Interactive Heat Transfer Simulations for Everyone

Charles Xie, The Advanced Educational Modeling Laboratory, The Concord Consortium, Concord, MA

Heat transfer is widely taught in secondary Earth science and physics. Researchers have identified many misconceptions related to heat and temperature.[1] These misconceptions primarily stem from hunches developed in everyday life (though the confusions in terminology often worsen them). Interactive computer simulations that visualize thermal energy, temperature distribution, and heat transfer may provide a straightforward method for teaching and learning these concepts. Through interacting with visual representations of the concepts and observing how they respond to manipulations, the misconceptions may be dispelled more effectively. This paper presents a new educational simulation tool called Energy2D developed to explore this idea.

The need for a new simulation tool

As there have been many programs for solving heat transfer problems (some were designed primarily for teaching[2,3]), the reader may be wondering why we had to reinvent the wheel. There are several reasons. First, despite their commercial availability, most programs are billed as professional engineering tools and not accessible to many cash-strapped schools. Second, even if they are available in schools, engineering programs aim at achieving high accuracy and, therefore, tend to run too slowly to allow for rapid experimentation. Third, educational tools need to present results with intuitive, dynamic graphics that learners can easily understand and manipulate. Last, simulations should be easily integrated into the web to leverage the power of e-learning. Our goal is to develop a free interactive computational tool that can run simulations in real time to provide students with a powerful online learning environment for the subject of heat transfer. To achieve this goal, our computational engine must be reasonably speedy and stable. We developed fast finite-difference algorithms for solving the heat equation[4] and the Navier-Stokes equation[5] to obtain acceptable performance to support smooth animation effects and responsive user interactions. This work has resulted in a Java program called Energy2D, which—in addition to its computational fluid dynamics (CFD) engine—has many features that support visualization, graphing, simulation authoring, online deployment, and so on.

Despite the fact that Energy2D models only 2D systems, it can be useful for teaching and learning basic concepts that are not sensitive to dimensionality. In the following section, we will see how Energy2D can be used to teach heat and temperature through a simple example.

A new approach to learning

Education standards require that students in high school introductory physics be able to "explain the relationships among temperature changes in a substance, the amount of heat transferred, the amount (mass) of the substance, and the specific heat of the substance."[6]

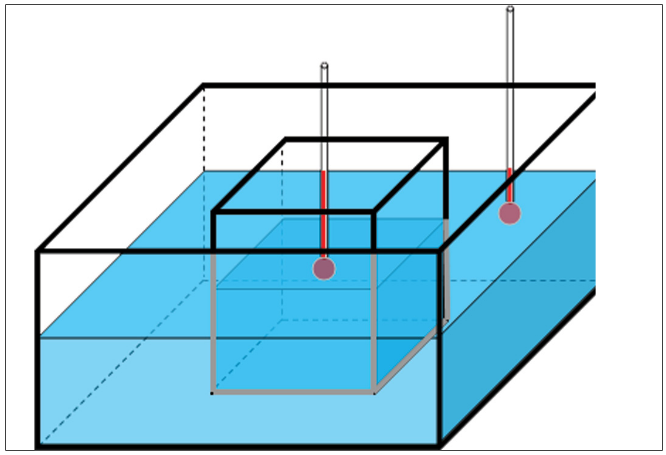

Fig. 1. A schematic drawing of a common experiment in which a cup of hot or cold liquid is placed into a container of cold or hot liquid.

An inquiry-based approach to teach these concepts commonly involves a hands-on experiment in which a cup of liquid is placed into an isolated container filled with liquid at a different temperature. A couple of thermometers are used to monitor their temperature changes (Fig. 1). Two fundamental physical laws dictate that the following things must happen:

- ***The first law of thermodynamics (i.e., the law of conservation of energy):*** Regardless of how much heat diffuses from hot liquid to cold liquid, the total energy of the system must stay the same as no thermal energy is lost or gained in an isolated system.
- ***The second law of thermodynamics:*** Regardless of how much heat diffuses from hot liquid to cold liquid, the temperature inside the system must finally become the same everywhere in an isolated system.

The change of thermal energy of the liquid in the container is:

$$Q_{con} = \rho_{con} V_{con} c_{con} (T_{fin} - T_{con}), \quad (1)$$

where T_{con} is the initial temperature of the liquid in the container, T_{fin} is the final temperature of the system, ρ_{con} is the density of the liquid, V_{con} is the volume of the liquid in the container, and c_{con} is the specific heat of the liquid.

The change of thermal energy of the liquid in the cup is:

$$Q_{cup} = \rho_{cup} V_{cup} c_{cup} (T_{fin} - T_{cup}), \quad (2)$$

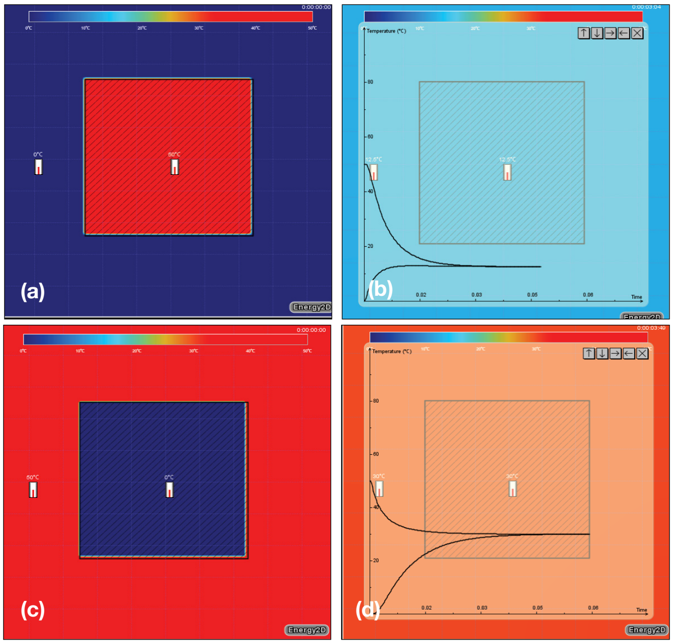

Fig. 2. (a) A cup of water (the shaded red area) at 50 °C is placed in a container filled with water at 0 °C. The size of the cup is a quarter of that of the container. (b) The temperatures recorded at locations inside and outside the cup show that they eventually approach the same value 12.5 °C. (c) An object at 0 °C (the shaded blue area) is placed in a container filled with water at 50 °C. Its size is a quarter of that of the container. Its density is twice as much as water, but the specific heats are the same. (d) The temperatures recorded at locations inside and outside the object show that they eventually approach the same value 30 °C. In both (b) and (d), the entire container appears to have exactly the same color everywhere, indicating a complete thermal equilibrium across the containers.

where T_{cup} is the initial temperature of the liquid in the cup, ρ_{cup} is its density of the liquid, V_{cup} is the volume of the liquid in the cup, and c_{cup} is the specific heat of the liquid.

The law of conservation of energy requires that $Q_{con} + Q_{cup} = 0$ for an isolated system. Therefore, the final temperature must be the following weighted mean of the initial temperatures:

$$T_{fin} = \frac{\rho_{con} V_{con} c_{con} T_{con} + \rho_{cup} V_{cup} c_{cup} T_{cup}}{\rho_{con} V_{con} c_{con} + \rho_{cup} V_{cup} c_{cup}}. \tag{3}$$

Equations (1)–(3) express the basic scientific ideas students are expected to learn. But these equations need to be linked to concrete experiences, such as hands-on experiments, to help students acquire the conceptual picture behind the mathematical formalism. Energy2D simulations provide supplementary learning experiences to hands-on labs.

To simulate real experiments with thermometers or temperature sensors, Energy2D provides virtual thermometers that show calculated temperatures at given locations. The user can drag a virtual thermometer to measure temperatures at different locations. Besides showing the current temperature value, a virtual thermometer also collects a sequence of temperature data like a data logger. Results from all the virtual thermometers in a model can be shown and compared in a graph.

The 3D problem shown in Fig. 1 can be approximated by a 2D model. Figure 2(a) shows the initial state in which a cup of warm water at 50 °C is placed into a container of cold water at 0 °C. The size of the cup is a quarter of that of the container. The boundary of the container is completely insulated so no heat can transfer across. When the user runs the simulation, thermal energy will transfer from the warm water in the cup to the cold water in the container, and the temperatures inside and outside the cup measured by the virtual thermometers will approach 12.5 °C [Fig. 2(b)], exactly as predicted using Eq. (3). Figure 2(c) shows the initial state in which a cold object at 0 °C is put into warm water at 50 °C. *The density of the cold object is twice as much as that of water.* The specific heats are the same. The water warms up the object and the final temperature settles at 30 °C, which agrees with the result given by Eq. (3).

Students can change a number of variables. For example, in addition to adjusting the initial temperatures, students can arbitrarily move or resize the rectangular object to test if and how the final temperature depends on the location and size. Students can also intervene anytime while a simulation is running (though some interventions in the middle of a simulation may not be desirable as they lead to implausible results).

Compared with their real counterparts, these simulations have the following advantages:

- They can be run quickly to allow many iterations in a short time—a single run normally takes less than a minute to complete as opposed to much longer time for preparing and conducting a real experiment.
- Thermal energy and temperature are visualized using holistic color maps that show how heat transfer actually occurs as opposed to being invisible in a real experiment.
- A wide range of properties can be explored as opposed to a limited number of materials available for comparison in a real experiment.

As recent studies have shown that virtual manipulations are just as effective as physical manipulations,[7-9] the advantages of these simulations may make them the preferred instructional materials in the classroom. In the following sections, we will present more examples to demonstrate the power of Energy2D.

Interactive visual simulations

Visual learning is a basic learning mechanism. The human brain is capable of quickly processing images, searching for patterns, and retaining impressions in the visual memory. Researchers have studied how scientific visualizations can foster learning.[10] Through salient scientific visualizations of heat and temperature, Energy2D could supplement classroom instructions with visual learning experiences. In this section, we will show a comprehensive set of examples for teaching conduction and convection.

Conduction

Thermal conduction is described by Fourier's law of heat conduction:

$$\frac{\Delta Q}{\Delta t} = -kA\frac{\Delta T}{\Delta x}, \tag{4}$$

where A is the area of the cross section through which heat is conducted, k is the thermal conductivity, ΔT is the temperature difference between the two points that are separated by a distance Δx, and ΔQ is the transferred amount of thermal energy within time Δt. Each of the factors on the right-hand side of Eq. (4) can be examined using an Energy2D simulation as shown in Fig. 3. Students can observe the rate at which heat is conducted between a heat source and a cold object through two "thermal bridges" with different properties for direct comparison. As Energy2D is easy to use, they can even be challenged to discover the circuit analogy of thermal conduction. Multiple rectangular objects with different R-values can be reshaped and dragged to form a "composite wall" between the heat source and the cold object. Students can learn how the arrangement of the objects—parallel or serial—affects the overall heat transfer rate.

Convection

Thermal convection from a surface into a fluid is described by Newton's law of cooling:

$$\frac{\Delta Q}{\Delta t} = -hA(T - T_{\text{env}}), \tag{5}$$

where T is the temperature of the surface, T_{env} is the temperature of the environment, A is the surface area, h is the convective heat transfer coefficient of the fluid, and ΔQ is the transferred amount of thermal energy within time Δt.

Convection is much more complex than conduction because it involves complicated behavior of fluids—even assumed incompressible. There are two types of convection: natural convection and forced convection, both of which can be simulated using Energy2D. Figure 4(a) shows a comparison between natural convection and conduction. The thermal conductivity of the solid on the right is set to that of the air on the left (it is hardly possible to find such a solid material, but this is something that can be easily done in a simulation). The simulation shows that convection transfers heat much faster than conduction. Figure 4(b) shows natural convection from two surfaces that have different temperatures. It approximately shows that the amount of heat transferred through convection is proportional to the difference between the surface temperature and the ambient temperature, as described by Eq. (5). Figure 4(c) shows the formation of a Rayleigh-Bénard convection pattern between a hot plate at the bottom and a cold plate at the top, indicating the accuracy of the CFD simulation. Figure 4(d) shows forced convection at different wind speeds. Clearly, the fluid motion is laminar under low wind speeds and becomes turbulent under high wind speeds.

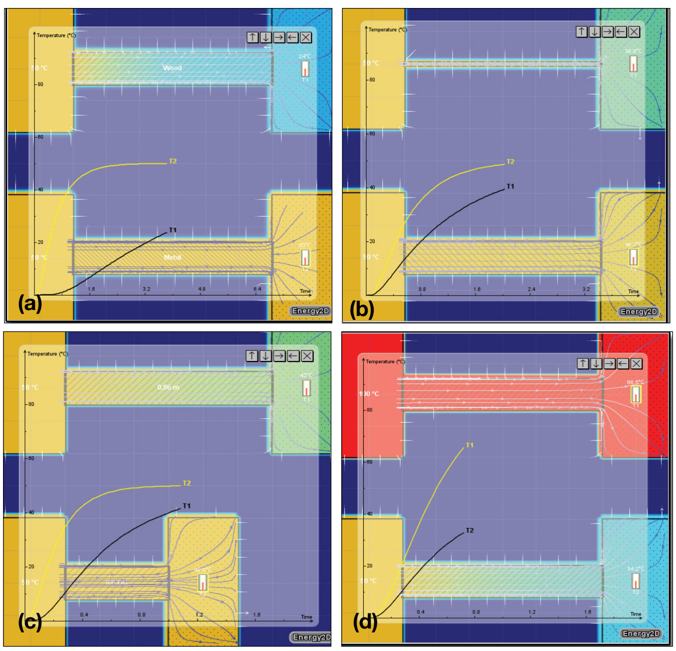

Fig. 3. These simulations compare conduction through two test pieces between a constant-temperature heat source on the left and an initially cold object on the right, under the following four conditions: (a) The test pieces are made of materials with different thermal conductivities (*k*). (b) The test pieces have different cross section areas (*A*). (c) The test pieces have different lengths (Δ*x*). (d) The temperature differences at the two ends are different (Δ*T*). In each image, the yellow curve corresponds to the highlighted thermometer. The arrow lines show the heat fluxes. These simulations allow students to study the effect of each individual factor in Fourier's law of heat conduction Eq. (4).

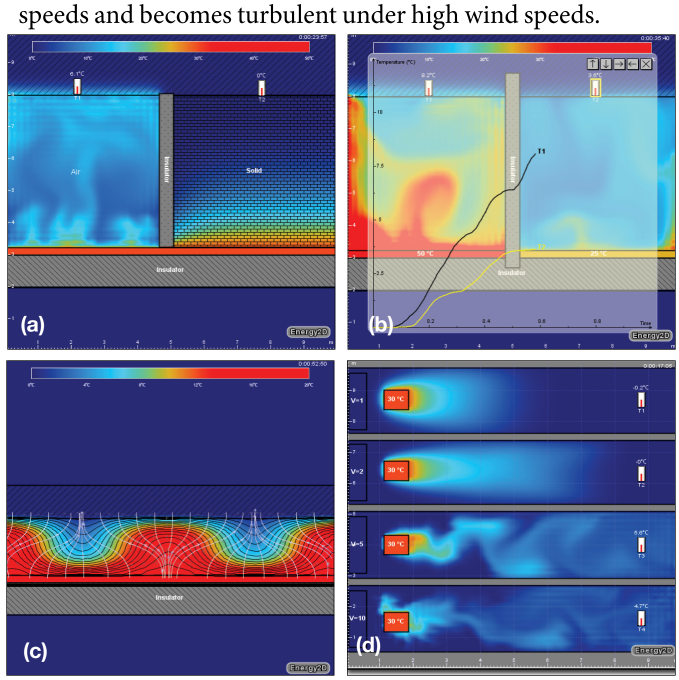

Fig. 4. (a) This simulates how heat transfers through an air chamber and a solid, both heated at the bottom. It provides a direct comparison between natural convection and conduction. (b) This simulation compares the convective heat transfer from two surfaces that have different temperatures. (c) The Rayleigh-Bénard convection cells emerge between a hot plate and a cold plate if the conditions are right (isotherm lines and heat fluxes are shown). (d) This simulation compares forced convection in separated chambers in which wind blows at different speeds. All these simulations produce realistic patterns of fluid flow.

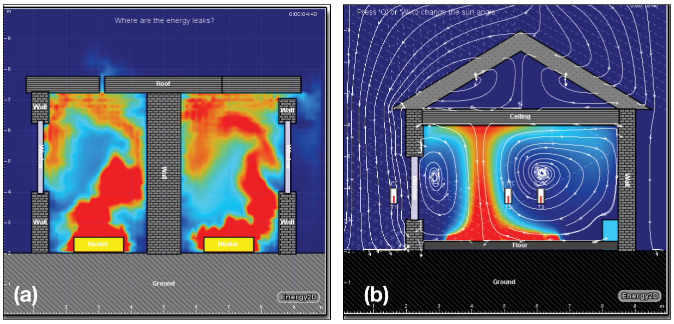

Fig. 5. (a) Infiltration of a house. The simulation shows how heat escapes through two openings. (b) Solar heating through a window. The dashed lines represent sunlight. The solid lines are the streamlines that show airflow.

Broader applications

Energy2D covers many topics in the science of heat transfer and its engineering applications. Thermal radiation is the topic that we have not mentioned so far. Energy2D has a simple ray-tracing engine that simulates the emission of photons from an object or from the Sun. The energy carried by each light particle is determined by the temperature and the emissivity of the source according to the Stefan-Boltzmann law. Each light particle travels in a straight line until it encounters an object, at which point it can be absorbed, reflected, or transmitted depending on the optical properties of the material. Due to the space constraint, we will omit examples of radiation simulations.

Energy2D may also be used to show how a building loses or gains thermal energy, a topic that may be increasingly taught in engineering and sustainability classes. Figure 5(a) shows infiltration in a house. Figure 5(b) shows how solar energy heats up the floor of a house through a window and how the heat is driven up through convection.

Concluding remarks

We have demonstrated the power of Energy2D for modeling complex thermal systems. Students now have a tool far better than a worksheet for crunching numbers to calculate heat transfer. Just like simulation-based research tools help scientists and engineers solve complicated problems, educational tools such as Energy2D have the potential to help teachers teach difficult concepts. Importantly, Energy2D is not just a collection of simulations. It is a full-fledged application that allows users to create new online simulations that can be embedded into their homepages, wikis, and blogs.

Just like many other tools, Energy2D has its own limitations. The computational engine trades accuracy for speed. As a result, its results should be considered as approximate solutions to the partial differential equations that may break down under certain circumstances. 2D simulations have some drawbacks that we did not address in this paper. Aware of the drawbacks, we are simultaneously developing a companion tool called Energy3D that will eventually make some of the simulation capabilities available in 3D. Some readers may question if the color-varying visualization could mislead students to develop an illusion that heat is a flowing substance—a common misconception dated back to the time of the caloric theory. This misconception cannot really be cleared at the macroscopic level. After all, Energy2D is a continuum simulation tool that does not involve the kinetic molecular picture of heat and temperature. Nevertheless, these visualizations are supported by infrared thermography that employs a similar coloring scheme for showing temperature.[11]

Website

Energy2D can be found at energy.concord.org/energy2d/. This site provides a link to download the stand-alone software and a collection of predesigned online simulations that can be readily run in the browser if Java has been installed on the computer.

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