LAB4 Multiple Linear Regression

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Stat 415/615 Regression, 2022

Table of Contents

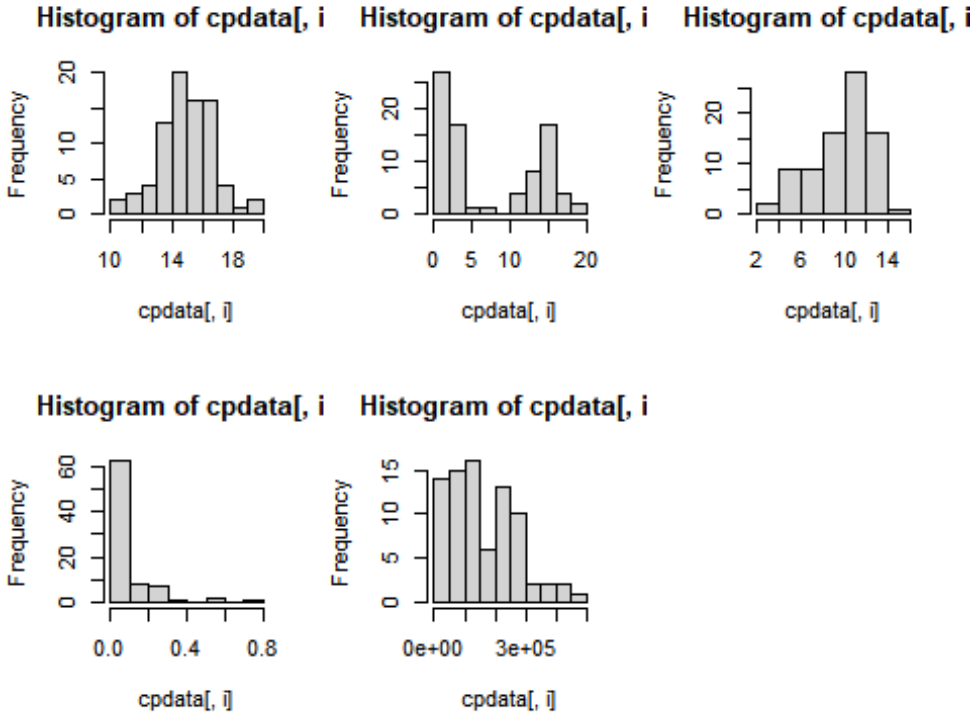
1 Commercial Property (polynomial and interactions)

```
cpdata<-read.table("../homework7/CommercialProperty.txt", header=T)
#cpdata
head(cpdata, 3) # first 3 observations

##   RentalRates Age Expense Vacancy Sfootage
## 1        13.5   1    5.02    0.14   123000
## 2        12.0  14    8.19    0.27   104079
## 3        10.5  16    3.00    0.00    39998

tail(cpdata, 3) # last 3 observations

##    RentalRates Age Expense Vacancy Sfootage
## 79       15.00  15   11.97    0.14   254700
## 80       15.25  11   11.27    0.03   434746
## 81       14.50  14   12.68    0.03   201930
```

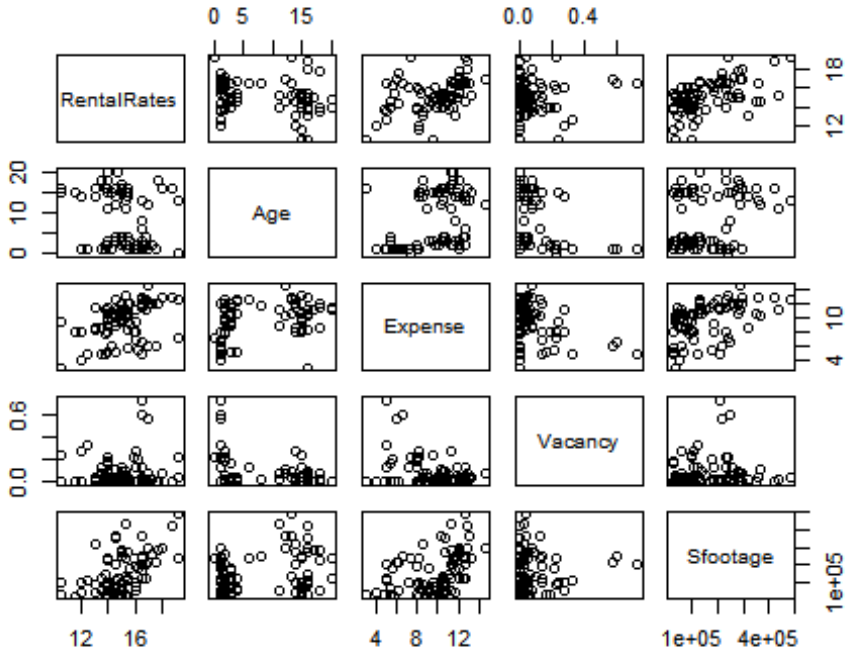
#1.1 Plot the data and comment. Also, get a numerical summary (mean,std.dev.) of all variables. Note that the mean of Age is 7.86.

```
par(mfrow=c(2,3))
for (i in 1:5) hist(cpdata[, i])
summary(cpdata)

##   RentalRates         Age            Expense          Vacancy
##  Min.   :10.50   Min.   : 0.000   Min.   : 3.000   Min.   :0.00000
##  1st Qu.:14.00   1st Qu.: 2.000   1st Qu.: 8.130   1st Qu.:0.00000
##  Median :15.00   Median : 4.000   Median :10.360   Median :0.03000
##  Mean   :15.14   Mean   : 7.864   Mean   : 9.688   Mean   :0.08099
##  3rd Qu.:16.50   3rd Qu.:15.000   3rd Qu.:11.620   3rd Qu.:0.09000
##  Max.   :19.25   Max.   :20.000   Max.   :14.620   Max.   :0.73000
##     Sfootage
##  Min.   : 27000
##  1st Qu.: 70000
##  Median :129614
##  Mean   :160633
```

```
##  3rd Qu.:236000
##  Max.   :484290
```




```
pairs(cpdata)
```




```
cor(cpdata)
```

```
##              RentalRates        Age    Expense     Vacancy   Sfootage
## RentalRates   1.00000000 -0.2502846  0.4137872  0.06652647 0.53526237
## Age          -0.25028456  1.0000000  0.3888264 -0.25266347 0.28858350
## Expense       0.41378716  0.3888264  1.0000000 -0.37976174 0.44069713
## Vacancy       0.06652647 -0.2526635 -0.3797617  1.00000000 0.08061073
## Sfootage      0.53526237  0.2885835  0.4406971  0.08061073 1.00000000
```

#1.2 Follow the instruction in Problem 8.8 from the text, we will consider a regression model with predictors Age, Age^2, Expense, and Square Footage.Fit the regression model.

```
cpdata$CentAge<-cpdata$Age-mean(cpdata$Age)
cpreg1 <- lm(RentalRates~Age+I(Age^2)+Expense+Sfootage, data=cpdata)
summary(cpreg1)
```

```
##
## Call:
## lm(formula = RentalRates ~ Age + I(Age^2) + Expense + Sfootage,
##     data = cpdata)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.89596 -0.62547 -0.08907  0.62793  2.68309
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)  1.249e+01  4.805e-01  26.000  < 2e-16 ***
## Age         -4.043e-01  1.089e-01  -3.712  0.00039 ***
## I(Age^2)     1.415e-02  5.821e-03   2.431  0.01743 *
## Expense      3.140e-01  5.880e-02   5.340 9.33e-07 ***
## Sfootage     8.046e-06  1.267e-06   6.351 1.42e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.097 on 76 degrees of freedom
## Multiple R-squared:  0.6131, Adjusted R-squared:  0.5927
## F-statistic:  30.1 on 4 and 76 DF,  p-value: 5.203e-15
```

#1.3 Add interaction terms to the previous model and examine their signifiance.

```
cpreg2<-lm(RentalRates~Age+I(Age^2)+Expense+Sfootage
+ Age:Expense + Age:Sfootage, data=cpdata)
summary(cpreg2)

##
## Call:
## lm(formula = RentalRates ~ Age + I(Age^2) + Expense + Sfootage +
##     Age:Expense + Age:Sfootage, data = cpdata)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.97236 -0.83548 -0.04637  0.68661  2.72955
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.296e+01  6.766e-01  19.151  < 2e-16 ***
## Age          -4.672e-01  1.211e-01  -3.856 0.000244 ***
## I(Age^2)      1.052e-02  6.079e-03   1.731 0.087564 .
## Expense       2.138e-01  8.127e-02   2.631 0.010357 *
## Sfootage      1.013e-05  2.370e-06   4.274 5.65e-05 ***
## Age:Expense   1.821e-02  9.962e-03   1.828 0.071539 .
## Age:Sfootage -3.125e-07  2.220e-07  -1.408 0.163392
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.085 on 74 degrees of freedom
## Multiple R-squared:  0.6319, Adjusted R-squared:  0.602
## F-statistic: 21.17 on 6 and 74 DF,  p-value: 2.672e-14

# Using centered-variables.
cpreg2b<-lm(RentalRates~CentAge+I(CentAge^2)+Expense+Sfootage
+ CentAge:Expense + CentAge:Sfootage, data=cpdata)
cpreg2b

##
## Call:
## lm(formula = RentalRates ~ CentAge + I(CentAge^2) + Expense +
```

```
##     Sfootage + CentAge:Expense + CentAge:Sfootage, data = cpdata)
##
## Coefficients:
##      (Intercept)           CentAge      I(CentAge^2)           Expense
##        9.935e+00        -3.016e-01         1.052e-02         3.570e-01
##         Sfootage   CentAge:Expense  CentAge:Sfootage
##        7.670e-06         1.821e-02        -3.125e-07

anova(cpreg1, cpreg2)

## Analysis of Variance Table
##
## Model 1: RentalRates ~ Age + I(Age^2) + Expense + Sfootage
## Model 2: RentalRates ~ Age + I(Age^2) + Expense + Sfootage + Age:Expense +
##     Age:Sfootage
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1     76 91.535
## 2     74 87.086  2    4.4488 1.8901 0.1583

anova(cpreg1, cpreg2)

## Analysis of Variance Table
##
## Model 1: RentalRates ~ Age + I(Age^2) + Expense + Sfootage
## Model 2: RentalRates ~ Age + I(Age^2) + Expense + Sfootage + Age:Expense +
##     Age:Sfootage
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1     76 91.535
## 2     74 87.086  2    4.4488 1.8901 0.1583
```

2 Example 2. Qualitative/Categorical Predictors and Interactions

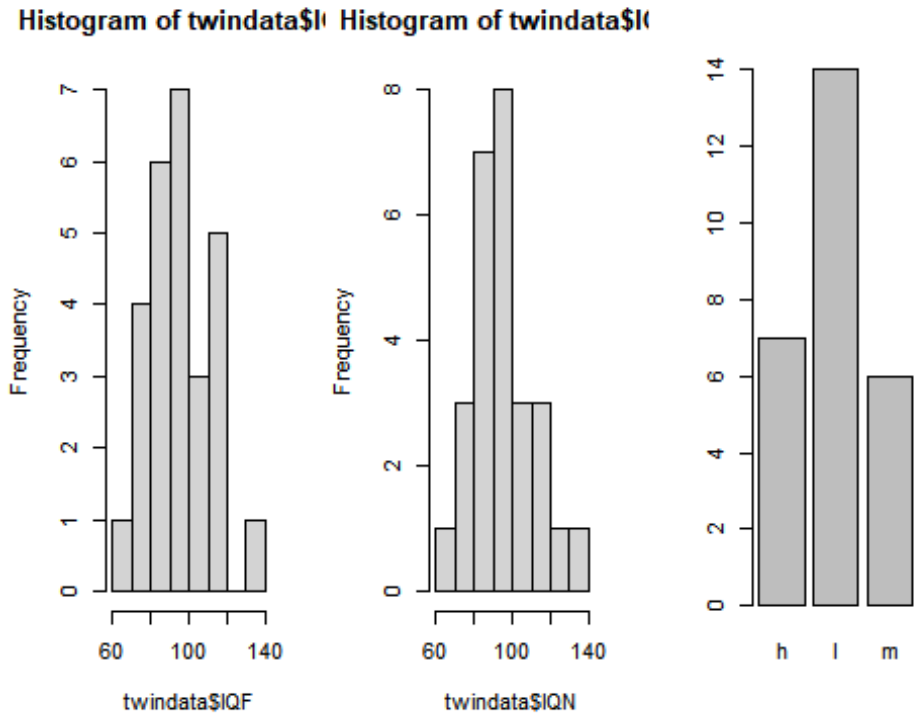
The data in file twins.txt and twins.sav are from a 1966 paper by Cyril Burt entitled "The genetic determination of differences in intelligence: A study of monozygotic twins reared apart". The data consist of IQ scores for identical twins, one raised by foster parents, the other by the natural parents. We also know the social class of natural parents (high, middle or low). We are interested in predicting the IQ of the twin with foster parents from the IQ of the twin with the natural parents and the social class of natural parents.

```
twindata<-read.table("../homework7/twins.txt", header=T)
head(twindata, 2)

##   IQF IQN status
## 1  82  82      h
## 2  80  90      h

tail(twindata, 2)
```

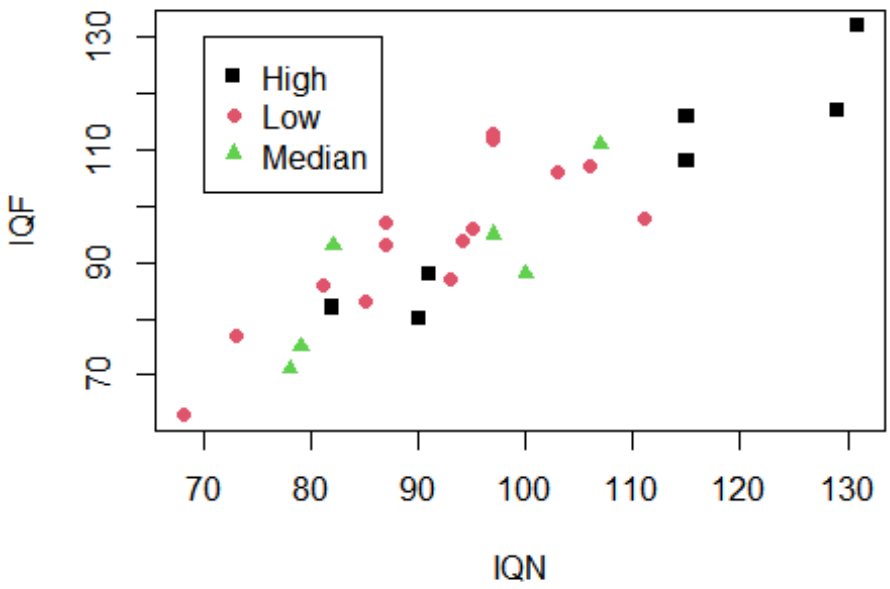
```
##    IQF IQN status
## 26 107 106      l
## 27  98 111      l

# Covert status to factor to simplify future code.
twindata$status <- as.factor(twindata$status)
```

2.1 Plot the data

```
par(mfrow = c(1, 3))
hist(twindata$IQF)
hist(twindata$IQN)
barplot(table(twindata$status))
```




```
plot(IQF ~ IQN, col=status, pch=14+as.numeric(status), data=twindata)
legend(70, 130, legend=c("High", "Low", "Median"), col=c(1:3), pch=14+c(1:3))
```



2.3 Fit regression models. Recall that we converted the categorical variable status into a factor variable earlier. Hence it can be used in the lm() function directly. If a categorical predictor is not converted into a factor, you must use as.factor(status) inside the lm() function.

```
twinreg<-lm(IQF~IQN+status, data=twindata)
# twinreg<-lm(IQF~IQN+as.factor(status), data=twindata)
summary(twinreg)

##
## Call:
## lm(formula = IQF ~ IQN + status, data = twindata)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -14.8235  -5.2366  -0.1111   4.4755  13.6978
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.6076    11.8551  -0.051    0.960
## IQN           0.9658     0.1069   9.031 5.05e-09 ***
## statusl       6.2264     3.9171   1.590    0.126
## statusm       2.0353     4.5908   0.443    0.662
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.571 on 23 degrees of freedom
```

```
## Multiple R-squared:  0.8039, Adjusted R-squared:  0.7784
## F-statistic: 31.44 on 3 and 23 DF,  p-value: 2.604e-08
```

```
twinreg2<-lm(IQF~IQN+status+IQN:status, data=twindata)
summary(twinreg2)
```

```
##
## Call:
## lm(formula = IQF ~ IQN + status + IQN:status, data = twindata)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -14.479  -5.248  -0.155   4.582  13.798
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.872044  17.808264  -0.105    0.917
## IQN          0.977562   0.163192   5.990 6.04e-06 ***
## statusl      9.076654  24.448704   0.371    0.714
## statusm      2.688068  31.604178   0.085    0.933
## IQN:statusl -0.029140   0.244580  -0.119    0.906
## IQN:statusm -0.004995   0.329525  -0.015    0.988
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.921 on 21 degrees of freedom
## Multiple R-squared:  0.8041, Adjusted R-squared:  0.7574
## F-statistic: 17.24 on 5 and 21 DF,  p-value: 8.31e-07
```

```
twinreg2<-lm(IQF~IQN+status+IQN:status, data=twindata)
summary(twinreg2)
```

```
##
## Call:
## lm(formula = IQF ~ IQN + status + IQN:status, data = twindata)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -14.479  -5.248  -0.155   4.582  13.798
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.872044  17.808264  -0.105    0.917
## IQN          0.977562   0.163192   5.990 6.04e-06 ***
## statusl      9.076654  24.448704   0.371    0.714
## statusm      2.688068  31.604178   0.085    0.933
## IQN:statusl -0.029140   0.244580  -0.119    0.906
## IQN:statusm -0.004995   0.329525  -0.015    0.988
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 7.921 on 21 degrees of freedom
## Multiple R-squared:  0.8041, Adjusted R-squared:  0.7574
## F-statistic: 17.24 on 5 and 21 DF,  p-value: 8.31e-07

anova(twinreg, twinreg2)

## Analysis of Variance Table
##
## Model 1: IQF ~ IQN + status
## Model 2: IQF ~ IQN + status + IQN:status
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1     23 1318.4
## 2     21 1317.5  2   0.93181 0.0074 0.9926

twindata$baseH<-relevel(twindata$status, ref="h")
twindata$baseM<-relevel(twindata$status, ref="m")
twindata$baseL<-relevel(twindata$status, ref="l")
as.factor(twindata$status)

##  [1] h h h h h h h m m m m m m l l l l l l l l l l l l l
## Levels: h l m

twindata$baseH

##  [1] h h h h h h h m m m m m m l l l l l l l l l l l l l
## Levels: h l m

twindata$baseM

##  [1] h h h h h h h m m m m m m l l l l l l l l l l l l l
## Levels: m h l

twindata$baseL

##  [1] h h h h h h h m m m m m m l l l l l l l l l l l l l
## Levels: l h m

twinreg.baseH <- lm(IQF~IQN+baseH, data=twindata)
twinreg.baseM <- lm(IQF~IQN+baseM, data=twindata)
twinreg.baseL <- lm(IQF~IQN+baseL, data=twindata)
```

#2.4 Interpret the regression coefficients in the context of the problem. the regression suggest that the child that was natured by a natural parent had a high IQ score compared to the other child.