Math 207A Ordinary Differential Equations: Ch. 1 Existence and uniqueness of Solutions

Charlie Seager

April 11, 2024

Chapter 1.4 Vector-Matrix Notation for Systems We can put derivatives in a matrix.

Chapter 1.5 The Need for a Theory The task of formulating a mathematical model for the motion of a physical system such as the mass-spring system or the simple pendulum leads to a differential equation; different physical approximations of the same system lead to different models (that is, different differential equations)

Chapter 1.6 Existence, Uniqueness and Continuity

Theorem 1.2 Let g, $\partial g/\partial y$ and $\partial g/\partial z$ be continous in a given region D. Let (t_0, y_0, z_0) be a given point of D. Then there exists an interval containing t_0 and exactly one solution ϕ, defined on this interval of the differential equations $y" = g(t, y, y')$ that passes through (t_0, y_0, z_0) (that is, the solution ϕ satisfies the initial conditions $\phi(t_0) = y_0, \phi'(t_0) = z_0$). The solution exists for those values of t for which the points $(t, \phi(t), \phi'(t))$ lie in D. Further, the solution ϕ is a continous function not only of t, but of t_0, y_0, z_0 as well (in fact, of the quadruple (t, t_0, y_0, z_0))

Theorem 1.3 Let h, $\partial h/\partial y_1, ..., \partial h/\partial y_n$ be continuous in a given region D. Let $(t_0, \backslash_1, ..., \backslash_n)$ be a given point of D. Then there exists on interval containing t_0 and exactly one solution ϕ defined on this interval of the differential equations $y^{(n)} = h(t, y, y', ..., y^{(n-1)})$ that passes through $(t_0, \backslash_1, ..., \backslash_n)$, [that is, the solution ϕ satisfies the initial conditions $\phi(t_0) = \backslash_1, \phi'(t_0) = \backslash_2, ..., \phi^{(n-1)}(t_0) = \backslash_n$]. The solution exists for those values of t for which the points $(t, \phi(t), \phi'(t), ..., \phi^{(n-1)}(t))$ lie in D. Further the solution ϕ is a continous function of the (n+2) variables $t, t_0, \backslash_1, ..., \backslash_n$.

Chapter 1.7 The Gronwall Inequality

Theorem 1.4 (Gronwall Inequality) Let K be a nonnegative constant and let f and g be continous nonnegative function on some interval $\alpha \leq t \leq \beta$ satisfying the inequality

$$f(t) \leq K + \int_\alpha^t f(s)g(s)ds$$

for $\alpha \leq t \leq \beta$. Then

$$f(t) \leq Kexp(\int_\alpha^t g(s)ds)$$

for $\alpha \leq t \leq \beta$