircraft type. Recent data for wings with a total of 34,700 rivets show 208 missing or deformed. The next wing contains 1070 rivets. What are the appropriate center line and control limits for plotting the $\hat{p}$ from this wing on a p chart?

26.35 School absenteeism. Here are data from an urban school district on the number of eighth-grade students with three or more unexcused absences from school during each month of a school year. Because the total number of eighth-graders changes a bit from month to month, these totals are also given for each month.

	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
Students	911	947	939	942	918	920	931	925	902	883
Absent	291	349	364	335	301	322	344	324	303	344

(a) Find $\overline{p}$. Because the number of students varies from month to month, also find $\overline{n}$, the average per month.

(b) Make a p chart using control limits based on $\overline{n}$ students each month. Comment on control.

(c) The exact control limits are different each month because the number of students n is different each month. This situation is common in using p charts. What are the exact limits for October and June, the months with the largest and smallest n? Add these limits to your p chart, using short lines spanning a single month. Do exact limits affect your conclusions?

CHAPTER 26 SUMMARY

- Work is organized in **processes,** chains of activities that lead to some result. Use **flowcharts** and **cause-and-effect diagrams** to describe processes. Other graphs such as **Pareto charts** are often useful.
- All processes have variation. If the pattern of variation is stable over time, the process is **in statistical control. Control charts** are statistical plots intended to warn when a process is **out of control.**
- Standard **3σ control charts** plot the values of some statistic Q for regular samples from the process against the time order of the samples. The **center line** is at the mean of Q. The **control limits** lie three standard deviations of Q above and below the center line. A point outside the control limits is an **out-of-control signal.** For **process monitoring** of a process that has been in control, the mean

and standard deviation are based on past data from the process and are updated regularly.

- When we measure some quantitative characteristic of the process, we use **$\bar{x}$ and s charts** for process control. The s chart monitors variation within individual samples. If the s chart is in control, the $\bar{x}$ chart monitors variation from sample to sample. To interpret the charts, always look first at the s chart.
- An ***R* chart** based on the **range** of observations in a sample is often used in place of an s chart. Interpret $\bar{x}$ and R charts exactly as you would interpret $\bar{x}$ and s charts.
- It is common to use **out-of-control signals** in addition to "one point outside the control limits." In particular, a **runs signal** for the $\bar{x}$ chart allows the chart to respond more quickly to a gradual drift in the process center.
- **Control charts based on past data** are used at the **chart setup** stage for a process that may not be in control. Start with control limits calculated from the same past data that you are plotting. Beginning with the s chart, narrow the limits as you find special causes, and remove the points influenced by these causes. When the remaining points are in control, use the resulting limits to monitor the process.
- Statistical process control maintains quality more economically than inspecting the final output of a process. Samples that are **rational subgroups** are important to effective control charts. A process in control is stable, so that we can predict its behavior. If individual measurements have a Normal distribution, we can give the **natural tolerances.**
- A process is **capable** if it can meet the requirements placed on it. Control (stability over time) does not in itself improve capability. Remember that control describes the internal state of the process, whereas capability relates the state of the process to external specifications.
- There are control charts for several different types of process measurements. One important type is the ***p* chart** for sample proportions $\hat{p}$.
- The interpretation of p charts is very similar to that of $\bar{x}$ charts. The out-of-control signals used are also the same.

STATISTICS IN SUMMARY

Here are the most important skills you should have acquired from reading this chapter.

A. Processes

1. Describe the process leading to some desired output using flowcharts and cause-and-effect diagrams.
2. Choose promising targets for process improvement, combining the process description with data collection and tools such as Pareto charts.
3. Demonstrate understanding of statistical control, common causes, and special causes by applying these ideas to specific processes.

4. Choose rational subgroups for control charting based on an understanding of the process.

B. Control Charts

1. Make $\bar{x}$ and s charts using given values of the process μ and σ (usually from large amounts of past data) for monitoring a process that has been in control.
2. Demonstrate understanding of the distinction between short-term (within sample) and longer-term (across samples) variation by identifying possible $\bar{x}$-type and s-type special causes for a specific process.
3. Interpret $\bar{x}$ and s charts, starting with the s chart. Use both one-point-out and runs signals.
4. Estimate the process μ and σ from recent samples.
5. Set up initial control charts using recent process data, removing special causes, and basing an initial chart on the remaining data.
6. Decide when a p chart is appropriate. Make a p chart based on past data.

C. Process Capability

1. Know the distinction between control and capability and apply this distinction in discussing specific processes.
2. Give the natural tolerances for a process in control, after verifying Normality of individual measurements on the process.

CHAPTER 26 EXERCISES

26.36 Enlighten management. A manager who knows no statistics asks you, "What does it mean to say that a process is in control? Is being in control a guarantee that the quality of the product is good?" Answer these questions in plain language that the manager can understand.

26.37 Special causes. Is each of the following examples of a special cause most likely to first result in (i) a sudden change in level on the s or R chart, (ii) a sudden change in level on the $\bar{x}$ chart, or (iii) a gradual drift up or down on the $\bar{x}$ chart? In each case, briefly explain your reasoning.

(a) An airline pilots' union puts pressure on management during labor negotiations by asking its members to "work to rule" in doing the detailed checks required before a plane can leave the gate.

(b) Measurements of part dimensions that were formerly made by hand are now made by a very accurate laser system. (The process producing the parts does not change—measurement methods can also affect control charts.)

(c) Inadequate air conditioning on a hot day allows the temperature to rise during the afternoon in an office that prepares a company's invoices.

David Frazier/Stone/Getty Images

26.38 Deming speaks. The quality guru W. Edwards Deming (1900–1993) taught (among much else) that[14]

(a) "People work in the system. Management creates the system."

(b) "Putting out fires is not improvement. Finding a point out of control, finding the special cause and removing it, is only putting the process back to where it was in the first place. It is not improvement of the process."

(c) "Eliminate slogans, exhortations and targets for the workforce asking for zero defects and new levels of productivity."

Choose one of these sayings. Explain carefully what facts about improving quality the saying attempts to summarize.

26.39 Pareto charts. You manage the customer service operation for a maker of electronic equipment sold to business customers. Traditionally, the most common complaint is that equipment does not operate properly when installed, but attention to manufacturing and installation quality will reduce these complaints. You hire an outside firm to conduct a sample survey of your customers. Here are the percents of customers with each of several kinds of complaints:

Category	Percent
Accuracy of invoices	25
Clarity of operating manual	8
Complete invoice	24
Complete shipment	16
Correct equipment shipped	15
Ease of obtaining invoice adjustments/credits	33
Equipment operates when installed	6
Meeting promised delivery date	11
Sales rep returns calls	4
Technical competence of sales rep	12

(a) Why do the percents not add to 100%?

(b) Make a Pareto chart. What area would you choose as a target for improvement?

26.40 What type of chart? What type of control chart or charts would you use as part of efforts to improve each of the following performance measures in a corporate personnel office? Explain your choices.

(a) Time to get security clearance.

(b) Percent of job offers accepted.

(c) Employee participation in voluntary health screening.

26.41 What type of chart? What type of control chart or charts would you use as part of efforts to improve each of the following performance measures in a corporate information systems department? Explain your choices.

(a) Computer system availability.

(b) Time to respond to requests for help.

(c) Percent of programming changes not properly documented.

26.42 Purchased material. At the present time, about 5 lots out of every 1000 lots of material arriving at a plant site from outside vendors are rejected because they are incorrect. The plant receives about 300 lots per week. As part of an effort to reduce errors in the system of placing and filling orders, you will monitor the proportion of rejected lots each week. What type of control chart will you use? What are the initial center line and control limits?

26.43 Pareto charts. Painting new auto bodies is a multistep process. There is an "electrocoat" that resists corrosion, a primer, a color coat, and a gloss coat. A quality study for one paint shop produced this breakdown of the primary problem type for those autos whose paint did not meet the manufacturer's standards:

Problem	Percent
Electrocoat uneven—redone	4
Poor adherence of color to primer	5
Lack of clarity in color	2
"Orange peel" texture in color	32
"Orange peel" texture in gloss	1
Ripples in color coat	28
Ripples in gloss coat	4
Uneven color thickness	19
Uneven gloss thickness	5
Total	100

Make a Pareto chart. Which stage of the painting process should we look at first?

26.44 Milling. The width of a slot cut by a milling machine is important to the proper functioning of a hydraulic system for large tractors. The manufacturer checks the control of the milling process by measuring a sample of 5 consecutive items during each hour's production. The target width for the slot is $\mu = 0.8750$ inch. The process has been operating in control with center close to the target and $\sigma = 0.0012$ inch. What center line and control limits should be drawn on the s chart? On the $\overline{x}$ chart?

26.45 p charts are out of date. A manufacturer of consumer electronic equipment makes full use not only of statistical process control but of automated testing equipment that efficiently tests all completed products. Data from the testing equipment show that finished products have only 3.5 defects per million opportunities.

(a) What is $\overline{p}$ for the manufacturing process? If the process turns out 5000 pieces per day, how many defects do you expect to see per day? In a typical month of 24 working days, how many defects do you expect to see?

(b) What are the center line and control limits for a p chart for plotting daily defect proportions?

(c) Explain why a p chart is of no use at such high levels of quality.

26.46 Manufacturing isn't everything. Because the manufacturing quality in the previous exercise is so high, the process of writing up orders is the major source of quality problems: the defect rate there is 8000 per million opportunities. The manufacturer processes about 500 orders per month.

(a) What is $\overline{p}$ for the order-writing process? How many defective orders do you expect to see in a month?

(b) What are the center line and control limits for a p chart for plotting monthly proportions of defective orders? What is the smallest number of bad orders in a month that will result in a point above the upper control limit?

Table 26.9 gives process control samples for a study of response times to customer calls arriving at a corporate call center. A sample of 6 calls is recorded each shift for quality improvement purposes. The time from the first ring until a representative answers the call is recorded. Table 26.9 gives data for 50 shifts, 300 calls total.[15] *Exercises 26.47 to 26.49 make use of this setting.*

TABLE 26.9 Fifty control chart samples of call center response times

SAMPLE	TIME (SECONDS)						SAMPLE MEAN	STANDARD DEVIATION
1	59	13	2	24	11	18	21.2	19.93
2	38	12	46	17	77	12	33.7	25.56
3	46	44	4	74	41	22	38.5	23.73
4	25	7	10	46	78	14	30.0	27.46
5	6	9	122	8	16	15	29.3	45.57
6	17	17	9	15	24	70	25.3	22.40
7	9	9	10	32	9	68	22.8	23.93
8	8	10	41	13	17	50	23.2	17.79
9	12	82	97	33	76	56	59.3	32.11
10	42	19	14	21	12	44	25.3	14.08
11	63	5	21	11	47	8	25.8	23.77
12	12	4	111	37	12	24	33.3	39.76
13	43	37	27	65	32	3	34.5	20.32
14	9	26	5	10	30	27	17.8	10.98
15	21	14	19	44	49	10	26.2	16.29
16	24	11	10	22	43	70	30.0	22.93
17	27	10	32	96	11	29	34.2	31.71
18	7	28	22	17	9	24	17.8	8.42
19	15	14	34	5	38	29	22.5	13.03
20	16	65	6	5	58	17	27.8	26.63
21	7	44	14	16	4	46	21.8	18.49
22	32	52	75	11	11	17	33.0	25.88
23	31	8	36	25	14	85	33.2	27.45
24	4	46	23	58	5	54	31.7	24.29
25	28	6	46	4	28	11	20.5	16.34
26	111	6	3	83	27	6	39.3	46.34
27	83	27	2	56	26	21	35.8	28.88
28	276	14	30	8	7	12	57.8	107.20
29	4	29	21	23	4	14	15.8	10.34
30	23	22	19	66	51	60	40.2	21.22
31	14	111	20	7	7	87	41.0	45.82
32	22	11	53	20	14	41	26.8	16.56
33	30	7	10	11	9	9	12.7	8.59

TABLE 26.9 continued

SAMPLE	TIME (SECONDS)						SAMPLE MEAN	STANDARD DEVIATION
34	101	55	18	20	77	14	47.5	36.16
35	13	11	22	15	2	14	12.8	6.49
36	20	83	25	10	34	23	32.5	25.93
37	21	5	14	22	10	68	23.3	22.82
38	8	70	56	8	26	7	29.2	27.51
39	15	7	9	144	11	109	49.2	60.97
40	20	4	16	20	124	16	33.3	44.80
41	16	47	97	27	61	35	47.2	28.99
42	18	22	244	19	10	6	53.2	93.68
43	43	20	77	22	7	33	33.7	24.49
44	67	20	4	28	5	7	21.8	24.09
45	118	18	1	35	78	35	47.5	43.00
46	71	85	24	333	50	11	95.7	119.53
47	12	11	13	19	16	91	27.0	31.49
48	4	63	14	22	43	25	28.5	21.29
49	18	55	13	11	6	13	19.3	17.90
50	4	3	17	11	6	17	9.7	6.31

26.47 Rational subgroups? The 6 calls each shift are chosen at random from all calls received during the shift. Discuss the reasons behind this choice and those behind a choice to time 6 consecutive calls.

26.48 Chart setup. Table 26.9 also gives $\bar{x}$ and s for each of the 50 samples.

(a) Make an s chart and check for points out of control.

(b) If the s-type cause responsible is found and removed, what would be the new control limits for the s chart? Verify that no points s are now out of control.

(c) Use the remaining 46 samples to find the center line and control limits for an $\bar{x}$ chart. Comment on the control (or lack of control) of $\bar{x}$. (Because the distribution of response times is strongly skewed, $\bar{s}$ is large and the control limits for $\bar{x}$ are wide. Control charts based on Normal distributions often work poorly when measurements are strongly skewed.)

26.49 Using process knowledge. Three of the out-of-control values of s in part (a) of the previous exercise are explained by a single outlier, a very long response time to one call in the sample. What are the values of these outliers, and what are the s-values for the 3 samples when the outliers are omitted? (The interpretation of the data is, unfortunately, now clear. Few customers will wait 5 minutes for a call to be answered, as the customer whose call took 333 seconds to answer did. We suspect that other customers hung up before their calls were answered. If so, response time data for the calls that were answered don't adequately picture the quality of service. We should now look at data on calls lost before being answered to see a fuller picture.)

26.50 Doctors' prescriptions. A regional chain of retail pharmacies finds that about 1% of prescriptions it receives from doctors are incorrect or illegible. The chain puts in place a secure online system that doctors' offices can use to enter prescriptions directly. It hopes that fewer prescriptions entered online will be incorrect or illegible. A p chart will monitor progress. Use information about past prescriptions to set initial center line and control limits for the proportion of incorrect or illegible prescriptions on a day when the chain fills 75,000 online prescriptions. What are the center line and control limits for a day when only 50,000 online prescriptions are filled?

You have just installed a new system that uses an interferometer to measure the thickness of polystyrene film. To control the thickness, you plan to measure 3 film specimens every 10 minutes and keep $\bar{x}$ and s charts. To establish control, you measure 22 samples of 3 films each at 10-minute intervals. Table 26.10 gives $\bar{x}$ and s for these samples. The units are ten-thousandths of a millimeter. Exercises 26.51 to 26.53 are based on this chart setup setting.

TABLE 26.10 $\bar{x}$ and s for 22 samples of film thickness

SAMPLE	$\bar{x}$	s	SAMPLE	$\bar{x}$	s
1	848	20.1	12	823	12.6
2	832	1.1	13	835	4.4
3	826	11.0	14	843	3.6
4	833	7.5	15	841	5.9
5	837	12.5	16	840	3.6
6	834	1.8	17	833	4.9
7	834	1.3	18	840	8.0
8	838	7.4	19	826	6.1
9	835	2.1	20	839	10.2
10	852	18.9	21	836	14.8
11	836	3.8	22	829	6.7

26.51 s chart. Calculate control limits for s, make an s chart, and comment on control of short-term process variation.

26.52 $\bar{x}$ chart. Interviews with the operators reveal that in Samples 1 and 10 mistakes in operating the interferometer resulted in one high outlier thickness reading that was clearly incorrect. Recalculate $\bar{x}$ and s after removing Samples 1 and 10. Recalculate UCL for the s chart and add the new UCL to your s chart from the previous exercise. Control for the remaining samples is excellent. Now find the appropriate center line and control limits for an $\bar{x}$ chart, make the $\bar{x}$ chart, and comment on control.

26.53 Categorizing the output. Previously, control of the process was based on categorizing the thickness of each film inspected as satisfactory or not. Steady improvement in process quality has occurred, so that just 15 of the last 5000 films inspected were unsatisfactory.

(a) What type of control chart would be used in this setting, and what would be the control limits for a sample of 100 films?

(b) The chart in (a) is of little practical value at current quality levels. Explain why.

NOTES AND DATA SOURCES

1. CNNMoney, "My Golden Rule," at **money.cnn.com**, November 2005.

2. Texts on quality management give more detail about these and other simple graphical methods for quality problems. The classic reference is Kaoru Ishikawa, *Guide to Quality Control,* Asian Productivity Organization, 1986.

3. The flowchart and a more elaborate version of the cause-and-effect diagram for Example 26.1 were prepared by S. K. Bhat of the General Motors Technical Center as part of a course assignment at Purdue University.

4. For more information and references on DRGs, see the Wikipedia entry "diagnosis-related group." Search for this term at **en.wikipedia.org**.

5. The terms "chart setup" and "process monitoring" are adopted from Andrew C. Palm's discussion of William H. Woodall, "Controversies and contradictions in statistical process control," *Journal of Quality Technology,* 32 (2000), pp. 341–350. Palm's discussion appears in the same issue, pp. 356–360. We have combined Palm's stages B ("process improvement") and C ("process monitoring") when writing for beginners because the distinction between them is one of degree.

6. It is common to call these "standards given" $\bar{x}$ and s charts. We avoid this term because it easily leads to the common and serious error of confusing control limits (based on the process itself) with standards or specifications imposed from outside.

7. Provided by Charles Hicks, Purdue University.

8. See, for example, Chapter 3 of Stephen B. Vardeman and J. Marcus Jobe, *Statistical Quality Assurance Methods for Engineers,* Wiley, 1999.

9. The classic discussion of out-of-control signals and the types of special causes that may lie behind special control chart patterns is the *AT&T Statistical Quality Control Handbook,* Western Electric, 1956.

10. The data in Table 26.5 are adapted from data on viscosity of rubber samples appearing in Table P3.3 of Irving W. Burr, *Statistical Quality Control Methods,* Marcel Dekker, 1976.

11. The control limits for the s chart based on past data are commonly given as $B_4\bar{s}$ and $B_3\bar{s}$. That is, $B_4 = B_6/c_4$ and $B_3 = B_5/c_4$. This is convenient for users, but avoiding this notation minimizes the number of control chart

constants students must keep straight and emphasizes that process-monitoring and past-data charts are exactly the same except for the source of μ and σ.

12. Simulated data based on information appearing in Arvind Salvekar, "Application of six sigma to DRG 209," found at the Smarter Solutions Web site, `www.smartersolutions.com`.

13. Micheline Maynard, "Building success from parts," *New York Times*, March 17, 2002.

14. The first two Deming quotes are from *Public Sector Quality Report*, December 1993, p. 5. They were found online at `deming.eng.clemson.edu/pub/den/files/demqtes.txt`. The third quote is part of the 10th of Deming's "14 points of quality management," from his book *Out of the Crisis*, MIT Press, 1986.

15. The data in Table 26.9 are simulated from a probability model for call pickup times. That pickup times for large financial institutions have median 20 seconds and mean 32 seconds is reported by Jon Anton, "A case study in benchmarking call centers," Purdue University Center for Customer-Driven Quality, no date.