Graphs

Sam Grayson

Cool school

January 27, 2017

This presentation is designed to be used in a review session for students already familiar with graphs but need to review the terminology. This is not designed to teach fundamental concepts of graphs.

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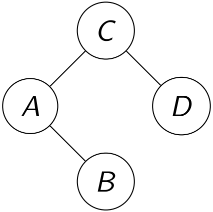
- We can draw graphs, but a drawing is just one interpretation of a graph
- Position doesn’t matter, vertex-names do

Undirected graph example

Example

$$G = (\quad \{A, B, C, D\}, \quad \{\{A, B\}, \{A, B\}, \{C, D\}\} \quad)$$

Remember the picture is just one way of thinking about the graph.



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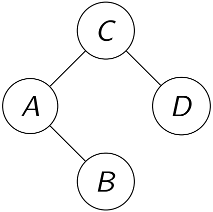

- Vertices:

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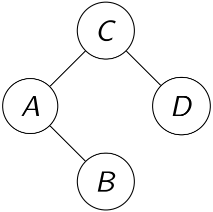

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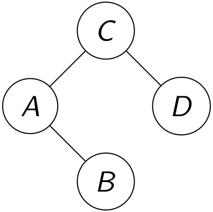

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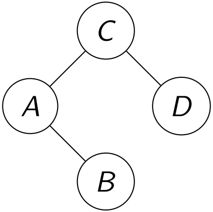

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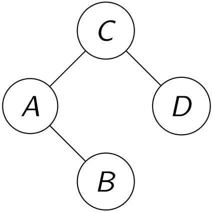

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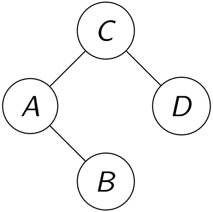

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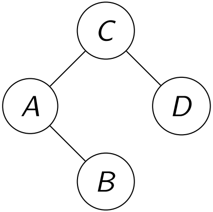

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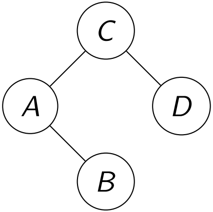

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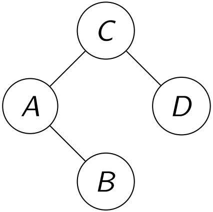

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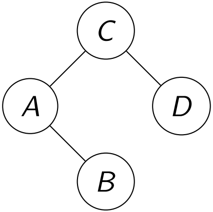

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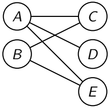

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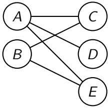

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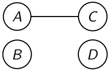

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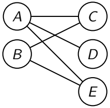

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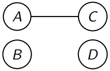

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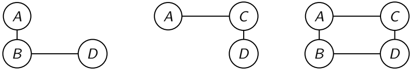
- Union: if $G_1 = (V_1, E_1), G_2 = (V_2, E_2)$ then $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$



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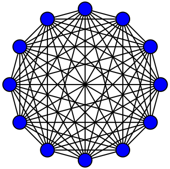

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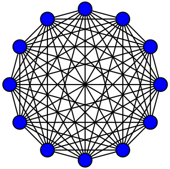
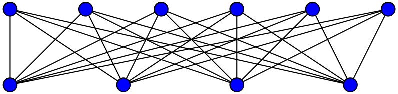


- Complete bipartite graph $K_{m,n}$: m vertices connected to all n vertices.



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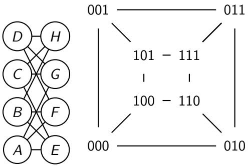

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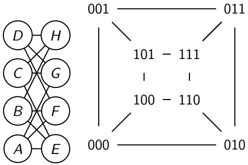
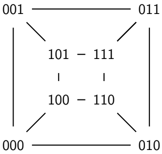


- *k*-coloring: Color a graph such that no two vertices with the same color are touching each other. Try to use as few colors as possible. Can you think of a graph that takes n colors to color?



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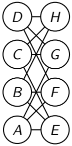

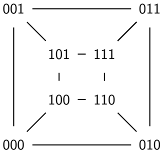

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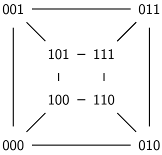

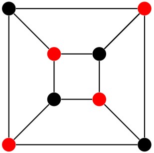

Handshaking lemma

Theorem

Given $G = (V, E)$

$$\sum_{v \in V} \deg v = 2E$$

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- $G = (V, E)$, just like old times

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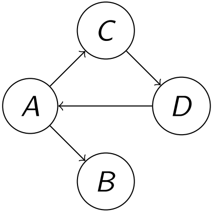

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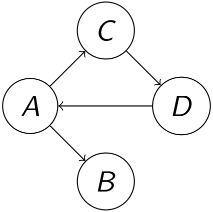

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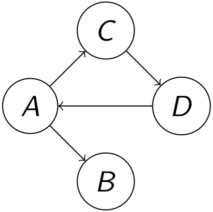

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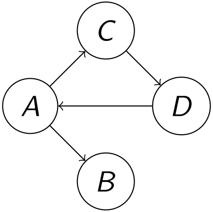

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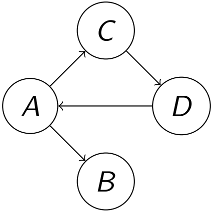

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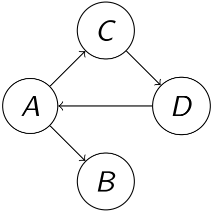

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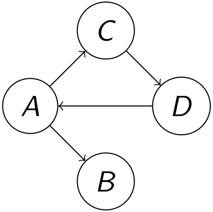

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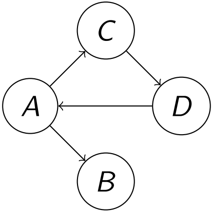

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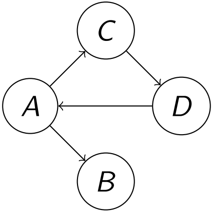

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- See a cycle? $[D, A, C, D]$ Path that starts where it stops.
  Paths can't go against the arrows

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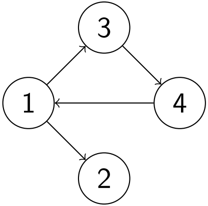

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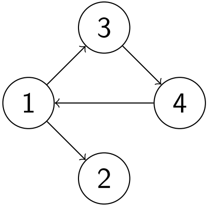

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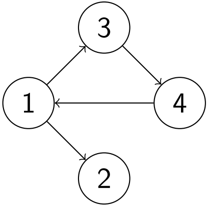

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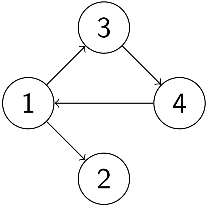

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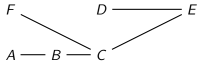

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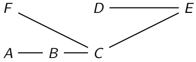

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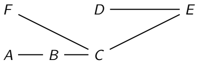

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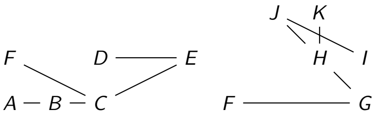

Don't cut down the trees

- Tree: a connected undirected graph with no cycles

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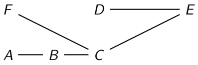

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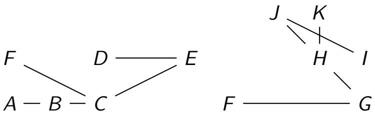

- Rooted tree: same, but one node is 'special'

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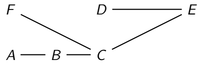

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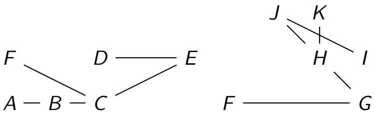

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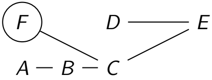

Our favorite kind of trees

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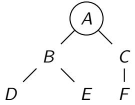

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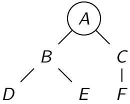

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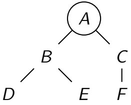

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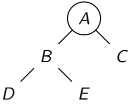

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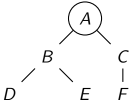

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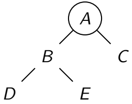

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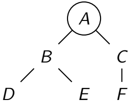

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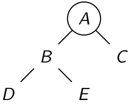

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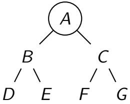

Dendrology

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- Subtree: subgraph that is also a tree
- Height: the longest path from the root to a leaf

Cool things trees can do

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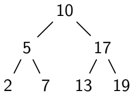
- Binary search



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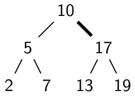

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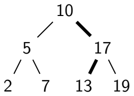

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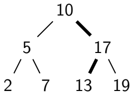

- 2 comparisons instead of 5 from linear search

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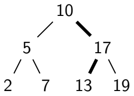

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- Huffman Coding