Problem 2

(a)

$$\int \frac{x^4+x^3-x^2+x+1}{x^2+x-2}dx = \int \frac{x^4+x^3-x^2+x+1}{(x-1)(x+2)}dx = \int \left(x^2+1-\frac{1}{x+2}+\frac{1}{x-1}\right)dx =$$
$$= \int x^2dx + \int xdx - \int \frac{1}{x+2}dx + \int \frac{1}{x-1}dx = \frac{x^3}{3}+\frac{x^2}{2} - \ln|x+2| + \ln|x-1| + C$$

(b)

$$\int \frac{4x^4-x+1}{x^3-x}dx = \int \frac{4x^4-x+1}{x(x-1)(x+1)}dx = \int \left(4x - \frac{1}{x} + \frac{2}{x-1} + \frac{3}{x+1}\right)dx =$$
$$= 4\cdot\int xdx - \int \frac{1}{x}dx + 2\cdot\int\frac{1}{x-1}dx + 3\cdot\int\frac{1}{x+1}dx = 2x^2 - \ln|x| + 2\ln|x-1| + 3\ln|x+1| + C$$

(c)

$$\int \frac{x+2}{x^2+6x+10}dx = \int \frac{\frac{1}{2}(2x+6)-1}{x^2+6x+10}dx = \frac{1}{2}\cdot\int\frac{2x+6}{(x+3)^2+1} - \int\frac{1}{x^2+6x+10}dx =$$
$$= [t = x^2+6x+10, \implies dt = t'dx = (2x+6)dx] \iff [t_1 = x+3, \implies dt = t_1'dx = dx] =$$
$$= \frac{1}{2}\cdot\int\frac{dt}{t} - \int\frac{dt}{t^3+1} = \frac{1}{2}\ln|t| - \arctan t + C = \frac{1}{2}\ln|x^2+6x+10| - \arctan{(x+3)} + C$$

Problem 6

(a)

$$\int \frac{2x+11}{x^2-6x+5}dx = \int\frac{2x+11}{(x-1)(x-5)}dx = \int\left(-\frac{13}{4(x-1)} + \frac{21}{4(x-5)}\right)dx =$$
$$= -\frac{13}{4}\cdot\int\frac{1}{x-1}dx + \frac{21}{4}\cdot\int\frac{1}{x-5}dx = -\frac{13}{4}\ln|x-1| + \frac{21}{4}\ln|x-5| + C$$

(b)

$$\int\frac{3x-7}{x^2+8x+19}dx = \int\frac{\frac{3}{2}\cdot(2x+8)-19}{x^2+8x+19}dx = \int\left(\frac{\frac{3}{2}\cdot(2x+8)}{x^2+8x+19} - \frac{19}{x^2+8x+19}\right)dx =$$
$$= \frac{3}{2}\cdot\int\frac{2x+8}{x^2+8x+19}dx - 19\cdot\int\frac{1}{(x+4)^2+3}dx =$$
$$= [t = x^2+8x+19, \implies dt = t'dx = (2x+8)dx] \iff [t_1 = x+4, \implies dt_1 = t_1'dx = dx] =$$
$$= \frac{3}{2}\cdot\int\frac{dt}{t} - 19\cdot\int\frac{dt}{t^2+(\sqrt{3})^2} = \frac{3}{2}\ln|t| - 19\cdot\frac{1}{\sqrt{3}}\cdot\arctan\frac{t}{\sqrt{3}} =$$
$$= \frac{3}{2}\ln|x^2+8x+19| - 19\cdot\frac{1}{\sqrt{3}}\cdot\arctan\frac{x+4}{\sqrt{3}} + C$$

(c)

$$\int \frac{5x+1}{\sqrt{1+2x-x^2}}dx = \int \frac{5((x-1)+1)+1}{\sqrt{2-(x-1)^2}}dx = [t=(x-1),\implies dt = t'dx = dx] =$$
$$= \int \frac{5(t+1)+1}{\sqrt{2-t^2}} = \int \frac{5t+6}{\sqrt{2-t^2}}dt = 5\cdot\int \frac{t}{\sqrt{2-t^2}}dt + 6\cdot\int \frac{1}{\sqrt{2-t^2}}dt =$$
$$= [z = 2-t^2,\implies dz = z'dt = -2tdt] = -\frac{5}{2}\cdot\int \frac{dz}{\sqrt{z}} + 6\cdot\int \frac{1}{\sqrt{2-t^2}}dt =$$
$$= -5\cdot\sqrt{2-t^2} + 6\cdot\arcsin\frac{t}{\sqrt{2}} + C = -5\cdot\sqrt{2-(x-1)^2} + 6\arcsin\frac{x-1}{\sqrt{2}} + C$$

Problem 4

(a)

$$\int \frac{x^2-3x-4}{x^3-4x^2+4x}dx = \int \frac{x^2-3x-4}{x(x-2)^2}dx = \int\left(-\frac{1}{x}+\frac{2}{x-2}-\frac{3}{(x-2)^2}\right)dx =$$
$$= -1\cdot\int\frac{1}{x}dx + 2\cdot\int\frac{1}{x-2} - 3\cdot\int\frac{1}{(x-2)^2}dx = -\ln|x| + 2\ln|x-2| + \frac{3}{x-2} + C$$

(b)

$$\int \frac{2x^2-4x-8}{(x^2-x)(x^2+4)}dx = \int \frac{2x^2-4x-8}{x(x-1)(x^2+4)}dx = \int\left(\frac{2}{x}-\frac{2}{x-1}+\frac{4}{x^2+4}\right)dx =$$
$$= 2\cdot\int\frac{1}{x}dx - 2\cdot\int\frac{1}{x-1}dx + 4\cdot\int\frac{1}{x^2+4}dx = 2\ln|x| - 2\ln|x-1| + 2\arctan\frac{x}{2} + C$$