A quasi-polynomial-time classical algorithm for Lindbladian evolution

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Motivations
Context

- Quantum computing could have big advantage over classical computing
  - Ex: Shor’s, Grover’s, etc… but what if QC is noisy?
- *Noisy intermediate-scale quantum (NISQ) era*: low noise, but not low enough for QEC in most situations

What is the complexity of a noisy quantum system without error correction?

Motivations

Continuous Time Evolution

- Currently known: Noisy quantum circuits are *easy* to simulate (Schuster et al 2025)
- What about **continuous time**?
  - Noisy analog quantum simulations — what is their computational power?
  - We would like to use QC to simulate quantum systems in Nature, which are usually open (i.e. noisy) — how hard or easy is it to simulate these systems?
    - Applications: chemistry, materials, fundamental physics, etc

Project Overview
Task

- Classically simulate the expectation value of any local observable O for a k-local, d-degree Hamiltonian H affected by depolarizing noise within ϵ errror

Input State

$$\mathrm{tr}(\rho O(t))$$

$$\partial_t O(t) = i[H, O(t)] + \gamma \mathcal{L}^\dagger(O(t))$$

Observable Hamiltonian Noise Strength

Noise Channel

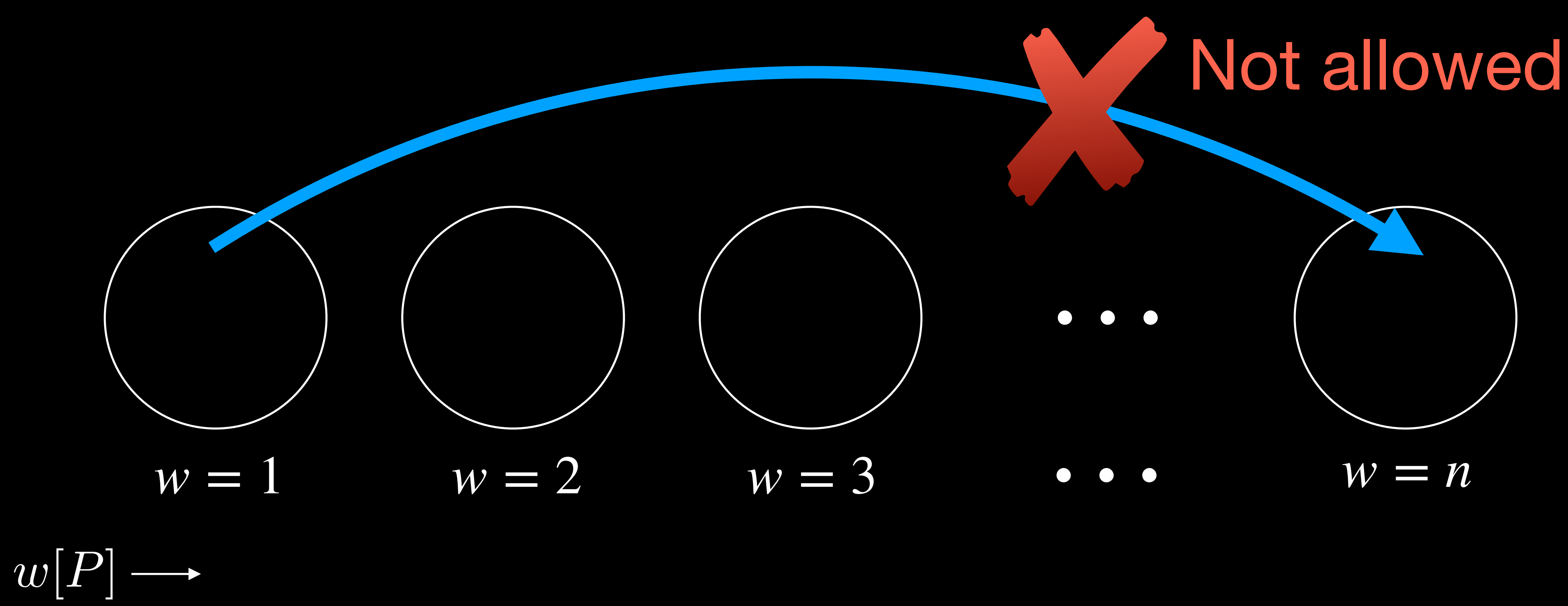
$$\mathcal{L}^\dagger : P \to -\mathrm{w}[P]P$$

$$P = \{I, X, Y, Z\}^{\otimes n}$$

Key Insight

Hard-to-simulate evolution is strongly damped by noise

- Decompose obserable in Pauli basis & group by weight
- k-local Hamiltonian can only increase weight by $k-1$ every time step



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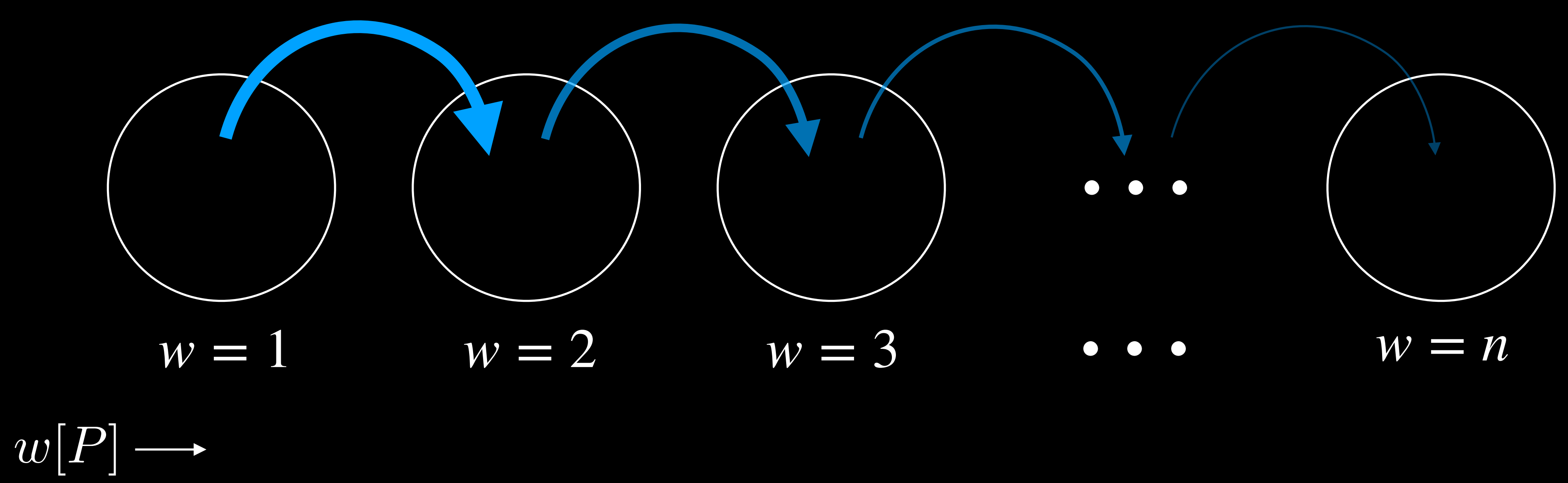
- “High-weight” components of the observable get exponentially dampened by noise under a local Hamiltonian



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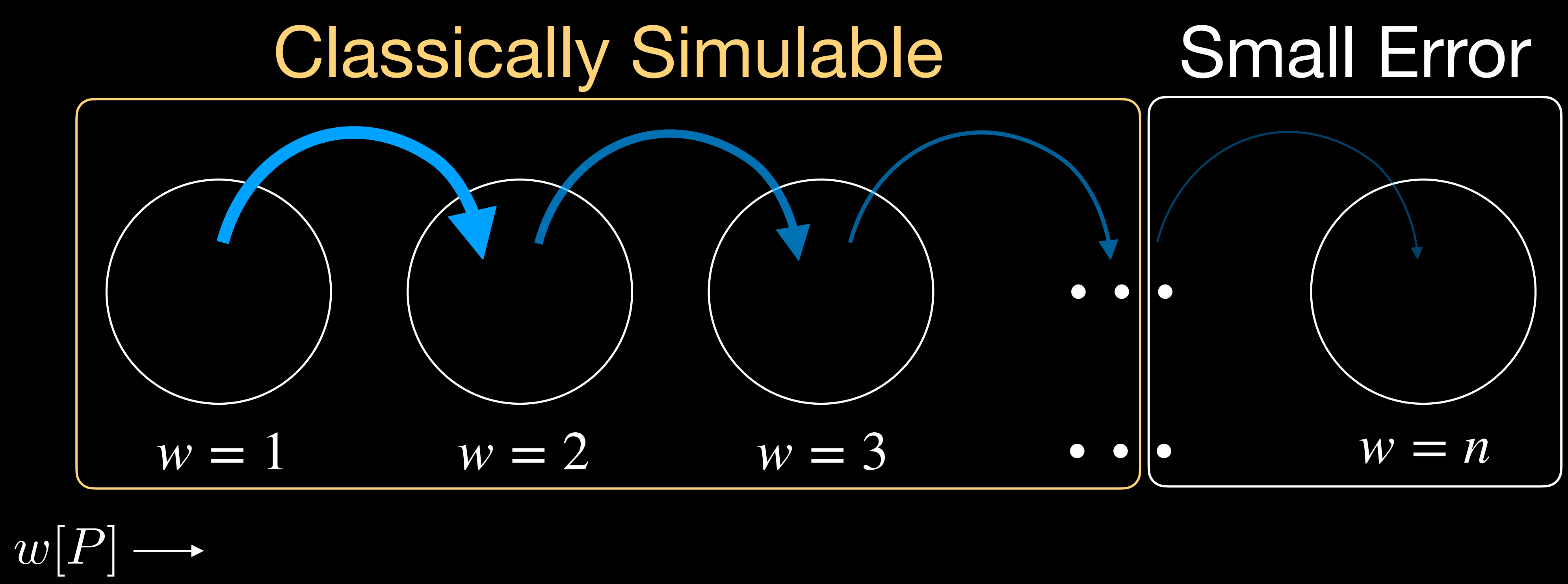
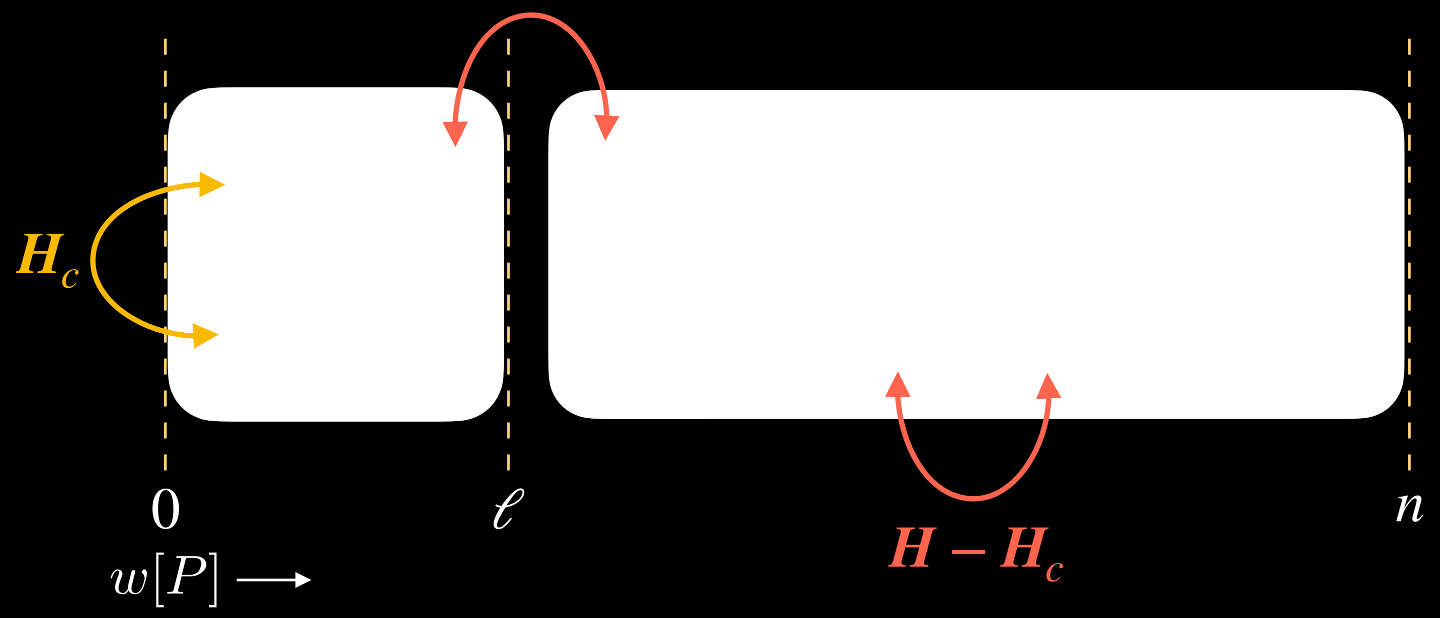


Our Algorithm

Only evolve the “low-weight” portions of the observable

- Define new matrix $\boldsymbol{H} \| O \rangle\!\rangle = \| [H, O] \rangle\!\rangle$
- Truncate matrix based on weight threshold ℓ: $\boldsymbol{H}_c = \mathcal{P}_{\leq \ell} \boldsymbol{H} \mathcal{P}_{\leq \ell}$
- Evolve observable under $\boldsymbol{H}_c$ instead of $\boldsymbol{H}$



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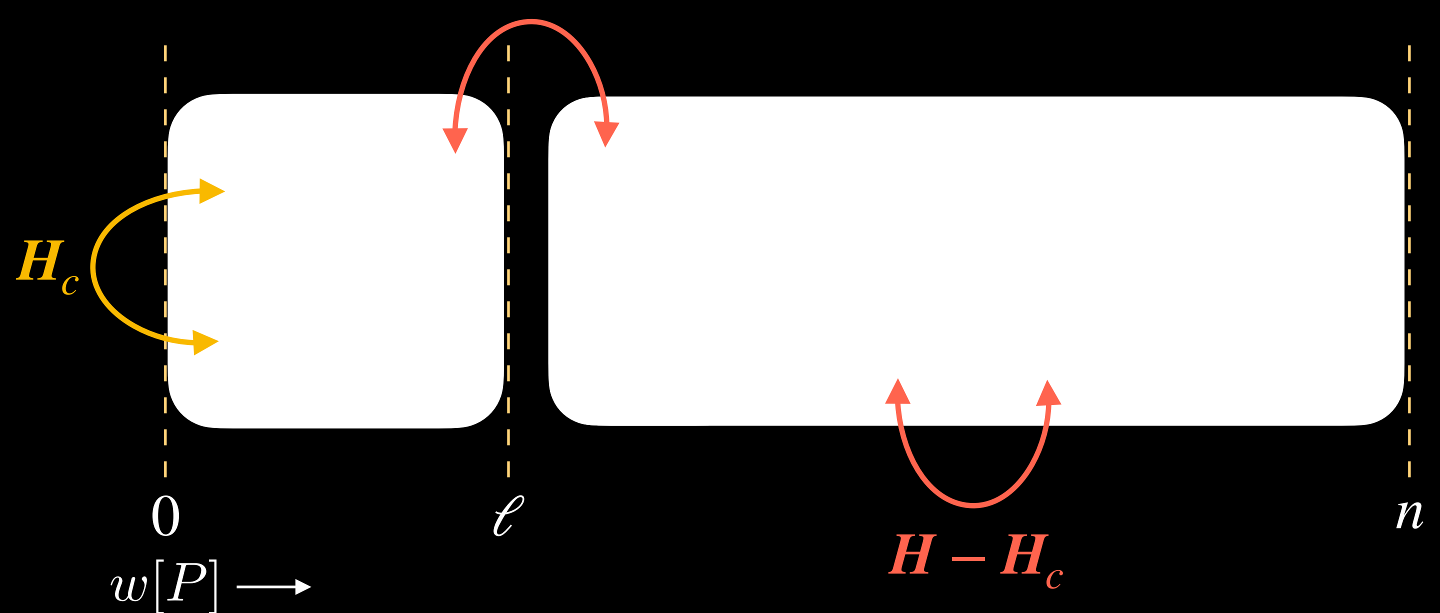


What is ℓ?

The Approach

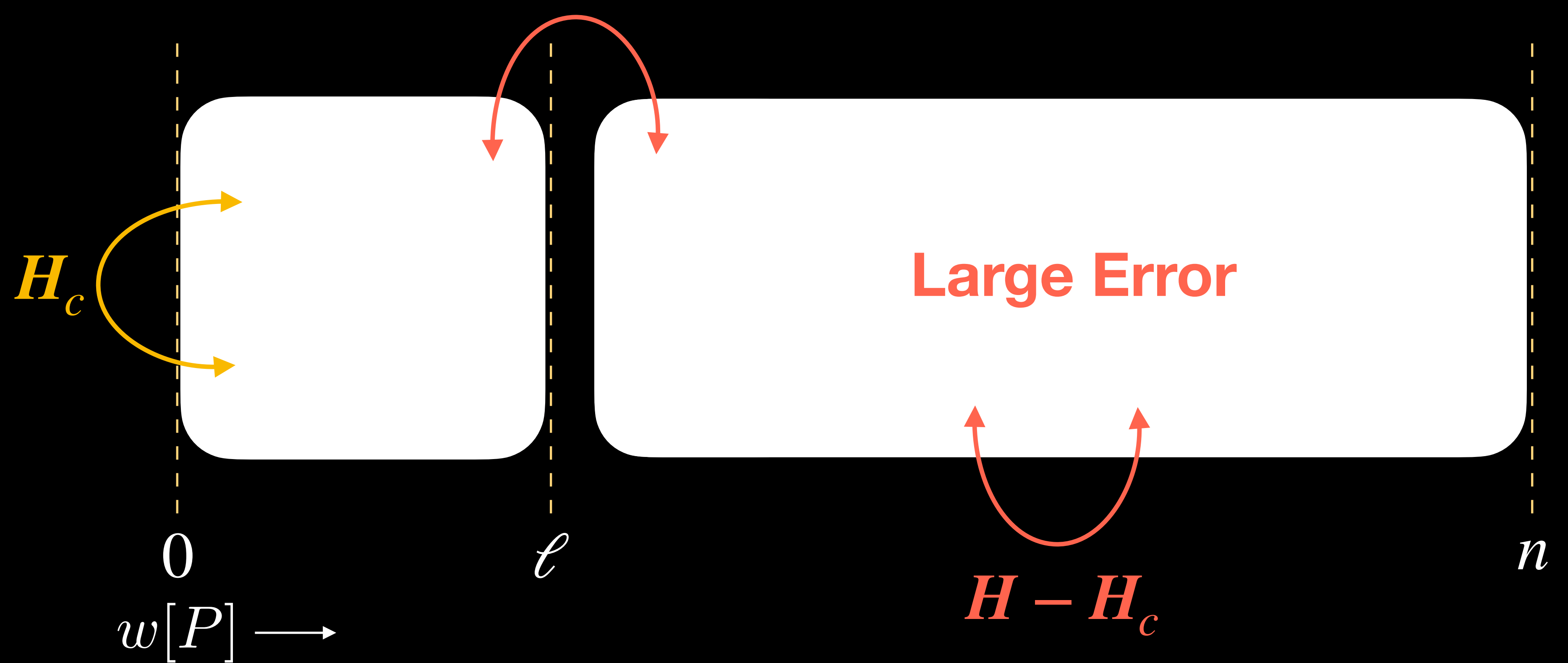
- Bound error ϵ in terms of ℓ
- Use bound to choose ℓ for a desired ϵ

Main Question: how do we tightly bound the error?

Key Techniques For Tight Bounds

Idea 1: Locality Restricts Movement in Weight Space

- Not evolving $\boldsymbol{H} - \boldsymbol{H}_c$ = Not evolving $\boldsymbol{H}_{mid}$



k-local

$$\boldsymbol{H}\|O\rangle\rangle = \|[H, O]\rangle\rangle$$

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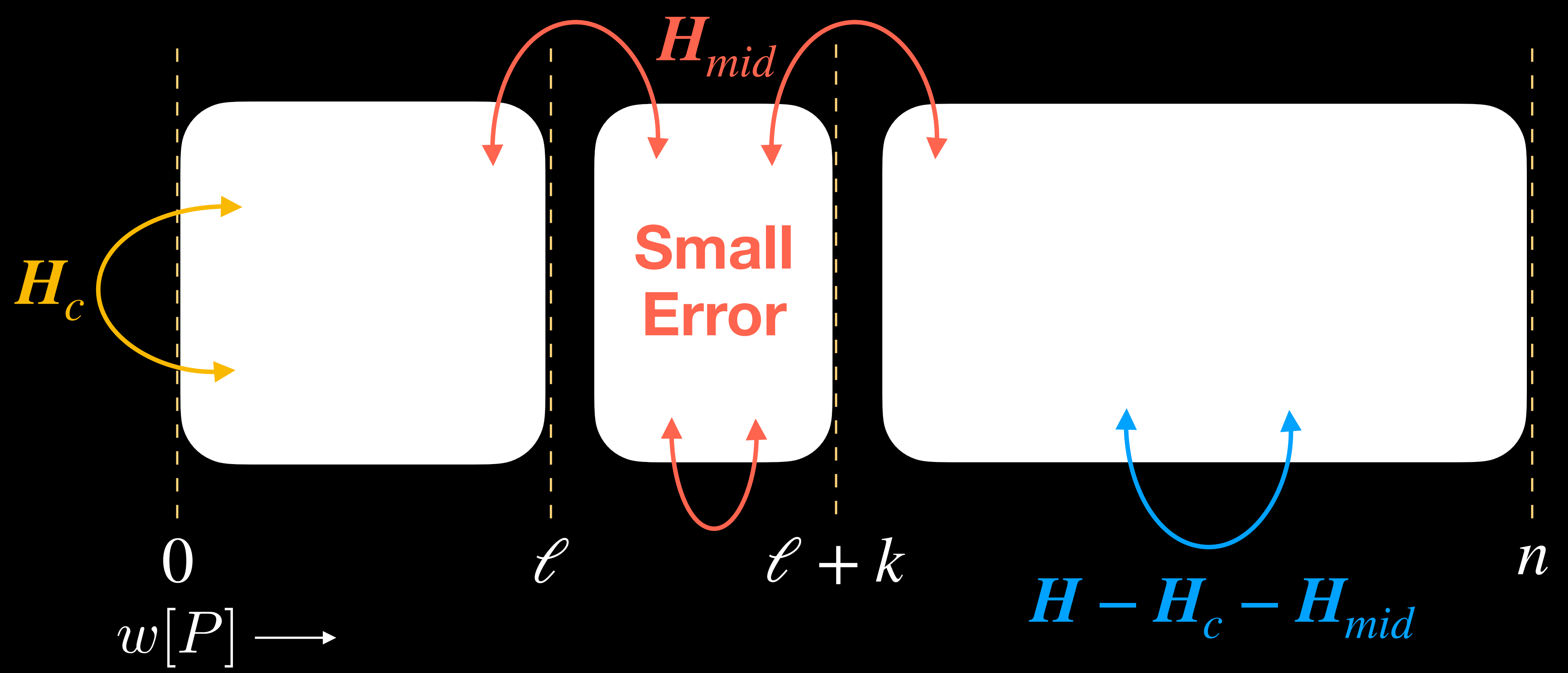

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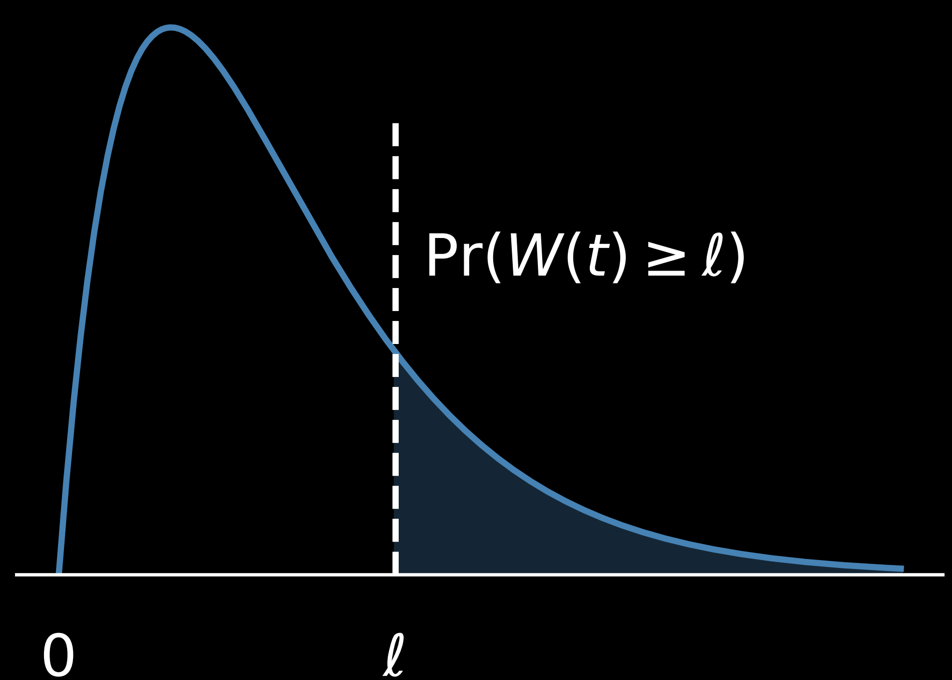
Key Techniques For Tight Bounds

Idea 2: Use Operator Weight Distribution

- How much of $O(t)$ is affected by the truncation? In other words, how high-weight does $O(t)$ become over time?

$$W(t) = \{w, P(w,t)\}$$
$$P(w,t) = \langle\langle O(t) \| \mathcal{P}_w \| O(t) \rangle\rangle$$

(Roberts et al 2018, Schuster et al 2022)



$$\Pr(W(t) \geq \ell) \leq \min_m \frac{\langle W(t)^m \rangle}{\ell^m}$$

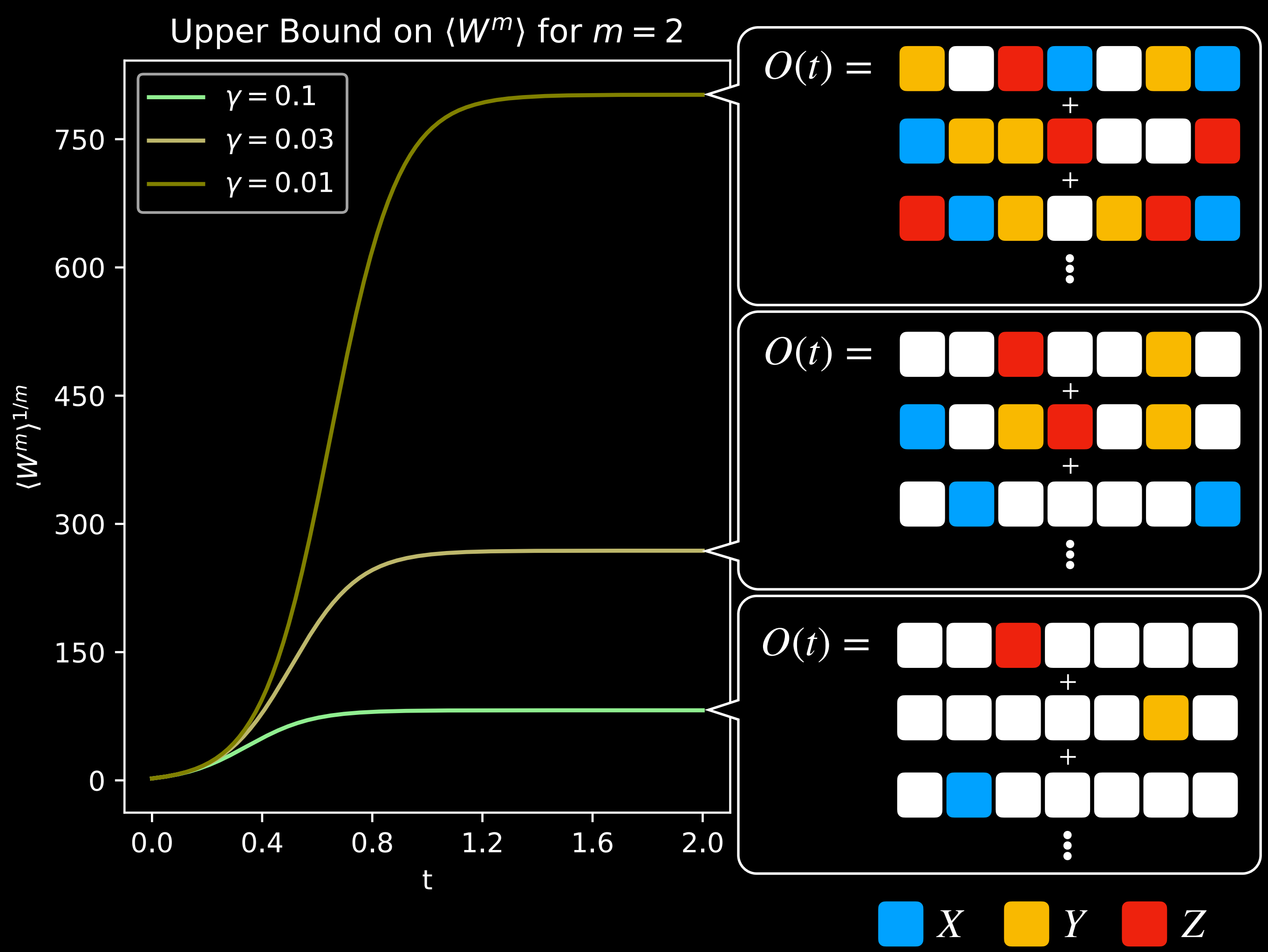
Markov's Inequality for Higher Moments

Key Techniques For Tight Bounds

Idea 3: Bound with Steady-State Solution

$$\partial_t \langle W(t)^m \rangle \leq -2\gamma u^{m+1} + 4dm(k-1)^2(u+k-1)^m$$

$$u = \langle W(t)^m \rangle^{\frac{1}{m}}$$



- The moments of $W(t)$ can not grow above an all-time constant upper bound
- Why? Differential equations have upper bounds with stable equilibirum solutions

Final Result

A Quasi-Poly Classical Algorithm

$\epsilon \leq \Theta\left(\ell t e^{-\gamma \ell}\right)$ where taking $\epsilon = \dfrac{1}{\text{poly}(n)}$ and $t = \text{poly}(n)$,

$$\ell \sim O\left(\gamma^{-1} \log \frac{t}{\epsilon}\right)$$

$$\text{time} = O(n^{\ell}) = O(n^{\log n})$$

Takeaway: Local, continuous time quantum systems can be simulated in quasi-polynomial time under any single qubit noise in the depolarizing noise class

Future Directions

- What is the complexity of quantum systems experiencing **thermal noise**? How do we distinguish classically easy and classically hard Gibbs states and time-evolutions?

- Pauli truncation methods have worked suprisingly well in practice, even without noise. Can we extend our analysis to the **noiseless case**?

Acknowlegdements

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Questions?

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