Chapter 7 Change of Basis

Chapter 7.1 Coordination and Change of Basis

Chapter 7.2 Matrix Representation and Similarity

Theorem 7.1 Similarity of Matrix Representatoins of T Let T be linear transformation of a finite-dimensional vector space V into itself, and let B and B' be ordered bases of V. Let R_B and $R_{B'}$ be the matrix representations of T relative to B and B', respectively. Then

$$R_{B'} = C^{-1} R_B C,$$

where C = $C_{B'B}$ is the change-of-coordinates matrix from B' to B. Conseqeuntly, $R_{B'}$ and R_B are similar matrices.

Significance of the Similarity Relationship for Matrices Two n X n matrices are simiilar if and only if they are matrix representations of the same linear transformation T relative to suitable ordered bases.

Similar matrices have the same eigenvalues

Theorem 7.2 Eigenvalues and Eigenvectors of Similar Matrices Let A and R be similar n X n matrices, so that R = $C^{-1}AC$ for some invertible n X n matrix C. Let the eigenvalues of A be the (not necessarily distinct) numbers $\lambda_1, \lambda_2, ..., \lambda_n$

1. The eigenvalues of R are also $\lambda_1, \lambda_2, ..., \lambda_n$
2. The algebraic and geometric multiplicity of each λ_i as an eigenvalue of A remains the same as when it is viewed as an eigenvalue of R.
3. If v_i in R^n is an eigenvector of the matrix A corresponding to λ_i, then $C^{-1}v_i$ is an eigenvector of the matrix R corresponding to λ_i.

Definition 7.2 Diagonalizable Transformation A linear transformation T of a finite-dimensional vector space V into itself is diagonalizable if V has an ordered basis consisting of eigenvectors of T.

Chapter 4.4 Linear Transformations and Determinants Let $a_1, a_2, ..., a_n$ be n independent vectors in R^m for n $\leq$m. The n-box in R^m determined by these vectors is the set of all vectors x satisfying

$$\text{x} = t_1 a_1 + t_2 a_2 + ... + t_n a_n$$

for $0 \leq t_i \leq 1$ and $i = 1, 2, ..., n$

Theorem 4.7 Volume of a box The volume of the n-box in R^m determined by independent vectors $a_1, a_2, ..., a_n$ is given by

$$\text{Volume} = \sqrt{det(A^T A)},$$

where A is the m X n matrix with a_j as the jth column vector.

Corollary Volume of an n-Box in R^n If A is an n X n matrix with independent column vectors $a_1, a_2, ..., a_n$, then —det(A)— is the volume of the n-box in R^n determined by these n vectors.

Theorem 4.8 Volume-Change Factor for T: $R^n - - > R^n$ Let G be a region in R^n of volume V, and let T: $R^n - - > R^n$ be a linear transformation of rank n with standard matrix representation A. Then the volume of the image of G under T is —det(A)—*V.