Chapter 8: Eigenvalues: Further Applications and Computations

Chapter 8.1 Diagonalization of Quadratic Forms

Every quadratic form in n variables x_i can be written as $x^T Ux$ where x is the column vector of variables and U is a nonzero upper-triangular matrix

Every quadratic form in n variables x_i can be written as $x^T Ax$, where x is the column vector of variables and A is a symmetric matrix.

Theorem 8.1 Principal Axis Theorem Every quadratic form f(x) in n variables $x_1, x_2, ..., x_n$ can be diagonalized by a substituion x = Ct, where C is an n X n orthogonal matirx. The diagonalized form appears as

$$\lambda_1 {\lambda_1}^2 + \lambda_2 {\lambda_2}^2 + ... + \lambda_n {\lambda_n}^2$$

where the λ_j are the eigenvalues of the symmetric coefficient matrix A of f(x). The jth column vector of C is a normalized eigenvector v_j of A corresponding to λ_j. Moreover, C can be chosen so that det(C) = 1