ISyE 4803: Advanced Simulation **Charu Thomas**

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Due 01/29/2018

Problem 1.

(a) We are looking to estimate $\mu = \int_a^b f(t)dt$, where a = -1, b = 3, f(t) = ln($1+\sqrt{|t|}$). Apply change of variables t = a + (b - a)u = -1 + (3 - (-1))u = -1 + 4u. New integral is

$\mu = \int_0^1 (3-(-1))f[-1 + (3-(-1))u]du = \int_0^1 4f[-1 + 4u]du = E[h(U)]$ where $U \sim$ Unif(0,1) and h(u) $\equiv$ (3 - (-1))f[-1 + (3 - (-1))u] = 4f[-1 + 4u] = 4 ln(1 + $\sqrt{|-1+4u|}$).

Since f(t) = ln($1+\sqrt{|t|}$), f[-1 + 4u] = ln(1 + $\sqrt{|-1+4u|}$)

We've generated a pseudorandom sample $U_i \;\; \forall\; i = 1, ..., 20$. Next, we compute X_i = h(U_i) and find point estimate.

$\overline{x_n} = \frac{1}{20}\sum_1^{20} X_i = 2.772628527$

$s_x^2 = \frac{1}{19}\sum_1^{20}(X_i - \overline{x_n})^2 = 0.8901038167 \Rightarrow s_x = 0.9434531344$

Therefore, 95% Confidence Interval for μ is given by:

$2.772628527 \pm t_{19,0.975}\frac{(0.9434531344)}{\sqrt{20}}$ = [2.331078869, 3.214178186].

(b) $\widehat{n} = \lceil n_0(\frac{H(n_0,\alpha)}{h^*})^2 \rceil$ where $n_0 = 20$, H(n_0 , α) = 0.4415496587, $h^* = 0.10$.

Therefore, $\widehat{n} = 390$.

Problem 2.

From Wolfram Alpha, the actual value of $\mu = 2.7422$. When recalculating 100 times, 94 of the repetitions' 95% confidence intervals contained μ. This means, in my experiment, 94% truly contained the mean, μ.

Problem 3.

M/G/1 Queue: Poisson Arrivals ⇔ Exponential Interarrival Times (A ~ Exp(0.5)), Gamma Service Times (S ~ Gamma(0.8,2)), 1 Server.

(a) Kingman's Formula implies that, in steady state,

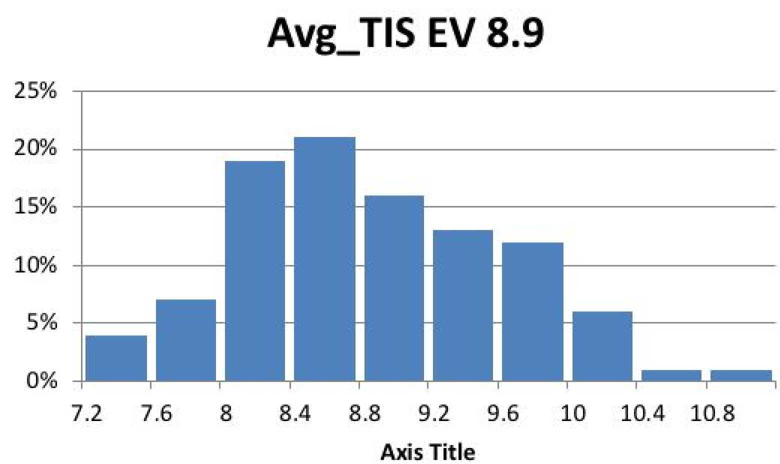
E[w_q] $\approx (\frac{\rho}{(1-\rho)})\left(\frac{c_a^2 + c_s^2}{2}\right)E[S]$.

In this scenario, $\lambda = 0.5$/per minute, $\rho = \frac{\lambda}{\mu} = \frac{0.5}{0.625} = 80\%$, $c_a^2 = \frac{Var(A)}{E[A]^2} = \frac{4}{(2)^2} = 1$, $c_s^2 = \frac{Var(S)}{E[S]^2}$ $= \frac{3.2}{(1.6)^2} = 1.25$.

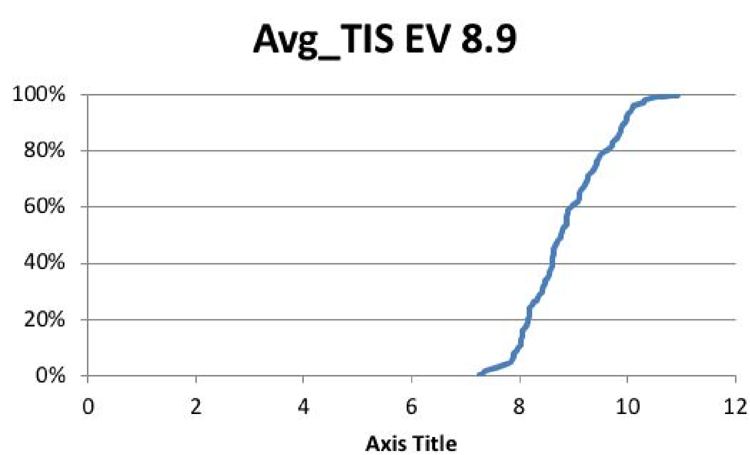
Therefore, in steady state, $E[w_q] = (\frac{0.8}{(1-0.8)})\left(\frac{1+1.25}{2}\right)(1.6) = 7.2$ minutes

(b) The 95% confidence interval for $E[\overline{x_i}] = [8.8650, 8.8700]$ minutes

(c) Histogram for Avg_TIS



Cumulative Histogram for Avg_TIS



(d) The 95% confidence interval for 90th Quantile = [23.737448, 23.750429] minutes