Problem 2

(a)

$$\int \left[(x^2+3x+2)\cos 4x\right] dx \implies [U=(x^2+3x+2), dU=(2x+3)dx \iff dV=\cos 4x dx, V=\frac{\sin 4x}{4}] \implies$$

$$\implies \frac{1}{4}(x^2+3x+2)\sin 4x - \frac{1}{4}\cdot\int \left[\sin 4x(2x+3)\right] dx \implies$$

$$\implies [U=2x+3, dU=2dx \iff dV=\sin 4x dx, V=-\frac{1}{4}\cos 4x] \implies$$

$$\implies \frac{1}{4}(x^2+3x+2)\sin 4x - \frac{1}{4}\cdot\left(-\frac{1}{4}(2x+3)\cos 4x+\frac{1}{2}\int\left[\cos 4x\right]dx\right)=$$

$$=\frac{1}{4}(x^2+3x+2)\sin 4x - \frac{1}{4}\cdot\left(-\frac{1}{4}(2x+3)\cos 4x+\frac{1}{8}\sin 4x\right)+C\in\mathbb{R}$$

(b)

$$\int\left[\frac{x}{\cos^2 x}\right]dx \implies [U=x, dU=dx \iff dV=\frac{1}{\cos^2 x}dx, V=\tan x] \implies x\cdot\tan x-\int\left[\tan x\right]dx=$$

$$=x\cdot\tan x+\ln|\cos x|+C\in\mathbb{R}$$

(c)

$$\int\left[x\ln(x+1)\right]dx \implies [U=\ln(x+1), dU=\frac{1}{x+1}dx \iff dV=xdx, V=\frac{x^2}{2}] \implies$$

$$\implies \frac{x^2\ln(x+1)}{2}-\frac{1}{2}\int\left[\frac{x^2}{x+1}\right]dx \implies -\frac{1}{2}\cdot\int\left[\frac{x^2}{x+1}\right]dx=[t=x+1, dt=dx]=-\frac{1}{2}\cdot\int\left[\frac{(t-1)^2}{t}\right]dt=$$

$$=-\frac{1}{2}\cdot\int\left[t-2+\frac{1}{t}\right]dt=\frac{-x^2+2x+3}{4}-\frac{\ln|x+1|}{2}+C\in\mathbb{R}\implies$$

$$\implies \frac{x^2\ln(x+1)}{2}+\frac{-x^2+2x+3}{4}-\frac{\ln|x+1|}{2}+C\in\mathbb{R}$$

(d)

$$\int\left[\frac{\arcsin\sqrt{x}}{\sqrt{1-x}}\right]dx \implies [U=\arcsin\sqrt{x}, dU=\frac{1}{2\sqrt{x}\sqrt{1-x}} \iff dV=\frac{1}{\sqrt{1-x}}dx, V=-2\sqrt{1-x}] \implies$$

$$\implies -2\sqrt{1-x}\arcsin\sqrt{x}+\int\left[\frac{1}{\sqrt{x}}\right]dx=-2\sqrt{1-x}\arcsin\sqrt{x}+2\sqrt{x}+C\in\mathbb{R}$$

(e)

$$\int\left[\arctan x\right]dx \implies [U=\arctan x, dU=\frac{1}{1+x^2}dx \iff dV=dx, V=x] \implies x\arctan x-\int\left[\frac{x}{x^2+1}\right]dx \implies$$

$$\implies \int\left[\frac{x}{x^2+1}\right]dx=[t=x^2+1, dt=2x]=-\frac{1}{2}\cdot\int\left[\frac{1}{t}\right]dt=-\frac{\ln|x^2+1|}{2}+C\in\mathbb{R}\implies$$

$$\implies x\arctan x-\frac{\ln|x^2+1|}{2}+C\in\mathbb{R}$$

(g)

$$\int [\exp(2x)\cdot\sin(3x)]\,dx \implies [U=\sin 3x, dU = 3\cos 3x dx \iff dV = \exp(2x)dx, V=\frac{\exp(2x)}{2}] \implies$$

$$\implies \frac{\exp(2x)\cdot\sin 3x}{2} - \frac{3}{2}\int [\exp(2x)\cdot\cos 3x]\,dx \implies$$

$$\implies [U=\cos 3x, dU=-3\sin 3x dx \iff dV=\exp(2x)dx, V=\frac{\exp(2x)}{2}] \implies$$

$$\implies \frac{\exp(2x)\cdot\sin 3x}{2} - \frac{3}{2}\cdot\left(\frac{\exp(2x)\cdot\cos 3x}{2} + \frac{3}{2}\cdot\int [\exp(2x)\cdot\sin 3x]\,dx\right) \implies [I=\int [\exp(2x)\cdot\sin 3x]\,dx] \implies$$

$$\implies I = \frac{\exp(2x)\cdot\sin 3x}{2} - \frac{3}{2}\cdot\left(\frac{\exp(2x)\cdot\cos 3x}{2} + \frac{3}{2}\cdot I\right) = \frac{\exp(2x)\cdot\sin 3x}{2} - \frac{3\exp(2x)\cdot\cos 3x}{4} - \frac{13}{4}I \implies$$

$$\implies I = \frac{4}{13}\cdot\left(\frac{\exp(2x)\cdot\sin 3x}{2} - \frac{3\exp(2x)\cdot\cos 3x}{4}\right) + C \in \mathbb{R}$$

Problem 4

(a)

$$\int \left[\frac{\ln\ln x}{x}\right] dx \implies [U=\ln\ln x, dU=\frac{1}{x\ln x}\cdot dx \iff dV=\frac{dx}{x}, V=\ln x] \implies$$

$$\implies \ln\ln x\cdot\ln x - \int\left[\frac{1}{x}\right]dx = \ln\ln x\cdot\ln x - \ln|x| + C \in \mathbb{R}$$

(b)

$$\int\left[\frac{x}{1+x^4}\right]dx \implies [t=1+x^4, \implies dt=4x^3dx] \implies \frac{1}{4}\cdot\int\left[\frac{1}{t}\right]dt = \frac{1}{4}\cdot\ln|t| + C = \frac{1}{4}\cdot\ln|1+x^4| + C \in \mathbb{R}$$