Using Wavelets to Reduce Gaussian Noise

Christopher Harth-Kitzerow - J11903099005
SJTU - SEIEE
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***Abstract*—In the frequency domain noise can be seperated from image content. As using wavelets can transform an image to the frequency domain it may be used to detect image noise. In this paper different Wavelet methods are tested to detect and then reduce Gaussian noise in an image.**

***Index Terms*—Wavelets, Gaussian Noise, Matlab, PSNR, MOS, HVS**

INTRODUCTION

When taking a picture in non ideal lighting or with the wrong equipment, noise might appear in the resulting image. One kind of noise is Gaussian Noise which is difficult to detect when performing image analysis. Sophisticated methods like wavelets transformation can split up an image into different sections in the frequency domain. The idea of this paper is to detect Gaussian noise with wavelets transormation and remove it. It will be evaluated whether wavelets function of Matlab can denoise images effectively.

I. DEFINITIONS

A. Gaussian Noise

During image aquisition Gaussian noise can appear. The probability density function of gaussian noise is similiar to the normal distribution. Basic Image Enhancement techniques like spatial filters and image smooting might be successful in reducing Gaussian noise but also leads to blurry images. [1]

B. Wavelets

By using high-pass, low-pass filters and other functions, Wavelets can partition any frequency signal into sub-signals. By seperating details from approximation at different levels of the decomposition, wavelets can be used to isolate certain signals. As images are 2D-signals, wavelets can also be used to decompose the image into the approximation, vertical and horizontal details and diagonal details. Each of those part make up one frame.

II. ANALYSIS OF APPLIED WAVELET FUNCTIONS

This paper analyses result of two wavelet-based noise reduction functions using Matlab. The Empirical Bayes function and Stein's Unbiased Risk Estimate function are both wavelet-based and have been used for experiements for this paper. Both methods are utilizing a threshold to detect noise or distortion. The Emperical Bayes Method asumes independent prior distributions of measurment, while the Stein's Unbiased Risk Estimate funcion uses a threshold based on risk estimate. [4]

During this paper several pictures were tested to analyse differences between Empericial Bayes function and Stein's Unbiased Risk Estimate function. In order to analyze the effects of the two different methods, three pictures where chosen, each of them with different characteristics. Those pictures were chosen to demonstrate the analysis results as they are all different in terms of region of interest, image brightness and contrast. The first image shows two people, has a broad region of interest and is dark with a low contrast. The second image shows a face of a woman and is bright with a smaller region of interest. The third picture shows multiple ships and is quite balanced in term of contrast, region of interest and brightness.

All images were noised with the same parameters to evaluate how the two wavelet function perform in denoising the image. Also, two different intensities of Gaussian noise were tested, one with a variance $\sigma_1 = 0.01$ and the other one with $\sigma_2 = 0.02$ variance. The originial and noised image can be found below in Figure 1.

Fig. 1. Originial and nosied images
(a), (d), (g) Originial images
(b), (e), (h) Gaussian-noised images with $\sigma_1 = 0.01$
(c), (f), (i) Gaussian-noised images with $\sigma_1 = 0.02$

III. EXPERMENTAL RESULTS

Figure 2 and Figure 3 show results of applying the two wavelet functions with the different above described parameters to the images.

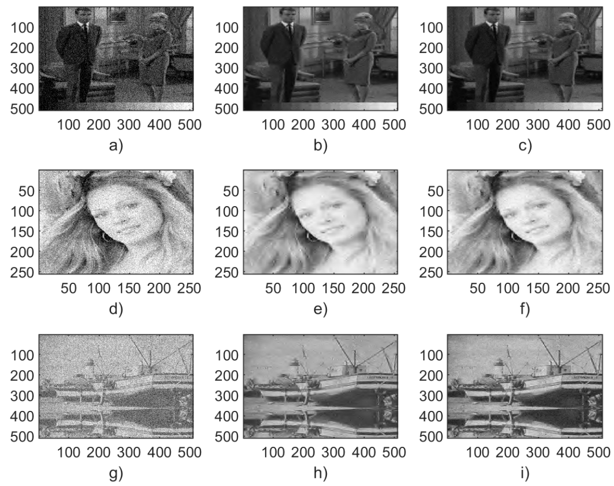

Fig. 2. Processed images with $\sigma_1 = 0.01$
(a), (d), (g) Gaussian-noised images with $\sigma_1 = 0.01$
(b), (e), (h) Processed images with Bayes' algorithm
(c), (f), (i) Processed images with Steins' algorithm

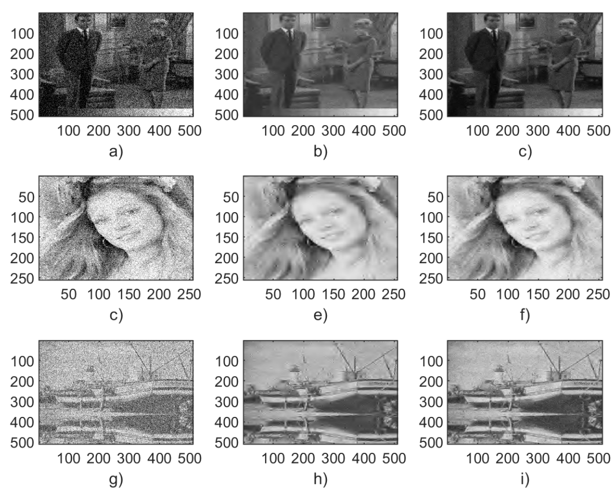

Fig. 3. Processed images with $\sigma_1 = 0.02$
(a), (d), (g) Gaussian-noised images with $\sigma_1 = 0.02$
(b), (e), (h) Processed images with Bayes' algorithm
(c), (f), (i) Processed images with Steins' algorithm

Both algortihms manage to denoise the image sucessfully to a satisfying result. When comparing the results of both algorithms, they look very similiar and only allow conclusions when inspecting closely on the HVS. At a closer look a first observation is however that Stein's algorithm performs slightly better when it comes to removing also slight noise and preserving details. In images e and f of Figure 2 one can see that Stein's algorithm both preserves more detail and is sharper than Bayes' algorithm. In images h and i however Bayes' algorithm both denoises the image more effectively and also leads to a more eye pleasing contrast. As the difference is not as strong as in the discussed images before though, Stein's algorithm still performs better overall. The potential hypothesis that Stein's algorithm could underperform in darker image is not supported by images b and c however. One can clearly see that image c is both sharper and has better contrast than image 1. This situation was also replicated when testing other images.
With increased variance as shown in Figure 3, the outperformance of Stein's algorithm becomes more clear. In images b and c of Figure 3 Bayes' algorithm shows leftover noise and less details than Stein's algorithm which denoises more effectively. Images e and f again show the biggest difference as the lady's face shows far less detail and leftover noise with Bayes' algorithm compared to Stein's algorithm. The most interesting change compared to the lower variance image samples are images h and i as in this Stein's algorithm performs better again than Bayes' algorithm. This leads to the conclusion that especially for images with a lot of noise Stein's algorithm consistently performs better.

To get an objective images assesment score, PSNR scores were computed for each image. Overall, PSNR scores of the two images are similiar and can therefore only be compared to with low statistical significance. However, in contrast to previous explained HVS results, PSNR shows a favour for Bayes' algorithm. This is surprising as especially the image of the lady showed a lot less detail with Bayes' algorithm compared to Stein's algorithm. Conducting PSNR on both images lead to results found in Table 1.

TABLE I
PSNR COMPARISON

	Denoising with Bayes'	Stein's algorithm
Couple σ_1	24.03	24.57
Lady σ_1	25.55	24.91
Boat σ_1	26.42	25.21
Couple σ_2	27.51	26.67
Lady σ_2	25.83	25.34
Boat σ_2	28.33	27.55

A Mean Opinion Score (MOS) was conducted with 5 people who rated each image after performing the algorithm with 1 to 5. Higher score means higher quality. The results can be found in Table 2 and show a statistically significant favour of Stein's algorithm over Bayes' algorithm. This result matches with the conducted analysis by this paper.

TABLE II
MOS EVALUATION

	Bayes' algorithm	Stein's algorithm
Couple σ_1	4.1	4.4
Lady σ_1	2.7	3.3
Boat σ_1	3.7	3.7
Couple σ_2	3.9	4.2
Lady σ_2	1.5	2.5
Boat σ_2	3.3	3.7

CONCLUSION

This paper analysed denoising iamges with Gaussian noise using Wavelts algorithms. Comparing MOS and PSNR of images denoised by Bayes' and Stein's algorithm one can conclude that Wavelets algorithms are sophisticated choicses when trying do denoise Gaussian noise. In addition, Wavelet's algorithms performed excellent on dark images and significantly worse in lighter images. When comparing Bayes' to Stein's algorithm, Stein's algorithm performs significantly better when evaluated by a human audience. While these results remained consistent over several tests, a higher sample size for images and human testers would be benefitial in the future to verify these results.

REFERENCES

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