Problem 2

(a)

$$\int \left[\cos^5(7x+3)\right] dx = [t = (7x+3), \implies dt = 7dx] = \frac{1}{7} \cdot \int \left[\cos^5 t\right] dt = \frac{1}{7} \cdot \int \left[\cos^4 t \cdot \cos t\right] dt =$$

$$= [U = \sin t, \implies dU = \cos t dt] = \frac{1}{7} \cdot \int \left[(1-U^2)^2\right] dU = \frac{1}{7} \cdot \int dU - \frac{2}{7} \cdot \int \left[U^4\right] dU + \frac{1}{7} \cdot \int \left[U^4\right] dU =$$

$$= \frac{U}{7} - \frac{2 \cdot U^3}{21} + \frac{U^5}{35} + C = \frac{\sin(7x+3)}{7} - \frac{2 \cdot \sin^3(7x+3)}{21} + \frac{\sin^5(7x+3)}{35} + C \in \mathbb{R}$$

(b)

$$\int \left[\cosh^4 x\right] dx = \int \left[\left(\frac{1+\cosh 2x}{2}\right)^2\right] dx = \int \left[\frac{3 + 4 \cdot \cosh 2x + \cosh 4x}{8}\right] dx =$$

$$= \frac{3}{8} \cdot \int dx + \frac{1}{2} \cdot \int \left[\cosh 2x\right] dx + \frac{1}{8} \cdot \int \left[\cosh 4x\right] dx = \frac{3}{8} \cdot x + \frac{\sinh 2x}{4} + \frac{\sinh 4x}{32} + C \in \mathbb{R}$$

(c)

$$\int \left[\sin^4 2x \cdot \cos^2 2x\right] dx = \int \left[(1-\cos^2 2x)^2 \cdot \cos^2 2x\right] dx = \int \left[\cos^2 2x - 2 \cdot \cos^4 2x + \cos^6 2x\right] dx$$

$$= \int \left[\cos^2 2x\right] dx - 2 \cdot \int \left[\cos^4 2x\right] dx + \cdot \int \left[\cos^6 2x\right] dx =$$

$$= \int \left[\frac{1+\cos 4x}{2}\right] dx - \int \left[\frac{3 + 4\cos 4x + \cos 8x}{8}\right] dx + \int \left[\frac{5 + 12\cos 4x + 6\cos 8x + \cos 12x}{32}\right] dx =$$

$$= -\frac{3x}{32} + \frac{\sin 4x}{16} + \frac{\sin 8x}{64} + \frac{\sin 12x}{384} + C \in \mathbb{R}$$

Problem 4

(a)

$$\int \left[\frac{\cos^3 x}{\sqrt{\sin x}}\right] dx = [t = \sin x, \implies dt = \cos x dx] = \int \left[\frac{(1-t^2)}{\sqrt{t}}\right] dt = \int \left[t^{-0.5}\right] dt - \int \left[t^{1.5}\right] dt =$$

$$= \frac{t^{0.5}}{0.5} - \frac{t^{2.5}}{2.5} + C = \frac{\sin^{0.5} x}{0.5} - \frac{\sin^{2.5} x}{2.5} + C \in \mathbb{R}$$

(b)

$$\int \left[\coth^3(2x+3)\right] dx = \int \left[\frac{\cosh^3(2x+3)}{\sinh^3(2x+3)}\right] dx = \int \left[\frac{\cosh(2x+3) + \cosh(2x+3)\cdot\sinh^2(2x+3)}{\sinh^3(2x+3)}\right] dx =$$

$$= \int \left[\frac{\cosh(2x+3)}{\sinh^3(2x+3)} + \frac{\cosh(2x+3)}{\sinh(2x+3)}\right] dx = \int \left[\frac{\cosh(2x+3)}{\sinh^3(2x+3)}\right] dx + \int \left[\frac{\cosh(2x+3)}{\sinh(2x+3)}\right] dx =$$

$$= [t = \sinh(2x+3), \implies dt = 2\cosh(2x+3)] =$$

$$= \frac{1}{2}\cdot\int \left[t^{-3}\right] dt + \frac{1}{2}\cdot\int \left[\frac{1}{t}\right] dt = -\frac{1}{4t^2} + \frac{\ln|t|}{2} + C = -\frac{1}{4\cdot\sinh^2(2x+3)} + \frac{\ln|\sinh(2x+3)|}{2} + C \in \mathbb{R}$$

(c)

$$\int \left[\cot^4(2-x)\right] dx = \int \left[\left(\frac{1}{\sin^2(2-x)} - 1\right)^2\right] dx = \int \left[\frac{1}{\sin^4(2-x)} - \frac{1}{\sin^2(2-x)} + 1\right] dx =$$

$$= \int \left[\frac{1}{\sin^4(2-x)}\right] dx - \int \left[\frac{1}{\sin^2(2-x)}\right] dx + \int dx = [t = 2-x, \implies dt = -dx] =$$

$$= 3\cot t + \frac{\cot^3 t}{3} - t + C = 3\cot(2-x) + \frac{\cot^3(2-x)}{3} - (2-x) + C \in \mathbb{R}$$

(d)

$$\int [\sin 5x\cdot\sin 7x]\, dx = \frac{1}{2}\int [\cos 2x - \cos 7x]\, dx = \frac{1}{2}\left(\int [\cos 2x]\, dx - \int [\cos 12x]\, dx\right) = \frac{\sin 2x}{4} - \frac{\sin 12x}{24} + C \in \mathbb{R}$$

(e)

$$\int \left[\frac{\sin x}{1 + 5\cos x}\right] dx = [t = 1 + 5\cos x, \implies dt = -5\sin x] = -\frac{1}{5}\cdot\int \left[\frac{1}{t}\right] dx = -\frac{\ln|1+5\cos x|}{5} + C \in \mathbb{R}$$

Problem 6

Problem A

$$\int \left[\frac{1}{2\sin x + \cos x + 3}\right] dx$$

Using Universal Trigonometric Substitution we get:

$$t = \tan\frac{x}{2} \implies \left[\sin x = \frac{2t}{1+t^2};\ \cos x = \frac{1-t^2}{1+t^2};\ dt = \frac{2}{1+t^2}dt\right] \implies \left[\frac{1}{2\sin x + \cos x + 3} = \frac{2(t^2+2t+2)}{1+t^2}\right]$$

$$\int \left[\frac{1}{2\sin x + \cos x + 3}\right] dx = \int \left[\frac{\frac{2}{1+t^2}}{\frac{2(t^2+2t+2)}{1+t^2}}\right] dx = \int \left[\frac{1}{(t+1)^2+1}\right] dx = \arctan\left(\tan\frac{x}{2} + 1\right) + C \in \mathbb{R}$$

Problem B

$$\int \left[\frac{1}{3\sin x - 4\cos x}\right] dx$$

Using Universal Trigonometric Substitution we get:

$$t = \tan\frac{x}{2} \implies \left[\sin x = \frac{2t}{1+t^2};\ \cos x = \frac{1-t^2}{1+t^2};\ dt = \frac{2}{1+t^2}dt\right] \implies \left[\frac{1}{3\sin x - 4\cos x} = \frac{4t^2+6t-4}{t^2+1}\right]$$

$$\int \left[\frac{1}{3\sin x - 4\cos x}\right] dx = \int \left[\frac{1}{4t^2+6t-4}\right] dx = \frac{1}{2}\cdot\int \left[\frac{1}{(t+0.75)^2-(1.25)^2}\right] dx = \frac{2}{5}\ln\left|\frac{\tan\frac{x}{2} - 0.5}{\tan\frac{x}{2} + 2}\right| + C \in \mathbb{R}$$

Problem 7

(a)

$$\int \left[\frac{1}{\sqrt{1-x^2}\cdot\sqrt[7]{\arcsin x}}\right]dx = \left[t = \arcsin x, \implies dt = \frac{1}{\sqrt{1-x^2}}dx\right] = \int \left[\frac{1}{\sqrt[7]{t}}\right]dt =$$
$$= \frac{7}{6}\cdot t^{\frac{6}{7}} + C = \frac{7}{6}\cdot(\arcsin x)^{\frac{6}{7}} + C \in \mathbb{R}$$

(b)

$$\int \left[\frac{\cos\left(\log_8(5x+8)\right)}{x}\right]dx = \left[t = \log_8(5x+8), \implies dt = \frac{5}{(5x+8)\ln 8}dt, \implies \frac{dx}{x} = \ln 8\cdot dt\right] =$$
$$= \int\left[\cos t\cdot\ln 8\right]dt = \ln 8\cdot\sin\left(\log_8(5x+8)\right) + C \in \mathbb{R}$$

(c)

$$\int\left[\frac{2x+3}{\sqrt{15-6x-x^2}}\right]dx = \int\left[\frac{2x+3}{\sqrt{24-(x+3)^2}}\right]dx = [t = x-3, \implies dt = dx] = \int\left[\frac{2t-3}{\sqrt{24-t^2}}\right]dt =$$
$$= \int\left[\frac{2t}{\sqrt{24-t^2}}\right]dt - \int\left[\frac{3}{\sqrt{24-t^2}}\right]dt = -2\cdot\sqrt{24-t^2} - 3\arcsin\left(\frac{t}{\sqrt{24}}\right) + C =$$
$$= -2\cdot\sqrt{24-(x-3)^2} - 3\arcsin\left(\frac{x-3}{\sqrt{24}}\right) + C \in \mathbb{R}$$