Problem 2

(a)

$$\int (x+1)(3x-2)dx = \int (3x^2+x-2)dx = 3\cdot\int x^2dx + \int xdx - 2\cdot\int 1\cdot dx =$$
$$= 3\cdot\frac{x^3}{3} + \frac{x^2}{2} - 2x + C = x^3 + \frac{x^2}{2} - 2x + C.$$

(b)

$$\int \frac{4}{6+x^2}dx = 4\cdot\int\frac{1}{6+x^2}dx = 4\cdot\int\frac{1}{x^2+(\sqrt{6})^2}dx = 4\cdot\frac{1}{\sqrt{6}}\cdot\arctan\left(\frac{x}{\sqrt{6}}\right) + C.$$

(c)

$$\int\frac{\cos 2x}{\sin^2 x\cdot\cos^2 x}dx = \int\frac{\cos^2 x - \sin^2 x}{\sin^2 x\cdot\cos^2 x} = \int\left(\frac{\cos^2 x}{\sin^2 x\cdot\cos^2 x} - \frac{\sin^2 x}{\sin^2 x\cdot\cos^2 x}\right)dx =$$
$$= \int\left(\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}\right)dx = \int\frac{1}{\sin^2 x}dx - \int\frac{1}{\cos^2 x}dx = -\cot x - \tan x + C.$$

(d)

$$\int x^{\frac{7}{8}}\cdot(2x^{\frac{3}{5}} - 3x^{-2} + \sqrt{x})dx = \int(2x^{\frac{59}{40}} - 3x^{-\frac{9}{8}} + x^{\frac{11}{8}})dx = 2\int x^{\frac{59}{40}}dx - 3\cdot\int x^{-\frac{9}{8}}dx + \int x^{\frac{11}{8}}dx =$$
$$= \frac{80x^2x^{\frac{19}{40}}}{99} + \frac{24}{x^{\frac{1}{8}}} + \frac{8x^2x^{\frac{3}{8}}}{19} + C = \frac{80\cdot\sqrt[40]{x^{99}}}{99} + \frac{24}{\sqrt[8]{x}} + \frac{8\cdot\sqrt[8]{x^{19}}}{19} + C.$$

(e)

$$\int\frac{\sqrt{x} - x^3\cdot e^x + x^2}{x^3}dx = \int\left(\frac{\sqrt{x}}{x^3} - e^x + x^{-1}\right)dx = \int x^{-\frac{5}{2}} - \int e^x dx + \int x^{-1}dx =$$
$$= -\frac{x^{-1,5}}{1,5} - e^x + \ln x + C.$$

(f)

$$\int\frac{x\sin 2x + \sqrt[5]{x^2}\cos x}{x\cos x}dx = \int\left(\frac{x\cdot 2\sin x\cos x}{x\cos x} + \frac{\sqrt[5]{x^2}\cos x}{x\cos x}\right)dx = \int(2\sin x + x^{-0,6})dx =$$
$$= 2\cdot\int\sin x\cdot dx + \int x^{-0,6}dx = -2\cos x + \frac{x^{0,4}}{0,4} + C.$$

(g)

$$\int\frac{\sqrt{3+x^2} - \sqrt{3-x^2}}{\sqrt{9-x^4}}dx = \int\left(\sqrt{\frac{3+x^2}{(3-x^2)(3+x^2)}} - \sqrt{\frac{3-x^2}{(3-x^2)(3+x^2)}}\right)dx =$$
$$= \int\left(\sqrt{\frac{1}{3-x^2}} - \sqrt{\frac{1}{3+x^2}}\right)dx = \int\left(\frac{1}{\sqrt{3-x^2}} - \frac{1}{\sqrt{3+x^2}}\right)dx = \arcsin\frac{x}{\sqrt{3}} - \ln|x + \sqrt{x^2+3}| + C.$$

(h)

$$\int (\tan x + \cot x)^2 dx = \int \left(\tan x + \frac{1}{\tan x}\right)^2 dx = \int \left(\tan^2 x + 2 + \frac{1}{\tan^2 x}\right) dx =$$
$$= \int \tan^2 x + 2 \cdot \int dx + \int \frac{1}{\tan^2 x} dx = \tan x - x + 2x - \cot x - x + C = \tan x - \cot x + C.$$

(i)

$$\int \frac{1 + 2x^2}{x^2(1 + x^2)} dx = \int \left(\frac{1}{x^2} + \frac{1}{1 + x^2}\right) dx = \int \frac{1}{x^2} dx + \int \frac{1}{x^2 + 1} dx = -\frac{1}{x} + \arctan x + C.$$

Problem 4

(a)

$$\int \frac{4x + 5}{2x^2 + 5x - 6} dx = \left[t = 2x^2 + 5x - 6, \implies dt = t' dx = (4x + 5) dx\right] =$$
$$= \int \frac{dt}{t} = \ln|t| + C = \ln|2x^2 + 5x - 6| + C.$$

(b)

$$\int \frac{\cos 2x}{\sin x \cdot \cos x} dx = \int \left(-\frac{\sin^2 x}{\sin x \cdot \cos x} + \frac{\cos^2 x}{\sin x \cdot \cos x}\right) dx =$$
$$= -1 \cdot \int \tan x dx + \int \cot x dx = \ln|\cos x| + \ln|\sin x| + C.$$

(c)

$$\int \frac{x^3}{\sqrt{1 - x^8}} dx = \left[t = x^4, \implies dt = t' dx = 4x^3 dx\right] = \int \frac{dt}{4\sqrt{1 - t^2}} = \frac{1}{4} \cdot \int \frac{dt}{\sqrt{1 - t^2}} =$$
$$= \frac{1}{4} \cdot \arcsin t + C.$$

(d)

$$\int \frac{\arctan x}{1 + x^2} dx = \left[t = \arctan x, \implies dt = t' dx = \frac{1}{1 + x^2} dx\right] = \int t dt = \frac{t^2}{2} + C = \frac{\arctan^2 x}{2} + C.$$

(e)

$$\int e^{\cos x} \cdot \sin x dx = [t = \cos x, \implies dt = t' dx = -\sin x dx] = \int -e^t dt = -e^t + C = -e^{\cos x} + C.$$

(f)

$$\int \frac{\cos \ln x}{x} dx = \left[t = \ln x, \implies dt = t' dx = \frac{1}{x} dx\right] = \int \cos t dt = \sin t + C = \sin \ln x + C.$$

Problem 6

(a)

$$\int e^{7x-2}dx = [t = 7x - 2, \implies dt = t'dx = 7 \cdot dx] = \int \frac{e^t}{7}dt = \frac{1}{7} \cdot \int e^t dt = \frac{e^{7x-2}}{7} + C.$$

(b)

$$\int (4-3x)^{3,5}dx = [t = 4 - 3x, \implies dt = t'dx = -3dx] = \int \frac{t^{3,5}}{3}dt = \frac{1}{3} \cdot \int t^{3,5}dt =$$

$$= \frac{1}{3} \cdot \frac{t^{4,5}}{4,5} + C = \frac{(4-3x)^{4,5}}{13,5} + C.$$

(c)

$$\int \frac{dx}{3 + (2x+5)^2} = [t = 2x + 5, \implies dt = t'dx = 2dx] = \int \frac{dt}{2(3+t^2)} = \frac{1}{2} \cdot \int \frac{dt}{3+t^2} = \frac{t}{2\sqrt{3}} \cdot \arctan \frac{t}{\sqrt{3}} + C =$$

$$= \frac{1}{2\sqrt{3}} \cdot \arctan \frac{2x+5}{\sqrt{3}} + C.$$