Math 207B Ordinary Differential Equations: Ch: 1 Introduction to PDE

Charlie Seager

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Chapter 1.1 Paritial Differential Equations A partial differential equation (PDE) is an equation involving an unknown function of two or more variables and certain of its partial deriviatives.

Definition An expression of the form

(1) $F(D^k u(x), D^{k-1}u(x), ..., Du(x), u(x), x) = 0 \qquad (x \in U)$

is called a kth-order partial differential equation, where

$$F : R^{n^k} x R^{n^{k-1}} x \dots R^n x R x U \to R$$

is given and

$$u : U \to R$$

is the unknown.

Definitions

(i) The partial differential equation (1) is called linear if it has the form

$$\sum |\alpha| \leq k a_\alpha(x) D^\alpha u = f(x)$$

for given functions $a_\alpha(|\alpha| \leq k), f$. This linear PDE is homogeneous if f = 0.

(ii) The PDE (1) is semilinear if it has the form

$$\sum_{|\alpha|=k} a_\alpha(x) D^\alpha u + a_0(D^{k-1}u, ..., Du, u, x) = 0$$

(iii) The PDE (1) is fully nonlinear if it depends nonlinearly upon the highest order derivatives.

Definition An expression of the form

(2) $F(D^k u(x), D^{k-1}u(x), ..., Du(x), u(x), x) = 0 \qquad (x \in U)$ is called a kth-order system of partial differential equations, where

$$F : R^{mn^k} x R^{mn^{k-1}} x \dots x R^{mn} x R^m x U \to R^m$$

is given and

$$u : U \to R^m, u = (u^1, ..., u^m)$$

is the unknown

1.2 Examples These are linear equations: Laplaces equation; Helmholtz (or eigenvalue) equation; Linear transport equation; Liouvill'es equation; heat (or diffusion) equation; Schrodinger's equation; Kolmogrorov's equation; Fokker-Planck equation; Wave equation; Klein-Gordon equation; Telegraph equation; General wave equation; Airy's equation; Beam Eqaution.
These are Nonlinear equations: Eikonal equation; nonlinear poisson equation; p-laplacian equations; minimal surface equation; Monge-Ampere equation; Hamilton-Jacobi equation; Scalar conservation law; Inviscid Burgers' equation; Scalar reaction-diffusion equation; Porous medium equation; nonlinear wave equation; kortewig-deVries (KdV) equation; nonlinear shrodinger equation;
These are linear systems: Equilibrium equations of linear elasticity; evolution equations of linear elasticity; maxwells equations;
These are nonlinear systems: System of conservation laws; reaction-diffusion system; Eulers equations for incompressible, invscid flow; Navier-Stokes equations for incompressible, viscous flow.

Chapter 1.3 Strategies for Studying PDE

Chapter 1.4 Overview