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ARTICLE

The negative theology of absolute infinity: Cantor, mathematics, and humility

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Abstract
Cantor argued that absolute infinity is beyond mathematical comprehension. His arguments imply that the domain of mathematics cannot be grasped by mathematical means. We argue that this inability constitutes a foundational problem. For Cantor, however, the domain of mathematics does not belong to mathematics, but to theology. We thus discuss the theological significance of Cantor's treatment of absolute infinity and show that it can be interpreted in terms of negative theology. Proceeding from this interpretation, we refer to the recent debate on absolute generality and argue that the method of diagonalization constitutes a modern version of the *via negativa*. On our reading, negative theology can evoke an attitude of humility with respect to the boundedness of the human condition. Along these lines, we think that the foundational problem of mathematics concerning its domain can be addressed through a methodological attitude of humility.

Keywords Georg Cantor · Absolute infinity · Negative theology · Absolute generality · Foundations of mathematics · Humility

Introduction

Cantor established mathematical theories of transfinite numbers and argued that, by contrast, absolute infinity is beyond mathematical comprehension. In the first part of the paper, we argue that Cantor's arguments lead to a foundational problem for mathematics. Given a suitable mathematical domain, we can always use Cantor's diagonal argument to transcend that domain, which implies that the domain

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of mathematics cannot be grasped using mathematical tools. However, due to his extensive engagement with metaphysics and theology, this conclusion was acceptable to Cantor. For him, the domain of mathematics does indeed not belong to mathematics, but to theology.

In the second part of the paper, we discuss the theological significance of Cantor's treatment of absolute infinity and argue that it can be interpreted in terms of negative theology. This interpretation seems to lead to a problem of incoherence though. Negative theology claims that God is beyond human comprehension, which itself appears to constitute a comprehensible assertion about God. For the same reason, claiming that absolute infinity cannot be comprehended seems to be self-defeating.

In order to resolve this problem, we argue in the third part of the paper that negative theology is not committed to asserting a negative claim but can be interpreted as a practice that performatively undermines our putative understanding of the absolute; it provides a *via negativa* that can yield an attitude of humility with respect to the boundedness of the human condition. Similarly, Cantor's method can be interpreted as performatively undermining the notion of absolute infinity and thereby yielding an attitude of humility that acknowledges the limits of mathematics.

Proceeding from this interpretation, in the fourth part of the paper we show that the incomprehensibility of absolute infinity not only pertains to the domain of mathematics, but also to the notion of absolutely everything. We refer to the recent debate on absolute generality and reinterpret the arguments subverting the possibility of universal quantification in terms of a practice that performatively undermines our putative understanding of the notion of the totality of everything that exists. In this view, we claim with respect to Neoplatonism that the method of diagonalization can be interpreted as a modern version of the *via negativa*. This reinforces negative theology by mathematical means and shows that mathematics can learn something from a performative reading of negative theology.

In this respect, in the final part of the paper we argue that the foundational problem concerning the domain of mathematics can be addressed through an adequate methodological attitude. This approach does not solve the foundational problem, but establishes a constructive way of dealing with it and might even suggest a new practice of mathematics. While contemporary mathematical practice seems to neglect the problem, we are claiming that philosophy of mathematics should adopt a version of the *via negativa* and explore the prospects of a methodological attitude of humility that acknowledges that mathematics cannot attain a full grasp of itself.

Cantor's paradox and the foundations of mathematics

Georg Cantor revolutionized our understanding of infinity. In particular, he originated mathematical theories of transfinite numbers and contrasted them with the notion of absolute infinity. Transfinite numbers are not finite, but still bounded, since absolute infinity is greater than all transfinite numbers. Absolute infinity, by contrast, is unbounded: nothing is greater than absolute infinity. However, while there is an arithmetic of transfinite numbers, absolute infinity cannot be grasped by mathematical means, which will be shown in detail in what follows. Cantor established these findings with straightforward arguments. The basic idea is that two sets, be they finite or infinite, have the same magnitude if there is a one-to-one correspondence between them.[1] This approach leads to Cantor's theorem, according to which the power set of a given set, i.e., the set of all subsets of that set (including the empty set and the set itself), is larger than the original set. Since this theorem holds for finite as well as for infinite sets, Cantor is commonly taken to have shown that there are infinitely many sizes of infinity. For Cantor, this result establishes the realm of transfinite numbers.

Cantor proved this theorem using a so-called diagonal argument. Suppose that there is a one-to-one correspondence S between a set A and its power set $P(A)$ that correlates each element a of A with a subset $S(a)$ of A. For each element a of A there are two possibilities: it may or may not be contained in the set $S(a)$ assigned to a in the given correspondence. Now, consider the subset D of A that consists of all elements a of A such that $S(a)$ does not contain a. This subset, the so-called diagonal set of the correspondence, differs from every $S(a)$ by at least the element a: if $S(a)$ contains a, D does not, while if $S(a)$ does not contain a, D does. D is thus not included in the correspondence which for this reason is not complete. Since the power set is at least as large as the set itself, this argument shows that for every set A, be it finite or infinite, the power set $P(A)$ is larger than A.

However, if such a diagonal argument is applied to the notion of the set of all sets, it leads to Cantor's paradox. According to Cantor's theorem, the power set of the set of all sets would be larger than this set, which is a contradiction, since, by definition, there is no larger set than the set of all sets. Hence, the notion of the set of all sets is logically incoherent: by definition, this set contains all sets and, as a consequence of Cantor's theorem, it does not contain all sets.[2] At the center of Cantor's paradox lies Russell's paradox, which stems from the question whether Russell's set, namely the set of all those sets that do not contain themselves, contains itself. If the identity relation is chosen as the assumed one-to-one correspondence between the set of all sets and its power set, the diagonal set D turns out to be Russell's set. This set is logically incoherent too, since it is neither contained nor not contained in itself. Either way, with respect to its power set or with respect to Russell's set, the notion of the set of all sets leads to a contradiction.

For this reason, ZFC set theory, the standard form of axiomatic set theory, was designed in such a way that its axioms permit neither unrestricted comprehension nor a universal set of all sets. By this means, Cantor's and Russell's paradox are ruled out. Accepting ZFC set theory as the axiomatic foundation of mathematics,

[1] In his *Two New Sciences* (1638), Galileo Galilei uses the notion of one-to-one correspondences to show that the number of positive integers equals the number of squares. This result is known as Galileo's paradox, and for Galileo, it shows that infinite sets cannot be compared with one another. Bernard Bolzano refers in his *The Paradoxes of Infinity* (1851) to Galileo's paradox and argues that it can be resolved through adequate concepts and definitions. Georg Cantor, then, compares infinite sets with each other and establishes a new paradox of (absolute) infinity.

[2] Proposed solutions to this problem of inconsistency, such as NF-style set theories and paraconsistent set theory, will be discussed below.

as it is widely done, entails that every mathematical object can be coded as a set.[3] Since the totality of all sets cannot be regarded as a set within ZFC set theory, this totality is not a mathematical object. Hence, the foundational approach implies that the totality of all sets embodies the realm of mathematical objects, but it also implies that this totality is not a mathematical object and, thus, cannot be studied by mathematics.

Of course, if the totality of all objects is missing among the objects of a theory but is needed to make sense of the theory, it can be added. NBG or MK set theory, for example, are extensions of ZFC set theory and allow for the notion of the class of all sets.[4] Against this background, set theory deals with 'set-theoretical universes', which in some contexts are even regarded as elements of a meta-structure, the 'set-theoretical multiverse' (cf. Hamkins, 2012). The procedure of adding the missing object generates a new realm of objects with a possibly new theory, which provides semantics for the old theory. The new theory has the same problem, though, since its realm of objects is not an object of this theory and can thus not be apprehended by means of this theory. For instance, strictly speaking, NBG and MK are not set theories, but rather class theories; their domains contain both sets and classes. But these theories do not allow for a universal class containing all classes and thus do not solve the problem of the missing object.[5] Obviously, this problem would be reiterated by introducing hyperclasses, hyper-hyperclasses and so forth.

Hence, Cantor's arguments lead to the conclusion that the realm of mathematical objects, be it in terms of sets or classes, is not a mathematical object, which implies that the domain of mathematics cannot be grasped by mathematical means. This poses a foundational problem, since set-theoretical practice implicitly depends on quantification over the totality of all sets. Take, for example, the ZFC-axiom of comprehension, which guarantees that for every set x and every formula F, one can form the set of all elements of x that satisfy F. If F only contains bounded quantifiers or no quantifiers at all, this comprehension has a clear meaning. However, if F contains unbounded quantifiers like "There is a set such that…" or "For all sets, it holds that…", F implicitly refers to the totality of all sets. Without reference to the totality of all sets, the set-theoretical operations of comprehension and, similarly, replacement would be restricted to formulas with bounded quantifiers, which would lead to a much weaker theory.[6] Hence, ZFC set theory crucially hinges on implicit quantification over the totality of all sets, which is not a set. In set theories that allow for proper classes, quantification over classes leads to the same problem. As long as

[3] Following Maddy (2019), accepting ZFC set theory as the axiomatic foundation of mathematics means that set theory provides (i) an ontological basis for mathematics in the sense that each mathematical object can be modelled as a set (Maddy's "Generous Arena", Maddy, 2019, 298), (ii) a unified methodology in the sense that what counts as a proof should ultimately be translatable into a formal derivation in set-theory (Maddy's "shared standard", Maddy, 2019, 298) and, as a side-effect of (ii), (iii) a basis for metamathematics in that the independence of a statement of set theory can be regarded as undecidability in currently accepted mathematics (roughly Maddy's "Meta-Mathematical Corral", Maddy, 2019, 301). In what follows, we only refer to claim (i).

[4] For NBG set theory, cf. Stoll/Enderton (2016), for MK set theory, cf. Morse (1986).

[5] Cf. Parsons, (1974); Boolos, (1998), 35–36; Shapiro/Wright (2006), 272–273, 282, 290–291.

[6] Such a theory, the power Kripke–Platek set theory KP(P), indeed exists; see, e.g., Rathjen (2020).

quantification is not possible without presupposing a definite domain, mathematics lacks the conceptual power to provide the semantics for expressions like 'all sets' or 'all classes.'[7]

This foundational problem has been frequently highlighted in the philosophy of mathematics. Gaisi Takeuti, for example, motivates the study of second-order logic by a criticism of set theory and argues that "the meaning of the axiom of replacement is still not very clear. The difficulty exists in the fact that the axiom of replacement involves formulas with quantifiers, hence, if the creation of sets is assumed to be endless, it is not clear what these quantifiers mean" (Takeuti, 1987, 162). Of course, it is quite common in mathematics that specific domains depend on something beyond themselves. In the case of Peano arithmetic, for example, which is the usual formalization of number theory, the axiom of induction refers to formulas quantifying over all natural numbers. Given that quantification requires some definite domain, number theory, in its semantics, relies on the totality of all natural numbers, but this totality cannot be thematized in number theory, as it is not a natural number. This is not a problem, though, since this totality is provided by set theory. However, if one accepts that ZFC set theory is the axiomatic foundation of mathematics, the totality of all sets is the all-embracing mathematical domain. Hence, the solution that works for a specific mathematical domain like number theory cannot be transferred to the domain of set theory, since there is no mathematical domain outside the totality of all sets.

In view of this problem, some attempts have been made to address Cantor's paradox. In what follows, we will briefly mention four of these attempts, along with their difficulties.

(1) Miniaturization. If the realm of objects under consideration is too large to be consistently considered as an object, simply restrict it. Finitism (restriction to finite objects) or ultrafinitism (restriction to small finite objects) are examples of foundational approaches that propose such restrictions. However, the totality of all finite objects is not finite, nor is the totality of all small objects small. For example, proponents of the claim that due to our limited capacity to give representations there is a largest natural number cannot at the same time keep up a notion of existence that relies on giving a representation. If such a representation were given of the supposed largest number, one could spoil it (diagonalize against it) by writing + 1 next to it. No matter how far one shrinks the domain, this structural problem reappears.

(2) Alternative ontologies. Not all foundational approaches to mathematics are based on set theory. If set theory is unable to capture its own domain, why not switch, for example, to category or type theory? However, the problem is a structural problem, which is independent from the specific character of the domain. For

[7] One might argue that quantification is not committed to the existence of an object that contains all the objects over which the quantifier ranges. Accordingly, some attempts have been made to understand and formalize quantification without this commitment (cf. Boolos, 1985 and Cartwright, 1994) or to simulate this domain, as in modal set theory (cf. Linnebo, 2013). Whether these attempts are successful is a matter of an ongoing debate that we will briefly touch on in the fourth part of the paper.

the case of category theory, Ernst (2015) shows that a corresponding foundational problem occurs and points out that the structure of this problem "is best understood by drawing parallels to a well known result: the proof that there can be no set of all sets using Cantor's theorem" (Ernst, 2015, 9). In the case of type theory, the notion of a type of all types leads to Girard's paradox, which can be understood as a type-theoretic analogue of Russell's paradox.[8]

(3) Blocking the contradiction. In ZFC set theory, the axioms are formulated in such a way that a universal set cannot be generated. Hence, the set-theoretic paradoxes are ruled out. However, there are also consistent theories that allow for a universal set. One example is Quine's NF ('new foundations'), which yields the existence of a universal set, but restricts comprehension to formulas that are stratifiable, which $x \notin x$ is not. While this constraint effectively blocks the set-theoretic paradoxes and still allows for a universal set, it seems to rather forbid the problem than to solve it. Banning some formulas from comprehension seems to be an ad hoc approach, not a clarification of the underlying notion.[9] This ad hoc character may be the reason why NF was never seriously used as a foundational theory. In addition, it is questionable whether it solves the underlying structural problem. The domain of mathematical discourse becomes an object of mathematical discourse, but only at the price of severely limiting this discourse in other ways. NF is not a framework that faithfully captures mathematical practice and, hence, does not resolve the problem that this practice cannot attain a full grasp of itself.[10]

(4) Accepting the contradiction. Paraconsistent set theory (cf. da Costa, 1986) accepts the existence of Russell's set, which is both contained and not contained in itself. To avoid the trivialization of mathematics by logical explosion, the underlying logic (i.e., the deduction rules) are weakened in such a way that it is not possible to deduce arbitrary statements from the contradiction. This strategy can be seen as a variation of (3), where rather than the axioms the inference rules are changed. It is arguably the most radical of all the approaches that we presented. However, as in the case of (3), it seems impossible to reconstruct mathematical praxis by the paraconsistent approach.[11] The attempts of doing so "seem to be all quite unconnected and apparently ad hoc" (Carnielli/Coniglio, 2013, 3). Moreover, even if one was willing to substantially revise mathematical practice, it is not clear whether paraconsistent approaches can solve the struc-

[8] See Girard (1972) and Hurkens (1995).

[9] However, such attempts have been made, cf. Forster (1995) and Holmes (1998).

[10] For a similar criticism of Quine's approach, see Weir (2006), 340, who refers to Williamson (2003), 425–426, in this respect; also see Boolos (1971), 219.

[11] With respect to a corresponding proposal by Graham Priest, Incurvati (2020), for example, argues: "To be sure, all of this does not rule out that alternative arguments for results whose standard proof does not go through in LP could be given. But in the absence of reasons for thinking that the proofs which seem to fail in NLP can be reconstructed in some other form, the prospects for developing a substantial amount of set theory in NLP look bleak, as Priest himself acknowledges" (Incurvati, 2020, 108. For Priest's acknowledgement, Incurvati refers to Priest, 2006, 250).

> tural problem in the end and allow mathematics to grasp its domain.[12] Although paraconsistent logic is certainly a bold and stimulating enterprise that has its applications, it thus does not fare better than (1)–(3) with respect to the foundational problem at hand.[13]

All in all, the diagonalization argument that we presented above is a clear-cut argument showing that the domain of mathematics is not a mathematical object and thus cannot be studied using mathematical means. We argued that this inability constitutes a foundational problem, which is still present even though it seems to be widely ignored in contemporary mathematical practice.

Absolute infinity and negative theology

As pointed out above, at the heart of Cantor's paradox lies Russell's paradox. For Russell himself, as for Frege and Dedekind, this paradox came as a shock; their quest for a secure foundation of mathematics was deeply challenged. By contrast, Cantor readily acknowledged the paradoxical nature of the set of all sets. For him, the totality of all sets does not belong to mathematics, but to metaphysics and theology. In contrast to Russell, Frege, or Dedekind, Cantor did not aim at a mathematical or logical foundation of mathematics. His mathematical studies were strongly influenced by metaphysics (cf. Bandmann, 1992) and his acquaintance with metaphysics and theology certainly was an important source of his confidence when he developed transfinite set theory (cf. Dauben, 1990, 299).

Against the prevailing Aristotelian tradition, Cantor claims that mathematical infinity is actual, not just potential, since any potential infinity would presuppose an actual infinity (GA 411).[14] Along these lines, he argues that both the transfinite numbers and absolute infinity are actual infinities (GA 375).[15] He also discusses this claim with respect to metaphysics. In particular, he refers to Spinoza and Leibniz (cf. Newstead, 2008 and 2009), and the intermediate realm of his transfinite numbers between finitude and absolute infinity strongly resembles Spinoza's infinite modes and Leibniz' infinite monads (cf. Tengelyi, 2014, 483–488). In his later writings, Cantor changed his terminology and replaced the distinction of transfinite and absolute infinity by the opposition of consistent and inconsistent multiplicities. In a

[12] Graham Priest, for example, points to this problem: "Is there a metatheory for paraconsistent logics that is acceptable on paraconsistent terms? The answer to this question is not at all obvious" (Priest, 2006, 258). He proposes that the classical metatheory could be "appropriated" (Priest, 2006, 259) by paraconsistent logic. We leave it open whether this proposal is tenable.

[13] See also the discussion of the dialethist 'resolution' of the set-theoretical paradoxes in Weir (2006), 341–342.

[14] In his major study of Cantorian set theory, Michael Hallett refers to this reasoning and calls it the 'domain principle' (Hallett, 1984, 7). In what follows, GA is used as shorthand for Georg Cantor's *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts* (Cantor, 1932).

[15] Against the background of Cantor's interest in theology (see below), Thomas-Buldoc (2016) contradicts the claim of Jané (1995) that the later Cantor conceived absolute infinity as potentially infinite. In Jané (2010), however, Jané weakened this claim.

letter to Dedekind, he defined inconsistent multiplicities as follows: “A multiplicity can be constituted in such a way that the assumption of the ‘togetherness’ of *all* their elements leads to a contradiction, with the result that it is impossible to regard this multiplicity as a unity, as a ‘completed object’. I call such multiplicities *absolutely infinite* or *inconsistent multiplicities*” (GA 443, our translation).

His prime examples of such multiplicities are the totality of all sets and the system of all numbers: “The system Ω of all numbers is an inconsistent, an absolutely infinite multiplicity” (GA 445, our translation). Due to the contradictory nature of such multiplicities, Cantor argues that absolute infinity can only be acknowledged, but never known, not even approximately (GA 205). For Cantor, thus, the inconsistent multiplicity of absolute infinity is not comprehensible in mathematical terms (GA 375, 405). According to the standard story told in textbooks of set theory, Cantor’s definition of a set as a gathering together into a whole of definite, distinct objects of our perception or of our thought (GA 282) leads to antinomies and, thus, ‘naïve set theory’ had to be replaced by axiomatic set theory. However, Cantor was well aware of these antinomies and points out in his letters to Dedekind that a set is supposed to be a collection of distinct objects into a whole without contradiction (GA 443, 448). He even criticized the usual reading of the Burali-Forti paradox for not distinguishing between proper sets and inconsistent multiplicities (cf. Hauser, 2013, 171–172). He did not emphasize this distinction in his now famous definition (his letters to Dedekind were not published until 1932), but for Cantor, sets are consistent multiplicities as opposed to inconsistent multiplicities (GA 443), and he even explained in a letter to Hilbert that his definition intended to avoid sets being antinomic (cf. Purkert, 1989, 61). Thus, Cantor’s set theory is not as naïve as the standard story has it (cf. Hallett, 1984, 38; Purkert, 1986, 323–325; Lavine, 1994, 1–3).

Despite being inconsistent and incomprehensible, for Cantor, inconsistent multiplicities exist; he considers them to be actual absolute infinities. In mathematical proofs concerning his transfinite arithmetic, he explicitly refers to such multiplicities and particularly to their very inconsistency (GA 446–447, cf. Jané, 1995, 388–389). The idea of employing the impossibility to mathematically determine the universe of all sets in mathematical proofs resembles the reflection principle that is used in modern theories of large cardinals. In a nutshell, this principle allows for the conclusion that “if we should reasonably expect that the universe possesses a property, then one can also reasonably expect that there exist sets which have it. And with regard to this property, the universe is *reflected* in these” (Hallett, 1984, 116). Otherwise, the property in question would be a property of the universe alone, which would make the universe of all sets characterizable. The very fact that the totality of all sets is not comprehensible in mathematical terms was thus employed by Cantor in his transfinite arithmetic and is also used in a similar way in modern set theory for arguing in favor of large cardinal principles.

However, this approach seems to neglect the foundational problem that we pointed out in the first part of the paper. Since the domain of mathematics, i.e., the universe of all sets, cannot be characterized mathematically, mathematics cannot attain a full grasp of itself. For Cantor, the domain of transfinite arithmetic is absolute infinity; he even argues that the notion of transfinite objects necessarily points to the notion of absolute infinity (GA 404–405). For Cantor, transfinite mathematics takes thus place within a domain that is incomprehensible in mathematical terms. This is not only acceptable for Cantor, but in fact essential for his approach. While modern mathematics seems to ignore this problem, for Cantor, the domain of mathematics can indeed not be grasped by mathematical means. For him, the domain of mathematical practice, i.e., the absolutely infinite and inconsistent multiplicity of all sets, does not belong to mathematics, but to theology (GA 405–406).

More precisely, Cantor argues that transfinite numbers can be studied in mathematics and metaphysics, but that absolute infinity belongs to 'speculative theology' (GA 378). Accordingly, Cantor identifies absolute infinity with God (GA 175–176, 376, 378, 386, 399) and with corresponding theological notions such as 'ens simplicissimum' and 'actus purissimus' (Meschkowski/Nilson, 1991, 454).[16] He was convinced that his mathematical treatment of absolute infinity should be recognized by Christian theology and, consequently, he started, although being Protestant, an extensive correspondence with Catholic theologians (cf. Tapp, 2005). In these letters, Cantor wanted to make sure that his theory of actual infinity would not be qualified as pantheism and therefore dismissed (cf. Newstead, 2009). Furthermore, he claimed in one of those letters that he "for the first time makes the true theory of infinity in its beginnings accessible to Christian philosophy" (Tapp, 2005, 312, our translation). He argued that his work can be used to prove that the world was created (Meschkowski/Nilson, 1991, 125) and that it helps to convince people of rational theism (Meschkowski/Nilson, 1991, 124–125). He even thought that his set theory was revealed to him by God (Dauben, 1990, 232). Be that as it may, Cantor's set theory has a strong religious dimension that should not be overlooked (Dauben, 1990, 291).

In what follows, we will argue that this dimension can be interpreted in terms of negative theology. This reading seems natural, since Cantor claims that absolute infinity is logically inconsistent and can thus not be known.[17] According to negative theology, God or the Divine is beyond human comprehension and can only be approached by negation, i.e., by the study of what God is not. In his letters, Cantor identifies absolute infinity—"in which everything is, which contains everything"—with the notion of God and claims that it cannot be grasped in mathematical terms, and, more generally, that it is "unmeasurable" and "incomprehensible for the human intellect" (Meschkowski/Nilson, 1991, 454, our translation).

Against this line of thought, recent studies contend that the above-mentioned reflection principle seems to allow for positive knowledge of the absolute. Horsten (2016), for example, argues that Cantor was familiar with some versions of negative theology and that he applied it to mathematics (Horsten, 2016, 109). But Horsten also refers to the reflection principle and claims that the "uncharacterisability of

[16] The identification of God and infinity goes back to Duns Scotus, who distinguished between *ens infinitum* (God) and *ens finitum* (creatures). Before Duns Scotus, the notion of infinity was rather negatively understood in terms of an amorphic indefiniteness and limitlessness (*apeiron*), which was not related to God (cf. Biard/Celeyrette, 2005). Four hundred years before Galileo, Duns Scotus compared the number of even and odd integers with the number of all integers (Parker, 2009, 89).

[17] As we argued above, this applies to Cantor's notion of absolute infinity as well as to his later concept of inconsistent multiplicities, cf. also Hallett (1984), 286. Cantor linked both notions to theology, cf. Hallett (1984), 166.

God" can be "transformed into a *positive* principle" (Horsten, 2016, 115). On that note, Russell (2011) shows with respect to Cantor and the reflection principle that there is a link between negative and affirmative theology (Russell, 2011, 283–285). But still, the premise of the reflection principle is that the universe of mathematics cannot be characterized mathematically. Accordingly, Russell (2011) points out that, in the end, absolute infinity is inconceivable, in spite of the reflection principle: "Mathematically, Absolute Infinity is known through the transfinites, and yet being so, it remains unknown in itself. Theologically, the God who is known is the God who is unknowable" (Russell, 2011, 285).

This tension within the reflection principle between knowing and not-knowing can be linked to the mathematical illustrations that Nicholas of Cusa employs in his theological writings. Hauser (2013), for example, refers to Cusa's thought experiment of the infinite line that coincides with the infinite circle or the infinite triangle, which, for Cusa, represents the 'coincidence of opposites' within absolute infinity. The corresponding transition from finite objects to infinity, Hauser (2013) argues, strongly resembles the reflection principle (Hauser, 2013, 176–178). For Cusa, the mathematically infinite objects of his thought experiments constitute a symbol of the absolute (Tengelyi, 2014, 479).[18] Cantor refers to Cusa with respect to the notion of actual infinity (GA 205; Tapp, 2005, 502) and says in the same footnote in which he mentions Cusa that the sequence of transfinite numbers appears to him to be a symbol of the absolute (GA 205). With respect to this notion, Hauser (2013) establishes a link between Cusa and Cantor through the reflection principle (Hauser, 2013, 176).

In the end, however, the reflection principle rests on the premise that the totality of all sets cannot be determined and that thus absolute infinity is not comprehensible. Correspondingly, Nicholas of Cusa's mathematical illustrations eventually show that absolute infinity is inconceivable. As we have argued above, for Cantor, absolute infinity is inconceivable due to its paradoxical nature; he speaks of inconsistent multiplicities in this respect. Similarly, Cusa uses the notion of the 'coincidence of opposites' (cf. Hauser, 2013, 169). Nicholas of Cusa is one of the main protagonists of negative theology and the notion of the 'coincidence of opposites' is supposed to express the incomprehensibility of God. Thus, as Hauser (2013) puts it, Cantor's "affinity with the central thesis of *Docta Ignorantia* is apparent: with regard to the Absolute, 'knowing is not-knowing'" (Hauser, 2013, 171, cf. also Tengelyi, 2014, 479–482). In addition, Cantor's treatment of absolute infinity strongly resembles Plotinus and Neoplatonism (Hauser, 2013, 165–167; Tengelyi, 2014, 474–476), particularly with respect to the concept of different sizes of infinity, which can be found already in Proclus (Hauser, 2013, 166). Beyond that, Plotinus' claim that the One is inconceivable is a major source of negative theology and strongly influenced Cusa. When Cantor argues that the absolute is not comprehensible in mathematical terms, he identifies it with Cusa's notion of the 'absolute maximum' (GA 391, cf. Tengelyi, 2014, 477) and with the 'One' (*ibid.*). While this seems to be the only explicit reference to Neoplatonism in Cantor's writings, his treatment of absolute infinity is

[18] Cusa's link between the absolute and infinity builds upon a theological debate on infinity that was initiated more than a hundred years earlier by Duns Scotus, cf. the footnote above.

presumably influenced by Neoplatonism, albeit indirectly through the works of Cusa (Hauser, 2013, 167; Tengelyi, 2014, 482).

The paradox of negative theology and a performative reading of Cantor

All in all, the discussion in the second part shows that the theological significance of Cantor's treatment of absolute infinity can be interpreted in terms of negative theology. Kreis (2015) agrees with this interpretation (Kreis, 2015, 402–403), but also points to a serious problem of this approach to infinity that we want to discuss now. For Kreis, the claim that the absolute exists (ontologically) without being conceivable (epistemically) constitutes the center of negative theology (*ibid.*). However, Kreis argues that making this claim is incoherent, since we cannot know that the absolute exists if we cannot comprehend the very idea of the absolute (Kreis, 2015, 404–405). In general terms, claiming that God is beyond human comprehension is self-defeating, since this claim appears to constitute a comprehensible proposition about God. This problem establishes a paradox at the center of negative theology. While Kreis agrees that Cantor's treatment of the paradox of infinity addresses a problem that is rather neglected in modern mathematics (Kreis, 2015, 393), he argues that the solution offered by Cantor is incoherent and therefore not tenable (Kreis, 2015, 406).

On our reading, the key to the resolution of the paradox of negative theology is that negative theology is not committed to contending a negative claim about our knowledge of God, which would indeed be self-defeating. Instead, negative theology can be conceived of as a practice that performatively undermines our putative knowledge of God. Indeed, for most of its protagonists, negative theology is not aiming at a negative theoretical statement about God, but at an experiential understanding of God's incomprehensibility that is achieved through the practice of negation. Nicholas of Cusa, for example, famously argued that we understand God through our non-understanding. More precisely, Cusa is not claiming in terms of a theoretical statement that God would be incomprehensible. In contrast, he employs different kinds of texts and rhetorical means to yield an experiential understanding that he describes as learned ignorance, as *docta ignorantia*. This form of understanding results from a new attitude, as Hans Blumenberg points out: "But the Cusan's procedure sees an essential difference between muteness and falling silent. The language and system of metaphor that he developed for *docta ignorantia* do not represent a state of knowledge but a praxis, a method, a path to a certain sort of attitude" (Blumenberg, 1983, 490).

Proceeding from this, we think that the practice of negative theology evokes the experience of failing to think the absolute. This experience is transformative, since it engenders an attitude of humility with respect to our boundedness. Even more, the experience is not just personally, but also epistemically transformative,[19] since the

[19] The link between personally and epistemically transformative experience is discussed in Paul, (2014), 5–15.

new attitude is linked to an experiential understanding of the incomprehensibility of the absolute. Of course, this is a non-propositional form of understanding. Otherwise, we would again face the paradox of negative theology. On our reading, the epistemic significance of this understanding is reflected in new values and goals and thereby goes beyond practical or phenomenal knowledge. We fail to think the absolute, but the experience of this failure does not end in brute ignorance, but in learned ignorance.[20] For Cusa, this notion describes a new attitude that is attained through the experiential understanding of God's incomprehensibility (Cusa, 1997, 91). On our reading, Cusa's concept of God "showing Godself to us as incomprehensible" (Cusa, 1997, 127) can thus be interpreted as referring to the epistemically transformative experience of the failure to grasp the absolute. In line with Blumenberg's interpretation, we thus think that negative theology is a practice, a *via negativa*, i.e., a path to a new attitude that yields a non-propositional form of understanding.

The practice of negative theology not only refers to the theological notion of God, but also to the philosophical problem of the totality of the world. Neoplatonism, for example, asks for the totality and the origin of everything that exists. The metaphysical quest for this absolute also leads to paradoxes. For Plotinus, everything emanates from the One, but the One itself is beyond comprehension since every concept attributed to the One would bring the One under the twofold structure of attribution and thus contradict its unitary structure (*Enn.* V 4, 1, 5–13). However, Plotinus' texts should not be understood as stating at the theoretical level that the One would be inconceivable. Instead, the texts show the incomprehensibility of the One performatively through a radical dialectic of negation (*Enn.* III 8, 10, 28–31). In the end, this performance aims at the mystical experience of the One, the *henosis*, yielding a form of experiential understanding that goes beyond thinking and is thus called *hyper-noesis* (*Enn.* VI 9, 17). On our reading, the practice of the negative theology of the One starts with the theoretical problem of the absolute in terms of the totality and the origin of everything that exists and becomes entangled in this problem in such a way that the failure of the attempt to grasp this totality and its origin evokes an epistemically transformative experience. This experience engenders an attitude of humility in the face of the boundedness of the human condition and thereby generates a non-propositional understanding of this boundedness.

The paradox of negative theology can thus be resolved: instead of incoherently claiming that the absolute is beyond human comprehension, negative theology evokes a transformative experience that yields a new attitude and a form of non-propositional understanding that is reflected in new values and goals. Against this background, the religious dimension of the problem of absolute infinity can be interpreted in a new way. We argued above that, for Cantor, absolute infinity exists but is beyond human comprehension. For Kreis (2015), this approach is in line with negative theology, but constitutes an incoherent position. However, we think that the diagonalization argument that leads to Cantor's paradox can be interpreted as a performative undermining of the notion of absolute infinity that evokes a transformative experience. Understood as a *via negativa*, the argument entangles our understanding

[20] Cf. Gutschmidt (2019) for a more detailed exposition of this interpretation of negative theology in terms of transformative experience.

into a paradox and can thereby lead to an attitude that acknowledges the incomprehensible character of the domain of mathematics. This reading goes beyond Cantor's theological interpretation of the problem of the domain. We argued in the second part that his treatment of absolute infinity is influenced by and intertwined with elements of negative theology, and we will sketch out in the fifth part how our performative reading of negative theology can lead to a constructive way of dealing with the foundational problem of mathematics that differs from Cantor's solution. In particular, we will develop some ideas for a methodological attitude of humility that parallels new values and goals with respect to the incomprehensibility of the domain of mathematics.

Via negativa and diagonalization: The problem of absolutely everything

Before discussing in more detail what mathematics can learn from a performative reading of negative theology, we want to show that negative theology can also be reinforced by mathematics.[21] In the previous part, we argued that the method of diagonalization can be interpreted as a *via negativa* with respect to the domain of mathematics. In what follows, we demonstrate that this interpretation can be extended with respect to the totality of everything that exists, i.e., absolutely everything. In Neoplatonism, this totality is understood in terms of unity as opposed to the plurality of beings. The concept of unity leads to the notion of the One, which is, according to Neoplatonism, beyond comprehension. Now, remarkably, the application of the set-theoretic paradoxes to the notion of absolutely everything shows that this notion is beyond comprehension too. In modern set theory, this raises the problem of absolute generality (cf. Rayo/Uzquiano, 2006a): how are we supposed to understand unrestricted quantification given that the notion of absolutely everything is incomprehensible?

To begin with, in his book on Frege's philosophy of language, Michael Dummett argues that "the one lesson of the set-theoretical paradoxes which seems quite certain is that we cannot interpret individual variables in Frege's way, as ranging simultaneously over the totality of all objects which could meaningfully be referred to or quantified over" (Dummett, 1981, 567). Against this background, he claims that "the overwhelming majority of logicians … do not think it possible intelligibly to quantify over all objects whatever" (Dummett, 1981, 229). Be that as it may, the set-theoretic antinomies, like the Cantor, Russell, and Burali-Forti paradoxes, are indeed widely understood as profoundly challenging the idea that quantifiers can range over absolutely everything (Cartwright, 1994, 2; Shapiro, 2003, 469; Williamson, 2003, 424; Shapiro/Wright, 2006, 255; Parsons, 2006, 205). Referring to the works of Michael Dummett and Charles Parsons, Øystein Linnebo summarizes the diagonalization arguments that undermine unrestricted quantification as follows: "It has been

[21] Such application of set theory and particularly of the method of diagonalization to philosophical issues related to negative theology can also be found in Becker (1973), King (1998), and Bova (2018). These studies discuss concepts of Husserl, Heidegger, Sartre, and Derrida.

argued that, whenever we form a conception of a certain range of quantification, this conception can be used to define further objects not in this range, thus establishing that the quantification wasn't unrestricted after all" (Linnebo, 2006, 149).[22]

But even beyond the debate on unrestricted quantification in the set-theoretic context, the coherence of the notion of absolutely everything is called into question. David Armstrong, for example, briefly discusses the 'paradox of totality' with respect to the problem of truth-makers (Armstrong, 2004, 78–79). Not least, in his *The Incomplete Universe* (1991), Patrick Grim transposes Cantor's diagonal argument from the set of all sets to the totality of everything that exists, which is not understood as a set. Grim's argument proceeds from the totality of all propositions, which he links through separate arguments to all matters of fact, to everything that exists, and to the totality of the world (cf. also Kreis, 2015, 414–423). In a nutshell, Grim discusses an analogue of Russell's set, namely a proposition about all propositions that are not about themselves, which leads to a semantic paradox established by the question whether this proposition is about itself or not (Grim, 1991, 120–122). While Cantor—inconsistently—claims that the absolutely infinite totality of all sets cannot be comprehended but still exists, Grim draws the conclusion that the totality of all propositions does *not* exist: "given fundamental logical principles, there cannot *be* any such totality" (Grim, 1991, 128). With reference to Wittgenstein's concept of the world as the totality of facts, he adds that "there can be no closed world of the form the *Tractatus* demands" (Grim, 1991, 126); the universe is, thus, incomplete.

This is a strong and potentially unsettling claim that deeply challenges our self-understanding as a part of this universe.[23] Of course, it seems that in everyday contexts it does not matter whether the universe is complete or not. As a matter of fact, regarding the problem of unrestricted quantification, it has been argued that we do not quantify over absolutely everything in everyday life anyway, even when we use terms like 'all' or 'everything' (Williamson, 2003, 415; Glanzberg, 2006, 48–49). Proceeding from this, Michael Glanzberg attempts to develop a contextualist position according to which quantifiers are always contextually restricted (Glanzberg, 2006, 49). Of course, this is a strong and apparently self-defeating claim about *all* quantifiers, and Glanzberg admits that it faces several difficulties (Glanzberg, 2006, 71). Beyond this self-criticism, Timothy Williamson points out that unrestricted quantification might be superficial in everyday contexts but is indispensable in metaphysics—in statements like "everything is natural" (Williamson, 2003, 415)—and in semantics, in order to make sense of statements like "no donkey talks" (Williamson,

[22] A detailed overview of arguments against unrestricted quantification can be found in Studd (2019), chapter 1, cf. also Rayo/Uzquiano (2006b), 4–12.

[23] For Grim, however, the main purpose of this line of thought is to show that there can be no omniscient being. Ironically, Grim is driven by an anti-religious affect. He wants to undermine the notion of an omniscient God by showing that the notion of the totality of all true propositions is logically incoherent. But negative theology also denies the existence of a transcendent being, be it omniscient or not. On our reading, negative theology undermines performatively the concept of a transcendent being and thereby evokes a transformative experience that may lead to an attitude of humility. Hence, when Grim shows that we cannot grasp the totality of everything that exists, this has—against his intentions—a religious dimension in terms of negative theology.

2003, 436).[24] Given the incomprehensibility of absolute generality, Williamson argues that "[p]erhaps both metaphysics and semantics need what they cannot have. That conclusion cannot easily be dismissed. Reality may be intrinsically unsystematic or mysterious, essentially resistant to full theoretical understanding" (Williamson, 2003, 449).

Even though this result about unrestricted quantification may not immediately affect everyday life, the mystery of reality in terms of an incomplete universe amounts to the potentially disturbing experience that we do not and cannot attain a full grasp of our own existence. In the first part of the paper, we argued that the inability of mathematics to grasp its own domain constitutes a foundational problem. Similarly, we think that the incomprehensibility of the notion of absolutely everything embodies an equally strong problem with respect to the human condition. The problem of absolute generality shows that human life takes place inside a 'domain'—the incomplete universe—that is beyond human comprehension. Correspondingly, Kreis (2015) points out in some detail that this problem undermines our grasp of the notion of the world and argues that it seems impossible to resolve this problem (Kreis, 2015, 435–460).

Of course, besides contextualism, more attempts have been made to address the problem of absolute generality. Richard Cartwright, for example, argues that the problem of unrestricted quantification stems from "the assumption that to quantify over certain objects is to presuppose that those objects constitute a 'collection,' or a 'completed collection'—some one thing of which those objects are the members" (Cartwright, 1994, 7). He calls this assumption the All-in-One Principle (*ibid.*) and argues that it is unwarranted and cannot be used to undermine unrestricted quantification (Cartwright, 1994, 17). Another attempt to enable unrestricted quantification has been made by George Boolos. His approach, known as 'plural quantification' (cf. Boolos, 1985), is to introduce new plural quantifiers motivated by considerations from natural language, so that, e.g., "There are some apples" does not refer to a non-empty set of apples, but merely to apples in the plural. This interpretation is supposed to block the derivation of Russell's paradox from the unrestricted comprehension principle (Boolos, 1985, 331). In addition, the position of schematism seeks to substitute unrestricted quantification by absolutely general commitments that are constituted of open-ended schemata (cf. Lavine, 2006; Studd, 2019, chapter 5).

However, we will neither discuss all the details of these accounts nor refer to the extensive debate surrounding them. While this debate is still open, its prospects are aptly resumed by Stewart Shapiro and Crispin Wright, who sum up their detailed discussion of these accounts by claiming "that every one of the available theoretical options has difficulties which would be justly treated as decisive against it, were it not that the others fare no better" (Shapiro/Wright, 2006, 293). In the first part of the paper, we argued that the diagonalization argument is a clear-cut argument that leads to a foundational problem. With respect to the problem of unrestricted quantification, Shapiro and Wright confirm this conclusion: "Frankly, we do not see a

[24] Other standard examples for the necessity of unrestricted quantification are logical truths like „absolutely everything is self-identical ", or „the empty set has absolutely no members" (Linnebo, 2006, 149).

satisfying position here. … Such situations are not unprecedented in philosophy, but this one seems particularly opaque and frustrating" (*ibid.*).

It is thus fair to say that the above-mentioned judgement of Dummett and others about unrestricted quantification seems appropriate: the method of diagonalization that leads to the set-theoretic paradoxes can be applied to totalities like all objects, which undermines our understanding of the notion of absolutely everything. However, it is not clear what this result amounts to. As in the paradox of negative theology, claiming that the notion of absolutely everything cannot be comprehended is self-defeating, since this claim appears to constitute a comprehensible proposition about this notion. With respect to unrestricted quantification, an often-quoted formulation of this problem goes back to David Lewis, who argues that the assertion "that some mystical censor stops us from quantifying over everything without restriction" cannot be stated coherently, since anyone who claims it "violates his own stricture in the very act of proclaiming it" (Lewis, 1991, 68; cf. also McGee, 2000, 55; McGee, 2006, 185). Timothy Williamson discusses this problem in some detail and sums up: "Generality-relativists seem to be unable to articulate their position in any coherent way" (Williamson, 2003, 433). Similarly, Patrick Grim concedes that his above-mentioned claim that the totality of all propositions does not exist is incoherent: "What we can't do, interestingly enough, short of falling victim to our own argument, is to draw as conclusion some universal proposition about all propositions" (Grim, 2000, 151).

With respect to this problem, it was argued that philosophers should not hold the position that unrestricted quantification is impossible (Weir, 2006, 335).[25] However, as in the case of negative theology, we think that the problem of incoherence can be resolved. We have argued in the third part of the paper that the diagonalization argument that leads to the set-theoretic antinomies can be interpreted as performatively undermining the notion of absolute infinity. As a matter of fact, such performative interpretation is discussed in the debate on absolute generality as well. Timothy Williamson, for example, describes a dialectical situation in which the skeptic about absolute generality brings forward Russell's paradox against the generality-absolutist in an indefinite iteration (Williamson, 2003, 434–435). Kit Fine discusses a similar dialectical situation and points out that it depends on an actual opponent (Fine, 2006, 25). More precisely, Fine argues that "all we can sensibly do, as enlightened limitavists, is to hope that our opponent will claim to be in possession of an absolutely unrestricted interpretation of the quantifier and then use the Russell argument to prove him wrong" (Fine, 2006, 28–29). In his discussion of Williamson's dialectical situation, Geoffrey Hellman argues that it can be summarized "in the form of a *reductio* of the absolutist position" (Hellman, 2006, 77). Not least, Patrick Grim also proposes to understand the diagonalization argument in a performative way. Instead of incoherently referring to notions like all truths or all propositions, he considers reinterpreting "the central abstract patterns of argument at issue as conceptual traps: as mazes bound to lead anyone who does take such notions seriously into the tangles

[25] Similarly, in the context of philosophy of religion, Alvin Plantinga, who defends the notion of an omniscient being, argues against Grim's position by showing that it is self-defeating (Grim/Plantinga, 1993, 284–287, 295–297).

of contradiction" (Grim, 1991, 123). In a later text, he describes this reinterpretation of the diagonalization argument in terms of a "logic bomb" that should not be used "to generate some general conclusion" but has to be applied case by case to single instances of claims about all propositions (Grim, 2000, 152).

This treatment of the problem strongly resembles the performative interpretation of negative theology that we sketched out above. As we have argued in the third part, the *via negativa* is not committed to making a theoretical statement about the incomprehensibility of the absolute, which would be self-defeating, but can be—and usually is—understood as a practice that performatively undermines our putative understanding of the absolute. We also pointed out that this practice concerns not only the theological notion of God, but also, particularly in Neoplatonism, the philosophical problem of the totality of the world. In terms of the One, Plotinus performatively undermines our putative understanding of this totality and its origin. Similarly, we just argued that the method of diagonalization can be interpreted as performatively undermining the notion of absolutely everything. Thus, we think that this method constitutes a modern version of the *via negativa*.

Cantor's engagement with metaphysics and 'speculative theology' was crucial for the development of his mathematics of the infinite, which particularly reflects his knowledge of central notions of negative theology. However, as we argued in the third part, he still fell prey to the paradox of negative theology. This paradox can be resolved through a performative reading and, accordingly, we think that Cantor's treatment of absolute infinity and, *a fortiori*, the problem of absolute generality can be reinterpreted in terms of the *via negativa*, understood performatively. We see that any attempted mathematical treatment of the absolutely infinite domain of mathematics falls short. Similarly, thus far the notion of absolutely everything is beyond comprehension. Like negative theology, mathematics with its formal methods thus encounters performatively the boundedness of the human condition.

Toward a methodological attitude of humility in mathematics

Proceeding from our reinterpretation of Cantor's arguments, we briefly discuss in this final part of the paper how the notion of an attitude of humility can be transposed from the context of negative theology to the foundational problem of mathematics that we outlined in the first part. On our reading, the attempts of grasping the notions of God, absolute infinity, or absolutely everything entangle our understanding into paradoxes, which can be understood as a transformative experience. We think that this experience can lead to an attitude that acknowledges the incomprehensible character of the human situation or of the domain of mathematics. The epistemic significance of this non-propositional attitude can manifest itself in new values and goals. Of course, the plain argument of diagonalization seems unable to evoke a transformative experience, let alone a mystical vision.[26] However, if we contemplate on the failing attempts of conceiving the notion of absolutely everything, if we immerse ourselves into the paradox of absolute infinity, this might indeed be epistemically transformative in generating an attitude of humility that acknowledges our inability of attaining a full grasp of our practices, including the practice of mathematics. Such transformative experience is a matter of the individual, but we think that the resulting attitude can be transposed from a personal stance to a methodological attitude shared by the mathematical community. Hence, we will discuss in what follows to what extent this approach implies a revaluation of mathematical practice and suggest new values and goals in mathematical research against the background of its fundamental limitations.

The common understanding of mathematics considers proved mathematical propositions absolutely certain, which even applies to empiricist positions (pace Mill and Quine). In the nineteenth century, this appeared to be not self-evident and figures like Frege and Russell aimed at securing the foundations of mathematics. As we argued above, for them Cantor's paradox came as a shock. Besides their logicist agenda, Hilbert's program aimed at securing mathematics through its formalization, i.e., through a provably consistent axiomatization of all of mathematics. It is generally accepted that this program failed due to Gödel's incompleteness theorems, which, like Cantor's paradox, rest upon diagonalization. However, while it seems impossible to study the domain of mathematics by mathematical means like any other mathematical object, it might be feasible to found mathematics from a wider perspective, e.g., through metaphysics (cf. Welch/Horsten, 2016), or through theology (which seems to be Cantor's approach). With respect to our interpretation of negative theology, we think that this approach reiterates the problem. The *via negativa* undermines our grasp of the domain of metaphysics, i.e., the totality of everything that exists, as well as our understanding of God. On our reading, the failure of securing mathematics thus shows that, like human practice in general, the practice of mathematics is not fully transparent to us. We cannot step outside our everyday practice and we cannot step outside mathematical practice by mathematical means. In both cases, we cannot fully oversee what we are doing; on that note, we are limited.

Bertrand Russell, for example, concedes that his engagement with mathematics was driven by his personal quest for absolute certainty, which was disappointed in the end, since, as a result of his thorough investigations of the foundations of mathematics, "the splendid certainty which I had always hoped to find in mathematics was lost in a bewildering maze" (Russell, 1959, 212). This experience led him to an attitude of humility that refrains from the quest for certainty: "What was lost was the hope of finding perfection and finality and certainty" (*ibid.*). A similar attitude is expressed by Paul Bernays, one of the most important figures of axiomatic set

[26] By contrast, Arthur Koestler reports in his autobiography a mystical experience that he underwent when reflecting on the proof of Euclid's theorem that the number of primes is infinite (Koestler, 1954, 350–354). This experience was not induced by a failure of thought, but it still shows that the notion of infinity can have a strong existential significance.

theory,[27] who argues with respect to epistemological considerations about the limits of knowledge that science should employ a methodological attitude of trust and humility (Bernays, 1948, 199). Trust is indeed an important aspect of the methodological attitude that we are seeking, since this attitude is supposed to acknowledge that we do not and cannot attain a full grasp of mathematical practice. In addition, one of Bernays' PhD students, Alexander Wittenberg, points to the foundational problems of mathematics and emphasizes the existential and experiential aspect of the acknowledgement of these problems (Wittenberg, 1957, 226–227).

Be they infused with humility or not, philosophically informed methodological attitudes have certainly influenced the development of mathematics. Russell's attempt to secure the foundations of mathematics was driven by his personal longing for absolute certainty. Gödel's mathematical practice was guided by his philosophical considerations (cf. van Atten, 2015), and Brouwer, the founding father of intuitionism, was even influenced by mysticism (cf. Brouwer, 1996; Pambuccian, 1992). The recent mathematical debate on set-theoretic pluralism is also deeply shaped by metaphysical thought (cf. Rittberg, 2020). Over and above, Cantor's revolution of set theory might not have been possible without his metaphysical background. His transfinite arithmetic might not appear to reflect an attitude of humility though. Indeed, it seems bold to establish a theory of transfinite numbers in face of the paradox of absolute infinity. However, Cantor readily acknowledged that his mathematical approach leads to this paradox. In contrast to Russell's demand for absolute certainty, we think Cantor was humble in conceding that a central notion of his theory is not only mathematically inaccessible but, in terms of negative theology, beyond human comprehension altogether.

It is important to notice, though, that we are not claiming that there is no way to solve the foundational problem that we outlined in the first part of the paper. Due to the structure of this problem, such a claim would be self-defeating. It is just that we see again and again that the attempts to solve this problem fall short. This is not to say that we should stop trying. To the contrary, being driven by a quest for certainty or at least for understanding the domain can motivate great mathematics. We should try to understand the sets as best we can even if we acknowledge that perfect understanding seems impossible.

In contemporary mathematical practice, however, the foundational problem of mathematics is rather ignored or even suppressed. It is extremely rarely mentioned in textbooks or research papers outside the context of foundational theories.[28] Even in works on set theory, it usually plays a minor part. This ignorance underestimates the significance of the problem. The foundational problem concerning the domain of mathematics that we outlined in the first part of the paper affects every field of mathematics, albeit indirectly, particularly since we do not know how unbounded

[27] The above-mentioned NBG set theory is named after von Neumann, Bernays, and Gödel.

[28] Sometimes, mathematicians ignore foundational problems explicitly. Here are two examples from textbooks of algebraic geometry: "We will ignore any set theoretic difficulties. These can be overcome with standard arguments using universes" (Fantechi et al., 2005, 10—however, the global reference to the concept of universes does not solve the problem at hand). "We will not concern ourselves with subtle foundational issues (set-theoretic issues, universes, etc.). It is true that some people should be careful about these issues. But is that really how you want to live your life?" (Vakil, 2017, 18).

quantifiers are supposed to work. Hence, we think that an attitude of humility toward mathematical practice that acknowledges the boundedness of mathematics would be more adequate, and that thus negative theology, which is present in Cantor's treatment of the problem of absolute infinity, should be applied to mathematics in a transformative way. This approach does not solve the foundational problem, but establishes a constructive way of dealing with it. As in negative theology, an attitude of humility would not be achieved once and for all, since there is no final coherent statement about the issue. Instead, as in the above-mentioned dialectical situation that performatively undermines the notion of absolutely everything, the struggle with the boundedness of the human condition is an endless task.

On our reading, the acknowledgment of the boundedness of mathematics implies a revaluation of mathematical practice that may lead to a methodological attitude of humility. A possible candidate of such an attitude can be found in the above-mentioned paper by Stewart Shapiro and Crispin Wright. They refer to Michael Dummett, who argues in his book on Frege's philosophy of mathematics "that we do not know how to accomplish the task at which Frege so lamentably failed, namely to characterise the domains of the fundamental mathematical theories so as to convey what everyone, without preconceptions, will acknowledge as a definite conception of the totality in question" (Dummett, 1991, 317). With respect to this problem, Shapiro and Wright advocate an attitude that acknowledges the failure of grasping the domain of mathematics and that incorporates this acknowledgment into the practice of mathematics. This attitude is described as follows:

> [I]t is to think of modern arithmetic, analysis, and set theory as exploring the consequences of a *working hypothesis* that the natural numbers, the real numbers, and other very large, infinite totalities allow of coherent conception as Definite. We cannot—yet, and maybe never will be able to—*justify* these hypotheses from first principles in the Philosophical Theory of Understanding, but we do not have to. Since Gödel, we have become used to flying without a safety net. In this way, the mathematician can in good conscience rest content with the theories in question, even without possessing the justification whose want the philosopher laments. *Pro tem*, he may let them stand or fall on the basis of the fruits they bear, wherever these fruits may lie. (Shapiro/Wright, 2006, 281).

This way of conceiving mathematics as exploring a working hypothesis without a safety net seems to represent an attitude of humility toward mathematical practice that acknowledges its limitations. With such an attitude, the foundational problem would not be neglected, but accepted and even applied to mathematics. Such methodological attitude might even alter mathematical practice, leading to new research goals and to a wider acceptance of alternative methods, such as experimental number theory (cf. Villegas, 2007) or enumerative induction (cf. Baker, 2007; Paseau, 2021). It may also motivate the exploration of new axioms and engender a pluralist understanding of the foundations of mathematics in such a way that classical and intuitionist positions do not preclude each other but interact fruitfully. Moreover, since mathematics does not attain a full grasp of itself, we cannot be certain that proved mathematical propositions are true. However, this position is not to be confused with fallibilism in mathematics.[29] Instead, the methodological attitude we are aiming at goes deeper and acknowledges the incomprehensibility of the domain of mathematics, which may even affect the standards of mathematical proofs. Such attitude may, for example, allow for computer-assisted proofs or for proofs by picture. As Shapiro and Wright put it, mathematical theories may be judged by their fruits. In general terms, a corresponding revaluation of mathematical practice implies that mathematics should not be understood as an example of absolutely certain knowledge. According to this reading, mathematics is rather a fragmentary construction site than a consummate building. This perspective can also alter the way in which mathematics is applied to other disciplines and may affect the principles of teaching mathematics.

By contrast, we think that an attitude of humility does not amount to abandoning those parts of mathematics that seemingly lead to foundational problems. Brouwer, for example, argued that Cantor's transfinite arithmetic should be precluded from mathematics. The corresponding positions of intuitionism, finitism, or ultrafinitism are motivated by the quest for absolute certainty, which, on our reading, does not reflect an attitude of humility. As we argued in the first part of the paper, the strategy of miniaturization does not solve the foundational problem anyhow. Following this strategy, mathematical practice is not flying without a safety net, but rather is staying on the seeming ground of absolute certainty. Against that, we are seeking for a methodological attitude of trust and humility in Bernays' sense that allows mathematical practice to serenely and reliantly lift off.

Conclusion

All in all, we think that philosophy of mathematics should consider the idea of a methodological attitude of humility toward mathematical practice. In the first part of the paper, we argued at length that the domain of mathematics cannot be grasped by mathematical means and that this inability constitutes a foundational problem. In the second part, we discussed extensively Cantor's engagement with metaphysics and theology, and we argued in the third part that this engagement can be reinterpreted in terms of a performative reading of negative theology. In the fourth part, we showed that the problem of absolute generality strongly resembles negative theology too and claimed that the method of diagonalization can be regarded as a modern version of the *via negativa*. This reinforces negative theology by mathematical means, and we think that, on the other hand, mathematics can learn something about the problem of the incomprehensibility of its domain from a transformative reading of negative theology. We do not propose a solution to this problem, but we argued in the fifth part that it can be addressed by means of an adequate methodological attitude.

Philosophy of mathematics recently turned toward the practice of mathematics (cf. Mancosu, 2008). It even considers a virtue theory of mathematical practice (Aberdein et al., 2021) and discusses intellectual humility in mathematics (Rittberg, 2021). Against this background, we think that an attitude toward mathematical

[29] Cf. De Toffoli (2021).

practice that parallels the attitude of humility resulting from the *via negativa* constitutes an adequate epistemic virtue. The investigation of such an attitude may lead to a new paradigm in the philosophy of mathematics that understands and alters mathematical practice through the acceptance of its boundedness. Such a paradigm would reflect the general boundedness of the human condition, which can be acknowledged through an attitude of humility towards human practice, including the practice of mathematics, that may be gained through the negative theology of absolute infinity.

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