



New robust forecasting models for exchange rates prediction

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ABSTRACT

This paper introduces two robust forecasting models for efficient prediction of different exchange rates for future months ahead. These models employ Wilcoxon artificial neural network (WANN) and Wilcoxon functional link artificial neural network (WFLANN). The learning algorithms required to train the weights of these models are derived by minimizing a robust norm called Wilcoxon norm. These models offer robust exchange rate predictions in the sense that the training of weight parameters of these models are not influenced by outliers present in the training samples. The Wilcoxon norm considers the rank or position of an error value rather than its amplitude. Simulation based experiments have been conducted using real life data and the results indicate that both models, unlike conventional models, demonstrate consistently superior prediction performance under different densities of outliers present in the training samples. Further, comparison of performance between the two proposed models reveals that both provide almost identical performance but the later involved low computational complexity and hence is preferable over the WANN model.

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1. Introduction

Prediction of various currency exchange rates is influenced by many factors such as economical, political and psychological and hence it is a complex task to predict these values by conventional methods. The advantage of currency exchange rate prediction is to get financial benefits and to facilitate strategic financial planning. In the literature different prediction methods have been reported for prediction of currency exchange rates. These methods can be broadly divided into technical analysis, fundamental analysis, traditional time series and machine learning. The market is predicted by the technical analysts by tracing patterns that are mostly achieved from the study of charts that describe past market data. Fundamental analysts examine the intrinsic value of currency and observe if its current value is lower than the intrinsic one. Traditionally the time series forecasting (Harvey, 1989) employs linear prediction models to trace patterns in historic data and still is in operation for the financial market prediction. In 1976, Box–Jenkins used statistical models to forecast the financial market

(Box & Jenkins, 1976). However, the statistical methods assume that data are linearly related and therefore is not true in real life situations. The newly introduced machine learning methods have emerged to be popular as they do not make such assumptions. Specially the artificial neural network (ANN) which is inherently a nonlinear network and therefore is well suited for prediction purpose. This technique has already been applied for exchange rate forecasting (Lubecke, Nam, Markland, & Kwok, 1998; Shady & Shazly, 1997). In a recent publication a low complexity artificial single layer nonlinear neural network model is proposed for exchange rate forecasting (Majhi, Panda, & Sahoo, 2009). It has proposed two ANN models involving nonlinear inputs and simple ANN structure with one or two neurons and are known as functional link artificial neural network (FLANN) and cascaded functional link artificial neural network (CFLANN).

Subsequently many hybrid models have been suggested using the ANN for exchange rate forecasting. A novel hybridization of artificial neural networks and ARIMA model is proposed (Khashei & Bijari, 2011) in order to overcome limitation of ANNs and has been demonstrated it to be a more accurate model than the traditional ones. This model has the unique advantages of ARIMA model to identify the linear structure in the data, and a neural network is then used into take care of the nonlinear part. In another communication an alternative methodology is proposed to construct a new class of hybrid forecasting models (Khashei & Bijari, 2012). In the first stage a time series based model is used as basis model

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and its estimated values are then used in classifier model stage. Recently a nonlinear ensemble forecasting model combining linear auto regression with artificial neural networks is suggested for accurate prediction of currency rate (Yu, Wang, & Laic, 2005). In (Khashei, Bijari, & Ardali, 2009), the ARIMA model is integrated with artificial neural networks (ANNs) and fuzzy logic to overcome the linear and data limitations of ARIMA model. Different two stage forecasting models employing parametric techniques such as ARIMA, vector autoregressive (VAR) and co-integration and nonparametric techniques such as support vector regression (SVR) and artificial neural networks (ANN) have been reported in (Ince & Trafalis, 2006). Some authors have combined the ANNs and fuzzy regression (Khashei, Hejazi, & Bijari, 2008) to overcome the limitations in both ANNs and fuzzy regression models and have shown improvement in forecasting accuracy. In another hybrid model (Pai, Chen, Huang, & Chang, 2010), the rough set theory (RST) based approach is used to extract the rules of exchange rate changes; and a SVM technique is employed to deal such situations which are not taken care of in the RST model. In addition, an immune algorithm and Tabu search (IA/TS) method have been used to select parameters of SVM. A combination of neural network and a nonparametric self-organizing modelling is suggested (Anastasakis & Mort, 2009) for the daily prediction of the exchange rate market. Another hybrid model combining neural network and genetic algorithm based training is used to forecast three-month spot rate of exchange for four different currencies (Shazly & Shazly, 1999). A non-linear model combining radial basis functions and the ARMA structure is proposed in (Silva, 2008). A fuzzy neural network with statistical interval input and output values has been reported (Zhang & Wan, 2007) for currency exchange rate prediction. In another interesting paper different training subsets have been generated from the original data sets using data sampling techniques. Using these training sets, different base prediction models using ANN have been

constructed. Subsequently the principal component analysis (PCA) technique has been used to prune the base model to obtain an optimal set. Finally, a neural-network-based nonlinear predictor is designed by learning from the selected base models (Yu, Wang, & Lai, 2009). A distance-based fuzzy time series (DBFTS) model is reported in (Leu, Lee, & Jou, 2009) to predict the exchange rate. The distance-based fuzzy time series model uses the distance between two fuzzy logic relationships (FLRs) in selecting prediction rules. A two factored distance-based fuzzy time series model is constructed to predict the exchange rate. The two factors used are exchange rate and the variables affecting the fluctuation of exchange rate. In another paper, an artificial neural network forecasting model is reported (Shin & Han, 2000) which employs significant parameters extracted from wavelet transform and GA based thresholding tools.

In many practical situations time series data are contaminated with outliers. The outliers are observations that are distinct from the rest of the data. Depending on their locations in the time series these have moderate to severe effects on the performance of the adaptive prediction model. A learning machine is robust if it is least affected by the presence of outliers in the data. The literature survey reveals that very few work has been reported on developing robust prediction models (McKean, 2004) particularly when outliers are present in the raw data. When outliers are present in the past data, the conventional learning algorithms used in the adaptive model exhibit poor performance. It means that the weights or coefficients of the model do not converge near to the optimal weights and many times the cost function diverges. This is because the learning algorithms employ cost function which is the mean square error obtained from the model. Such cost function is not immune to the outliers in the data and hence the corresponding learning algorithms lead to poor prediction performance. To alleviate this problem such cost functions which are robust to outliers

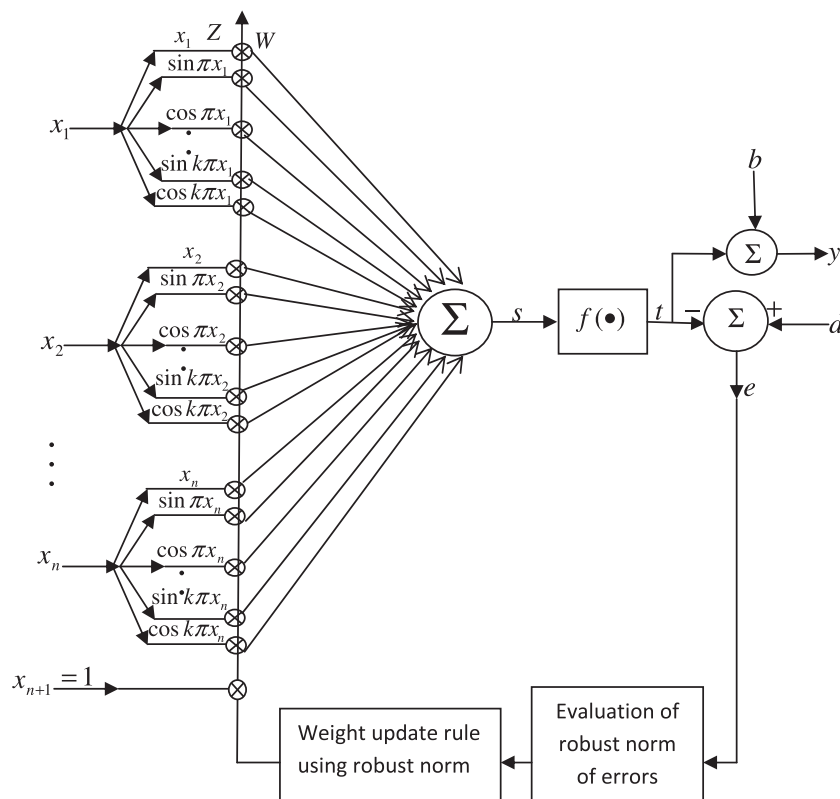


Fig. 1. Wilcoxon functional link artificial neural network (WFLANN) based robust prediction model.

need to be chosen. In statistics many cost functions have been defined which offer robustness to the outliers in the data. The Wilcoxon norm (Hsieh, Lin, & Jeng, 2008), is one of such norms. Thus there is requirement to develop a robust forecasting model which is least affected by outlier while predicting currency exchange rates. To achieve this objective a low complexity single layer functional link artificial neural network (FLANN) is used as the basic model and its weights are trained by a robust learning algorithm which is being developed in this paper. This new learning algorithm for the FLANN derived in this paper has employed the Wilcoxon norm of the errors of the model as the cost function. The new prediction model so developed is termed in this paper as Wilcoxon FLANN (WFLANN) model.

The paper is organized into seven sections. Section 1 deals with literature review, problem formulation and motivation behind the problem selection. The details of Wilcoxon norm and Wilcoxon artificial neural network (WNN) are given in Sections 2 and 4 respectively. The development of a new robust WFLANN model is dealt in Section 3. The design of input data of the model and the simulation study carried out are presented in Section 5. A comprehensive discussion on results obtained in the previous section has been made in Section 6. Finally the conclusion of the paper is dealt in Section 7.

2. Robust norm used as cost function

To explain the robust norm let a score function is first defined as an increasing function $\phi(u)$ such that

$$\int_0^1 \phi^2(u) du < \infty \quad (1)$$

A score function has the characteristics

$$\int_0^1 \phi(u) du = 0 \quad \text{and} \quad \int_0^1 \phi^2(u) du = 0 \quad (2)$$

The score associated with the score function ϕ is defined as

$$a_\phi(i) = \phi\left(\frac{i}{I+1}\right), \quad i \in I \quad (3)$$

where I is a fixed positive integer.

From (2) it may be observed that $a_\phi(1) \leq a_\phi(2) \leq \dots \leq a_\phi(I)$. The Wilcoxon norm (Hsieh et al., 2008) is defined as

$$C_1 = \sum_{i=1}^I a(R(v_i)) v_i = \sum_{i=1}^I a(i) v_i, \quad v = [v_1, v_2, \dots, v_I]^T \in \mathcal{R}' \quad (4)$$

where $R(v_i)$ denotes the rank of v_i among v_1, v_2, \dots, v_I . Let $v_{(1)} \leq v_{(2)} \leq \dots \leq v_{(I)}$ are the ordered values of v_1, v_2, \dots, v_I , and $a(i) = \phi[i/(I+1)]$. In statistics, different types of score functions have been defined but a commonly used one which is given as $\phi(u) = \sqrt{12}(u - 0.5)$ is chosen in this paper.

3. Development of robust prediction model using Wilcoxon functional link artificial neural network (WFLANN)

This section deals with the development of robust exchange rate prediction model using a FLANN structure with parallel inputs and robust Wilcoxon norm based training scheme to tackle the outliers present in the data. A block diagram of the WFLANN is proposed in Fig. 1. The predictor consists of an input layer with $(n+1)$ nodes, no hidden layer and one output node. From the time series of the exchange rate n number of features are extracted for each day. In the proposed scheme each of the n inputs undergo trigonometric based functional expansion to generate $(2k+1)$ nonlinear terms. The trigonometric expansion is chosen because such expansion based models have been shown to provide improved performance for various applications (Patra, Pal, Baliarsingh, & Panda, 1999; Patra, Panda, & Baliarsingh, 1994; Patra, Pal, Chatterji, & Panda, 1999). The $(2k+1)$ expanded version of each term (say x_1) using trigonometric expansion may be written as

$$\{x_1 \sin \pi x_1 \cos \pi x_1 \dots \sin k \pi x_1 \cos k \pi x_1\}$$

and which may be represented as $\{z(1) z(2) \dots z(2k) z(2k+1)\}$. In the similar manner other elements of the input pattern are expanded. Since there are n inputs, the total number of terms after expansion becomes $n(2k+1)$. In addition, there is a bias term, x_{n+1} with unity input. Hence including the bias term, the total number of terms becomes $M = n(2k+1) + 1$ and the nonlinear input vector is given by

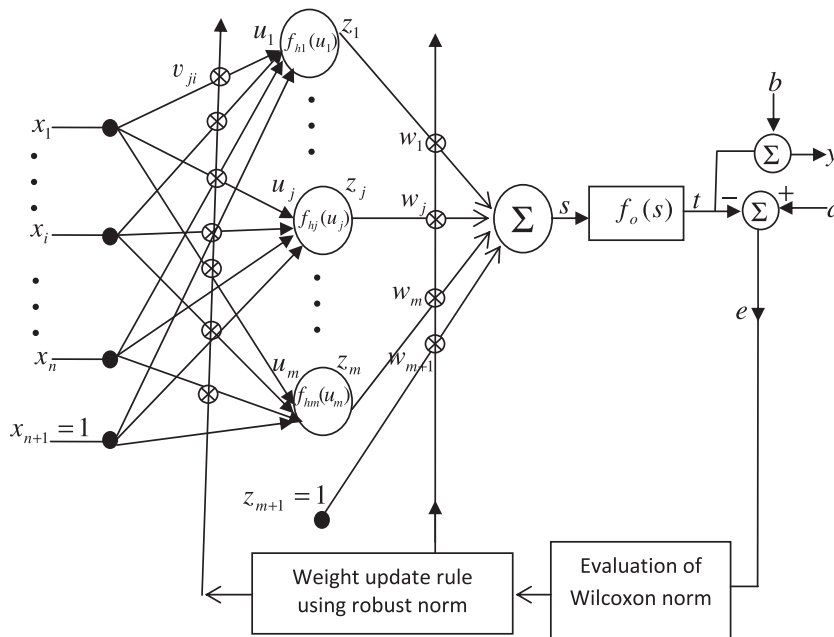


Fig. 2. A WANN model for robust exchange rate forecasting.

$$Z = [z_1 \ z_2 \ \dots \ z_m \ \dots \ z_M]^T; \quad 1 < m \leq M \quad (5)$$

where $z(M) = 1$

Let the corresponding weight vector of the prediction model is represented as

$$W = [w_1 \ w_2 \ \dots \ w_m \ \dots \ w_M]^T \quad (6)$$

Then the weighted sum output is given by

$$s = W^T Z \quad (7)$$

Referring to Fig. 1, the output after applying to the activation function which is a nonlinear sigmoid function is obtained as

$$t = f(s) = \frac{1 - \exp(-s)}{1 + \exp(-s)} \quad (8)$$

where $\exp(\bullet)$ denotes exponential operation.

Then the predicted output of the model is computed as

$$y = t + b \quad (9)$$

where b = bias value to be added to the output. The bias term is computed as

$$b = \text{median}\{d_q - t_q\}_{1 \leq q \leq Q} \quad (10)$$

Suppose Q patterns; $Q = \{x_q, d_q\}_{q=1}^Q$ are available to train the forecasting model and x_{qi} denoted the i th component of q th pattern;

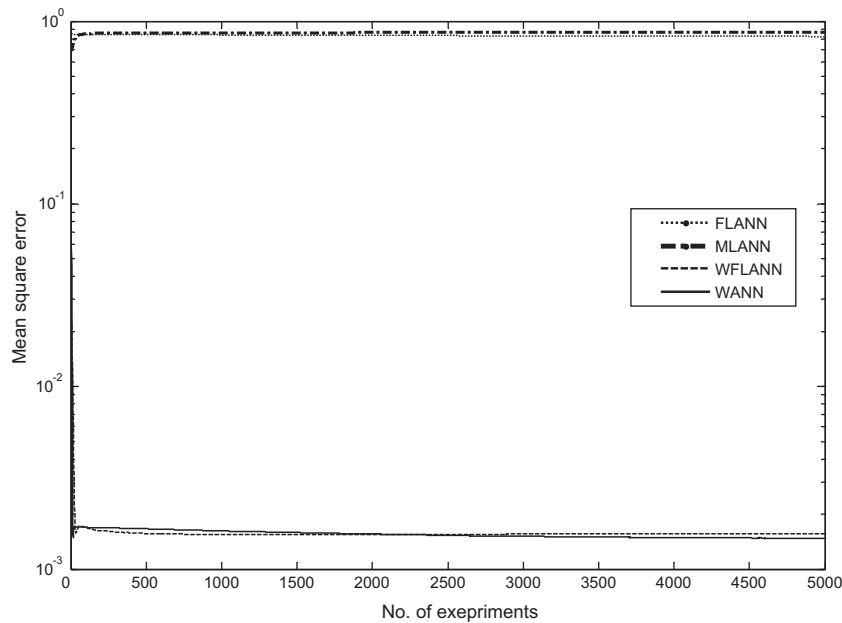


Fig. 3a. Comparison of convergence characteristic of Rupees data for 12 month ahead prediction with 50% outlier with magnitude -2 to 2 .

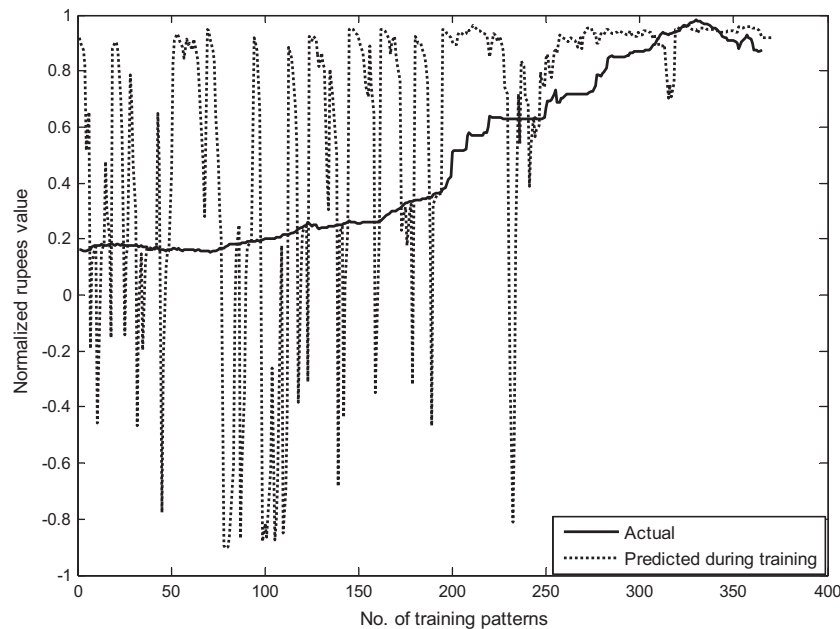


Fig. 3b. Comparison of actual and predicted value for 12 months ahead prediction using Rupees data with 50% outlier having magnitude -2 to 2 during training using MLANN model.

$q \in Q$ and $i \in n$. The variable d_q denotes the corresponding target exchange rate. In the proposed WFLANN model the optimal weights are achieved by minimizing the Wilcoxon norm of the total error values defined as

$$\xi = \|e\|_W = \sum_{i=1}^Q a\{R(e_q)\} e_q \quad (11)$$

where e = error vector $= \{e_1 \ e_2 \cdots e_q \cdots e_Q\}^T$ obtained due to application of Q number of patterns. For any q th pattern the error term is given by

$$e_q = d_q - t_q \quad (12)$$

$a\{R(e_q)\}$ = score associated with the score function defined in (3).

$R(e_q)$ = the rank of the error e_q among the error values $\{e_1 \ e_2 \cdots e_q \cdots e_Q\}$ and is obtained as its position number after arranging the Q number error values in increasing order. If p , denotes the rank of an error e_q , then $a\{R(e_q)\}$ is computed as

$$a\{R(e_q)\} = a(p) = \sqrt{12} \left(\frac{p}{Q+1} - 0.5 \right) \quad (13)$$

Using (13), (11) may be rewritten as

$$\xi = \sum_{q=1}^Q \sqrt{12} \left(\frac{p}{Q+1} - 0.5 \right) \times e_q \quad (14)$$

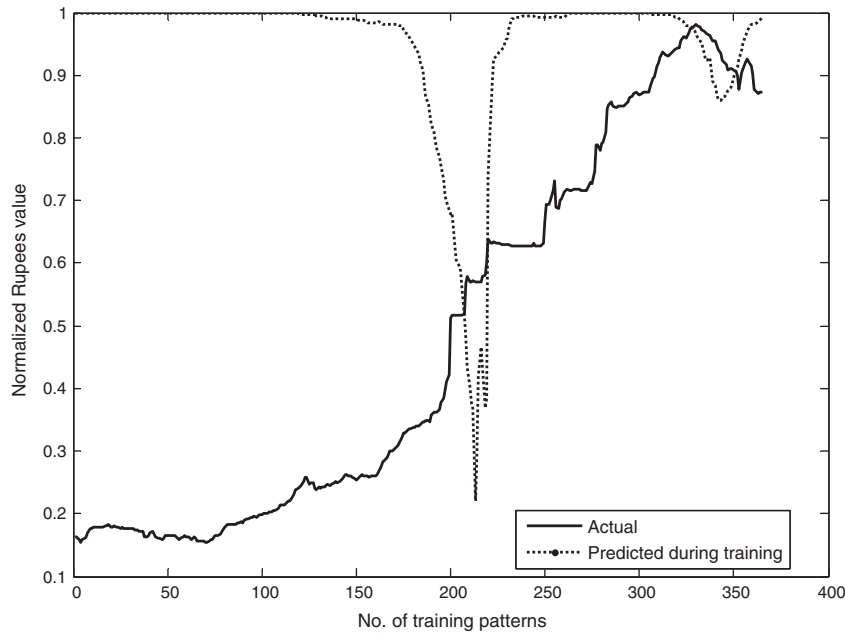


Fig. 3c. Comparison of actual and predicted value for 12 months ahead prediction using Rupees data with 50% outlier having magnitude -2 to 2 during training using FLANN model.

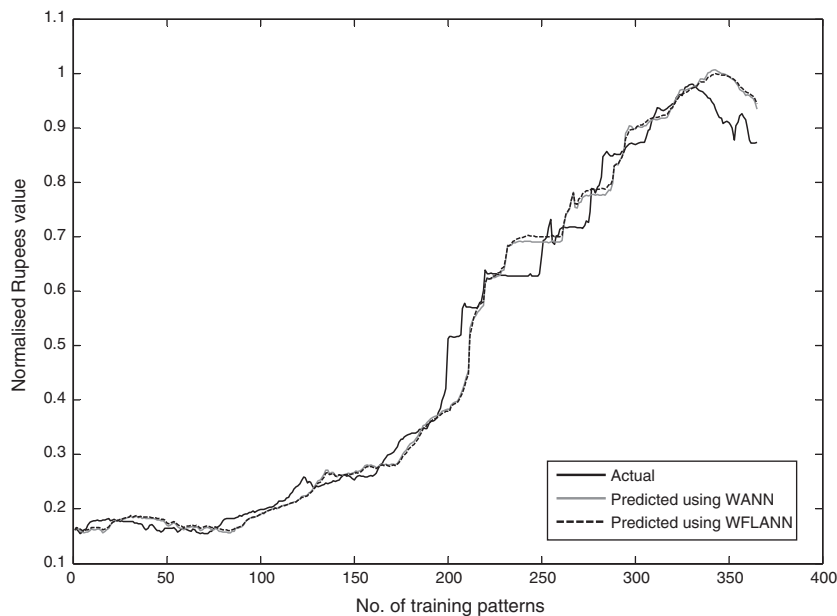


Fig. 3d. Comparison of actual and predicted value for 12 months ahead prediction using Rupees data with 50% outlier having magnitude -2 to 2 during training using WANN and WFLANN models.

where p , $1 \leq p \leq Q$, denotes the rank of e_q . The weight update rule of the m th weight of the model is given by

$$w_m = w_m - \eta \frac{\partial \xi}{\partial w_m}; \quad 1 \leq m \leq M \quad (15)$$

where η = learning parameter within the range of 0 to 1. Using Eqs. (7), (8), (12) and (14), the derivative term in (15) may be obtained as

$$\frac{\partial \xi}{\partial w_m} = - \sum_{q=1}^Q \sqrt{12} \left(\frac{p}{Q+1} - 0.5 \right) f'(s) z_m \quad (16)$$

where $f'(s)$ denotes the derivative of $f(s)$ defined in (8) with respect to the connecting weights of the model. Using (16), (15) can be rewritten as

$$w_m = w_m + \eta \sqrt{12} \sum_{q=1}^Q \left(\frac{p}{Q+1} - 0.5 \right) f'(s) \cdot z_m \quad (17)$$

where $f(s)$ and z_m are defined in (8) and (5), respectively. Eqs. (7), (8), (9), (12) and (17) represent the key equations of the WFLANN forecasting model which are used in the simulation section.

4. Wilcoxon artificial neural network (WANN) based forecasting model

Fig. 2 depicts a WANN based robust exchange rate forecasting model which is robust against outliers present in the training samples due to uncertainty of the data. This model has been adopted from (Hsieh et al., 2008) and has been suitably modified to suit

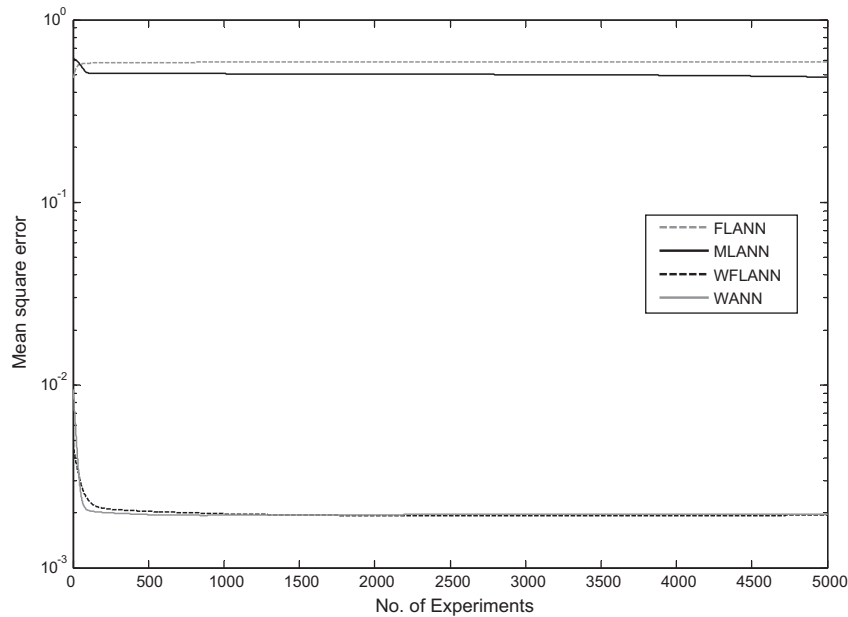


Fig. 4a. Comparison of convergence characteristic of Pound data for 9 month ahead prediction with 40% outlier with magnitude -2 to 2 .

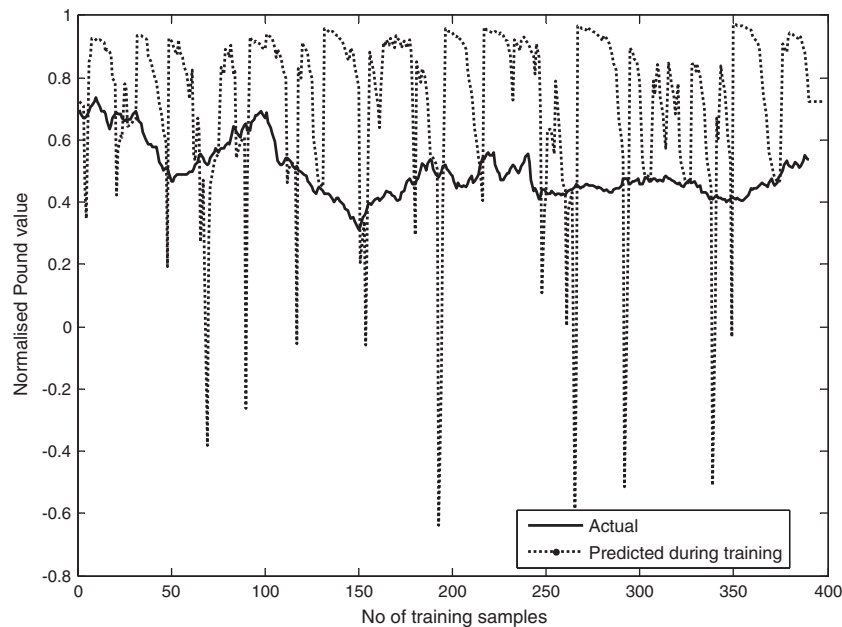


Fig. 4b. Comparison of actual and predicted value for 9 months ahead prediction using Pound data with 40% outlier having magnitude -2 to 2 during training using MLANN model.

for the prediction purpose. It has one input layer with $(n + 1)$ nodes, one hidden layer with $(m + 1)$ nodes and one output layer with one node. The final output is obtained by adding a bias term to the output node. The input feature vector is extracted from the exchange rate series and is represented as $X = [x_1 \ x_2 \ \dots \ x_n \ x_{n+1}]^T$. Let v_{ji} represents the connecting weights between input and the hidden layers and w_j , $(1 \leq j \leq m + 1)$ denotes the j th weight between j th hidden node and the output. The values of input u_j and output z_j of the hidden node are calculated as

$$u_j = \sum_{i=1}^{n+1} v_{ji}x_i, \quad x_{n+1} = 1, \quad z_j = f_{hj}(u_j) = \frac{1 - \exp(-u_j)}{1 + \exp(-u_j)}, \quad j \in \underline{m} \quad (18)$$

Similarly let w_j represents the connecting weights from j th hidden node to the output node. In the same way the input s and output t of the output node are given as

$$s = \sum_{j=1}^{m+1} w_j z_j, \quad z_{m+1} = 1, \quad t = f_o(s_k) = \frac{1 - \exp(-s)}{1 + \exp(-s)}, \quad k \in \underline{p} \quad (19)$$

where $\exp(\bullet)$ denotes exponential operation. Unlike the normal neural network models the final output of the network is given as

$$y = t + b \quad (20)$$

where b is the bias defined in (10).

Suppose Q patterns, $Q = \{(x_q, d_q)\}_{q=1}^Q$ are available for training purpose. Let, x_{qi} denotes the i th component of q th pattern, $q \in Q$.

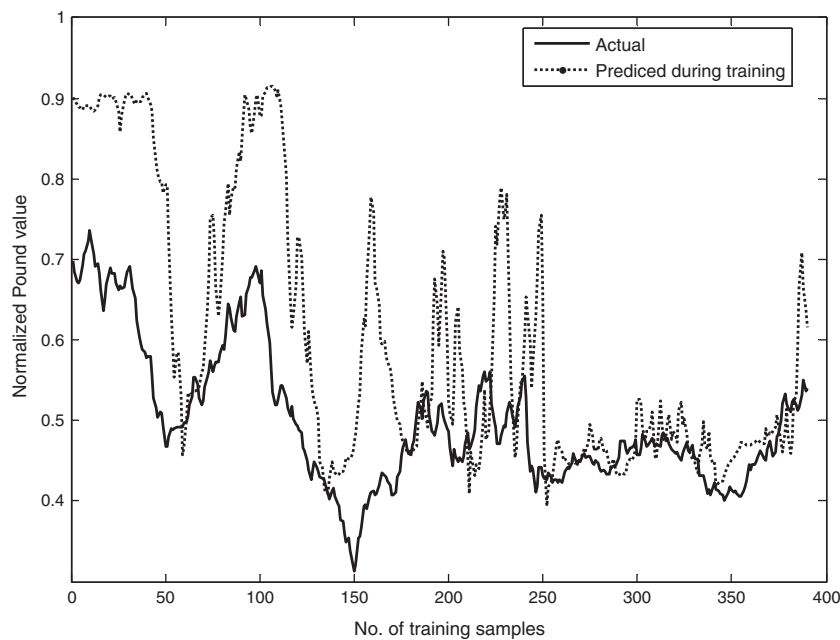


Fig. 4c. Comparison of actual and predicted value for 9 months ahead prediction using Pound data with 40% outlier having magnitude -2 to 2 during training using FLANN model.

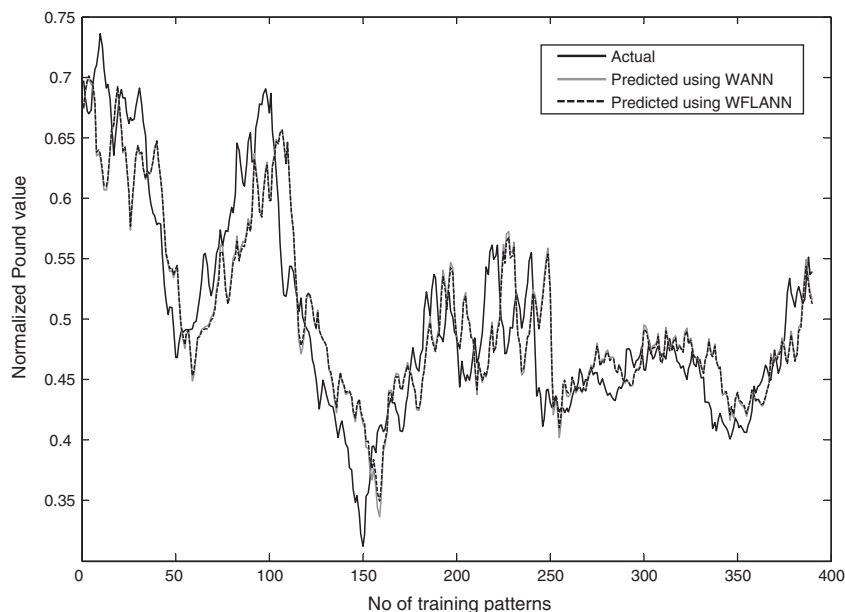


Fig. 4d. Comparison of actual and predicted value for 9 months ahead prediction using pound data with 40% outlier having magnitude -2 to 2 during training using WANN and WFLANN models.

$i \in \underline{n}$ and d_q the corresponding target exchange rate. In a WANN model, the required weights are obtained by suitably minimizing the robust norm of the error terms defined as

$$\xi = \sum_{q=1}^Q a(R(e_q)) e_q \quad (21)$$

$$e_q = d_q - t_q, \quad q \in Q \quad (22)$$

where $R(e_q)$ denotes the rank of the error e_q among $\{e_1, e_2, \dots, e_q, \dots, e_Q\}$ and computed as the position of the q th error when the error vector is arranged in an increasing order. Following (Hsieh et al., 2008), if p , $1 \leq p \leq Q$ denotes the rank of error e_q , the update rule for j th weight of the output layer is derived to be

$$w_j = w_j + \eta \sqrt{12} \sum_{q=1}^Q \left(\frac{p}{Q+1} - 0.5 \right) f'(s) z_j \quad (23)$$

where $f'_o(\bullet)$ denotes the derivative of $f_o(\bullet)$. In a similar manner the weight update equation of the input layer is derived as

$$v_{ji} = v_{ji} + \eta \sum_{q=1}^Q a(R(e_q)) f'_o(s_q) w_j f'_{hj}(u_{qj}) x_{qi}, \quad j \in m+1, \quad i \in n+1 \quad (24)$$

where $f'_{hj}(\bullet)$ denotes the derivative of $f_{hj}(\bullet)$ and u_{qj} is the j th component of the q th vector u_q . Eqs. (18)–(20), (22)–(24) describe the key equations pertaining to WANN forecasting model.

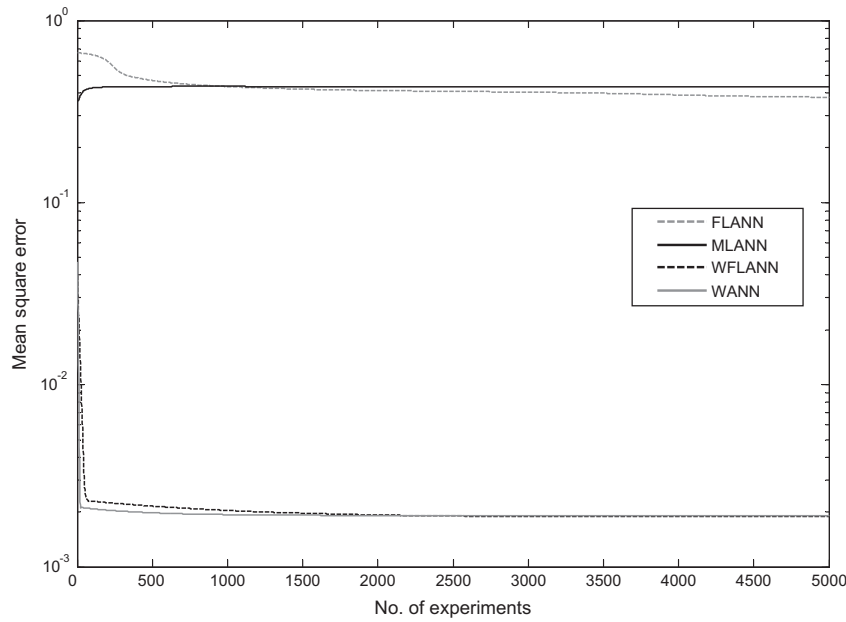


Fig. 5a. Comparison of convergence characteristic of Yen data for 6 month ahead prediction with 30% outlier with magnitude -2 to 2 .

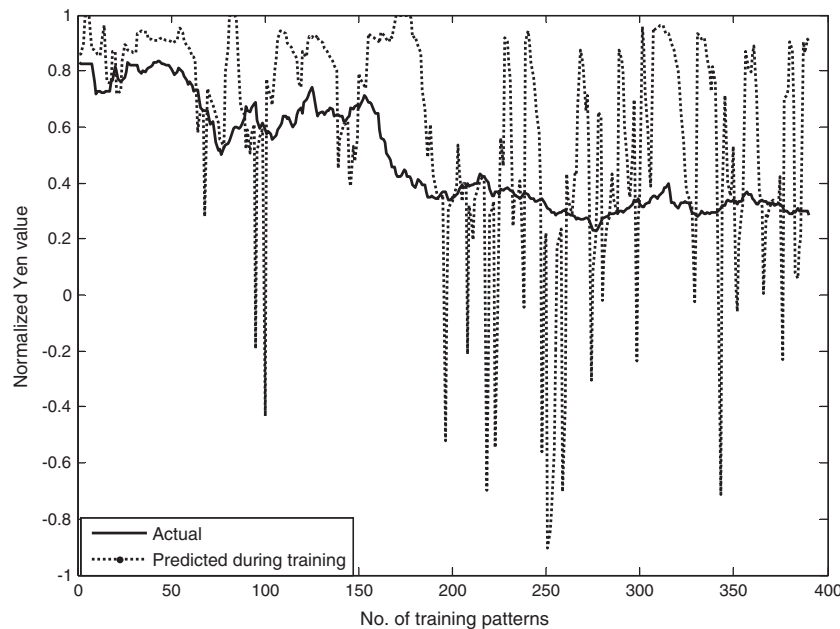


Fig. 5b. Comparison of actual and predicted value for 6 months ahead prediction using Yen data with 30% outlier having magnitude -2 to 2 during training using FLANN model.

5. Simulation study

In this section different exchange rate prediction experiments are conducted to assess the performance of the proposed robust model using practical sets of data. The robust performance is evaluated by contaminating the training sets with outliers. To compare the performance of the new model, the WANN, MLANN and FLANN prediction models are also simulated under identical conditions. The simulation experiments include prediction of conversion rates from 1US\$ to Pound, Rupees and Yen in presence of outliers in the training sets. The experiments on forecasting models consists of three major steps:

- Design of input patterns.
- Training of model.
- Validation of the model.

5.1. Design of input patterns

It involves the following steps:

- Collection of raw data.
- Normalization of the data.
- Feature extraction from the data.
- Design of input patterns for training and testing.

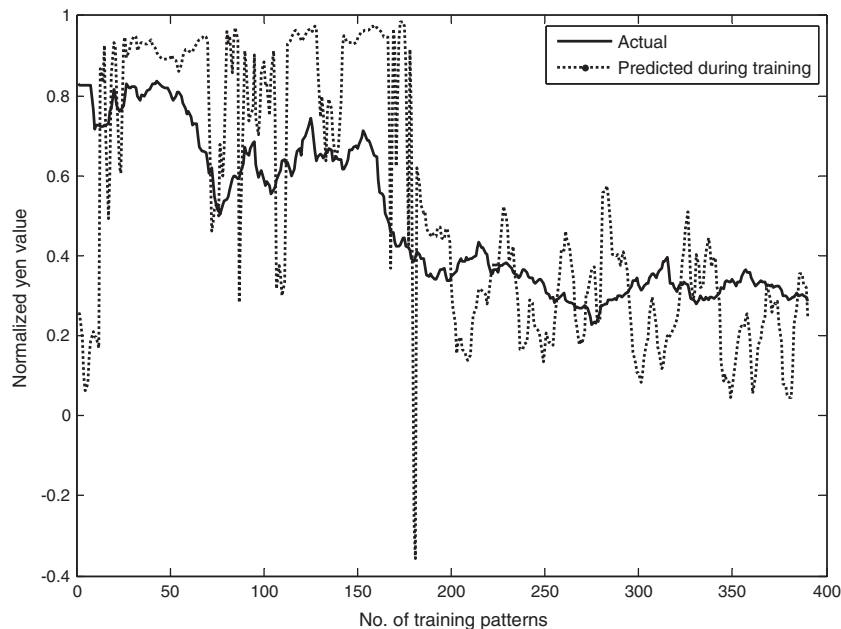


Fig. 5c. Comparison of actual and predicted value for 6 months ahead prediction using Yen data with 30% outlier having magnitude -2 to 2 during training using MLANN model.

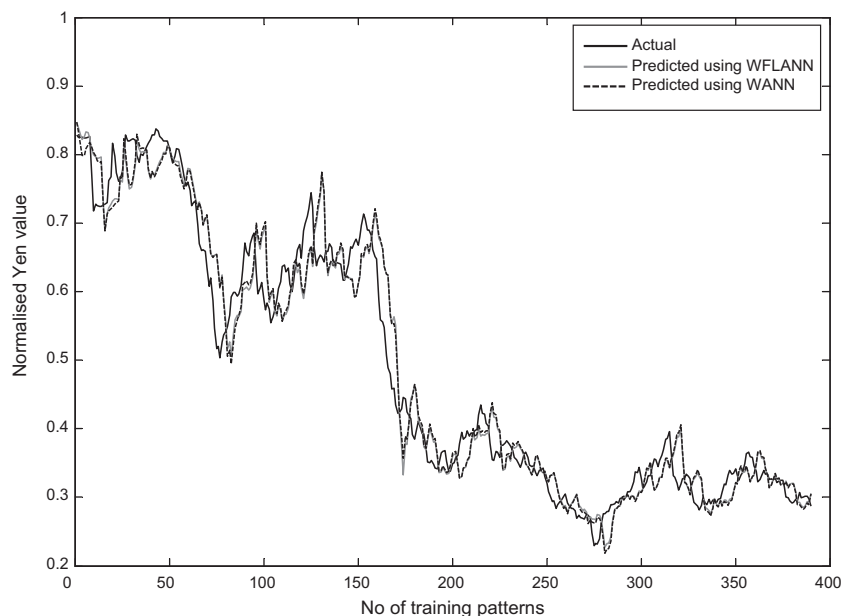


Fig. 5d. Comparison of actual and predicted value for 6 months ahead prediction using Yen data with 30% outlier having magnitude -2 to 2 during training using WANN and WFLANN models.

Real life data of three different exchange rates are collected from <http://www.forecasts.org>. The data show the average of daily figures (noon buying rates in New York city) on the 1st day of each month. The number of data for Indian Rupees, British Pound and Japanese Yen collected during 1-1-1973 to 1-10-2005, 1-1-1971 to 1-1-2005 and 1-1-1971 to 1-1-2005, respectively are 393, 418 and 418. Each set of data is normalized by dividing individual value by the maximum value of that set such that each normalized value lies between 0 and 1. Normalization of input data has been made to avoid the saturation problem associated with the sigmoid function of neural network models. Some fundamental statistical features from the exchange rate series are extracted for training and testing purposes. The normalized exchange rate corresponding to the first day of a month, the mean and variance value computed up to this month using a running window of 12 values are considered as the input to model. The total number of input patterns obtained by this procedure are 382 for rupees and 407 each for pound and yen conversions. Out of the total 382 patterns, 365 patterns are used for training purpose and the remaining 17 are used for testing the prediction performance. Similarly in case of Pound and Yen exchange rates out of the 407 patterns, 390 are used for training the model and 17 patterns are kept aside for testing the performance.

5.2. Training of the model

The prediction model as shown in Figs. 1 and 2 are used for training. Each input pattern contains three values. After generation of the input patterns the target values corresponding to the required number of days ahead prediction are selected and then contaminated with outliers within magnitude of $(-0.5 \text{ to } 0.5)$, $(-1 \text{ to } 1)$ and $(-2 \text{ to } 2)$ at randomly selected 10–50% locations. The first input pattern is fed to the WANN/WFLANN model and the output is obtained by using (7)–(10) for WFLANN and (18)–(20) for WANN model, respectively. The error is obtained by comparing the output with the corresponding target value. In this way total training patterns are applied one by one and the corresponding errors are obtained. For example in case of Rupees data, for 365 input pattern 365 errors are obtained in the first experiment. The structures used for WANN and WFLANN are 3:17:1 and 3:15:1, respectively. Each input of WFLANN is expanded to five terms using trigonometric expansion. For each input pattern 15 expanded values are obtained. Using each error vector the Wilcoxon norm is computed and then used for updating the weights of the WANN and WFLANN models. The corresponding weight update equations used are (17), (23) and (24), respectively. The above procedure constitutes one experiment and this process is repeated until the Wilcoxon norm is minimized after few experiments. The total number of experiments carried out is 5000 and the value of learning parameter η is chosen to be 0.3 and 0.02 for WANN and WFLANN models, respectively. To compare the robust performance of the proposed models other standard models such as the multilayer artificial neural network (MLANN) and functional link artificial neural network (FLANN) models trained with conventional learning algorithm. The training samples are generated by mixing randomly 10% to 50% of outliers for predicting currency rate for 1, 3, 6, 9 and 12 months ahead prediction. The structure of MLANN and FLANN is chosen to be same as that of WANN and WFLANN models. The comparison

Table 1
Comparison of computational complexity of WANN and WFLANN forecasting models.

Types of models	No. of tanh ()	No. of Cos/Sin	No. of weights	No. of adds.	No. of muls.
WANN	18	00	86	68	68
WFLANN	01	15	16	15	15

Table 2

Comparison of APE of Indian Rupees for one month ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	2.7443	2.2512	4.5522	0.9649
20	2.8180	2.8777	3.3368	0.7947
30	5.2526	2.3390	7.1657	0.8118
40	3.0719	1.7438	2.1061	0.7742
50	1.5098	1.0081	4.2674	1.6055
–1 to 1				
10	3.6891	2.2579	6.8331	0.9586
20	4.3254	3.6425	3.9593	0.7950
30	8.8790	2.6009	3.8051	0.8110
40	4.6942	2.2346	7.3028	0.7791
50	2.0881	1.0397	8.6209	1.7373
–2 to 2				
10	4.6337	4.5208	8.6644	0.8063
20	7.7973	2.8267	3.8858	0.7938
30	9.2689	2.5706	11.0269	0.8129
40	6.5338	2.2379	9.2391	0.7801
50	4.7294	1.0430	11.1944	1.7972

Table 3

Comparison of APE of Indian Rupees for three months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	3.5977	2.7804	6.7263	1.9141
20	3.8314	2.5786	4.2884	1.8394
30	6.3342	2.7076	3.5947	1.8462
40	4.0441	2.0061	6.9035	1.8629
50	3.5763	2.8308	8.1155	3.3868
–1 to 1				
10	4.1940	2.7750	7.7795	1.9137
20	4.8986	2.5603	3.9156	1.8380
30	9.5517	2.7055	3.5353	1.8422
40	5.5418	2.0735	7.9138	1.8759
50	3.0895	2.0015	8.7323	3.6252
–2 to 2				
10	4.9511	3.6639	9.2118	1.9108
20	7.3524	2.5662	3.8476	1.8391
30	9.6727	2.6990	10.9926	1.8404
40	7.3560	1.9558	9.6162	1.8823
50	4.9625	3.0705	11.4617	3.7272

Table 4

Comparison of APE of Indian Rupees for six months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	4.9414	3.1716	8.3925	3.3134
20	5.1705	2.9003	5.8537	2.9994
30	8.2132	3.2360	4.7023	3.0877
40	6.2779	2.7418	8.4577	3.0242
50	8.1943	5.2336	9.3292	5.8449
–1 to 1				
10	5.1938	4.1461	9.1665	3.4158
20	6.0720	2.9005	4.0823	2.9801
30	11.3591	3.2546	2.0656	3.1019
40	7.9725	2.7316	8.5908	3.0521
50	5.9811	2.6083	7.8854	6.2863
–2 to 2				
10	5.8427	4.1147	10.2977	3.4228
20	8.1167	2.9003	3.1176	2.9720
30	10.8980	3.2477	11.3298	3.1145
40	10.4598	2.7387	10.6156	3.0569
50	5.7591	5.7603	12.4585	6.4984

Table 5

Comparison of APE of Indian Rupees for nine months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	5.6741	4.4698	9.2915	4.8024
20	5.9272	3.9918	7.4350	4.0495
30	9.3873	4.2111	6.4180	4.1964
40	7.9273	3.6064	9.1740	3.6989
50	2.5542	6.5483	9.8343	7.2635
–1 to 1				
10	5.6383	4.4758	9.9186	4.8238
20	6.7173	3.9741	5.6803	4.0300
30	14.5541	4.1262	2.9838	4.0545
40	9.8789	3.5358	7.2370	3.6763
50	7.4851	1.6649	8.9696	8.1992
–2 to 2				
10	6.0699	4.5039	10.9005	4.8331
20	8.5877	3.9623	2.9395	4.0111
30	11.8579	4.1344	9.9914	4.0703
40	12.3490	3.5322	11.0316	3.7183
50	7.8500	5.9774	13.0717	8.4173

Table 8

Comparison of APE of British Pound for three months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	3.1172	2.8496	4.6580	2.9702
20	4.3864	2.7957	5.4132	3.1026
30	8.9858	2.8741	6.5898	3.0740
40	6.1354	2.9035	8.3081	3.9554
50	25.6721	2.7665	5.9250	3.5274
–1 to 1				
10	22.7478	2.8527	4.4195	3.0311
20	29.8791	2.7999	5.6184	3.0966
30	38.7897	2.8640	8.1677	3.3080
40	6.8435	2.9235	11.2811	3.1629
50	47.8793	2.7686	6.9362	3.9088
–2 to 2				
10	72.4354	2.8525	7.8847	2.9700
20	64.2499	2.8006	6.4362	3.1100
30	70.0393	2.8629	11.0604	3.5949
40	28.7774	2.9432	31.3027	3.6284
50	70.0102	2.7772	10.4573	3.7334

Table 6

Comparison of APE of Indian Rupees for twelve months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	5.5802	5.4680	9.4289	5.6999
20	6.0424	4.4849	8.0766	4.5154
30	9.4569	4.6012	7.1243	4.5784
40	8.2349	4.0037	8.8528	4.2358
50	7.5535	7.6712	8.9892	8.1351
–1 to 1				
10	5.3448	5.5060	9.9584	5.7759
20	6.6711	4.4727	6.8590	4.5248
30	14.7335	4.5543	4.5306	4.6143
40	10.2173	3.8973	5.5996	4.0900
50	10.4169	8.6697	10.7736	8.9847
–2 to 2				
10	5.7614	5.5037	10.8282	5.7761
20	10.7432	4.4691	7.6695	4.5183
30	11.9911	4.5993	6.0919	4.6641
40	12.3530	3.9976	10.8603	4.1617
50	15.4099	9.2553	12.9945	9.5156

Table 9

Comparison of APE of British Pound for six months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	5.4543	4.5389	6.6142	4.5357
20	4.7808	4.6106	5.4615	4.1744
30	8.9421	4.5615	4.5804	4.4329
40	10.0750	4.4421	4.8959	4.6459
50	23.9612	4.3035	9.3080	5.0680
–1 to 1				
10	18.1558	4.5356	4.2459	4.2008
20	30.6959	4.6220	4.9480	4.5075
30	34.0412	4.5574	8.2661	4.4181
40	6.4553	4.4575	7.8624	4.5612
50	47.4443	4.3058	6.4078	4.1723
–2 to 2				
10	72.1019	4.5360	5.1883	4.6099
20	64.5498	4.6241	19.5272	4.2705
30	69.3513	4.5564	9.5038	4.5605
40	31.1808	4.4576	7.5140	4.3901
50	69.2501	4.2992	8.6270	6.7562

Table 7

Comparison of APE of British Pound for one month ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	3.6916	1.8342	3.1669	1.8545
20	5.9159	1.8340	3.9334	1.9400
30	12.8696	1.8486	5.0388	1.9365
40	6.5351	1.8821	7.0485	1.7667
50	26.8399	1.8112	4.6279	2.2092
–1 to 1				
10	24.8113	1.8341	3.4548	1.8846
20	31.4696	1.8339	4.0992	1.9199
30	40.2844	1.8490	6.3580	1.8329
40	7.4162	1.8948	10.3057	1.9216
50	47.6346	1.8156	5.4767	2.7956
–2 to 2				
10	73.5708	1.8343	7.7278	1.8379
20	63.4184	1.8342	5.4854	1.9116
30	70.1588	1.8490	9.0571	2.1465
40	24.4233	1.9206	32.2762	2.4643
50	69.7882	1.8231	10.1837	2.3572

Table 10

Comparison of APE of British Pound for nine months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	6.0962	3.6478	4.5608	3.7258
20	9.0584	3.8483	3.7638	3.4857
30	13.5767	3.7264	4.9213	2.9076
40	6.9268	3.4894	4.8553	4.1951
50	26.8870	3.3832	4.5981	3.3712
–1 to 1				
10	20.7940	3.6352	3.4381	3.3261
20	34.2156	3.8368	4.9361	3.6202
30	37.1751	3.7330	4.1899	2.2229
40	3.9092	3.4501	4.3005	3.4872
50	49.8262	3.3778	12.7937	3.4412
–2 to 2				
10	73.5566	3.6335	8.8096	3.5623
20	66.6014	3.8375	7.3072	3.3284
30	69.2070	3.7330	8.9327	2.5697
40	38.6128	3.4294	25.1949	4.5725
50	71.3296	2.8767	7.5863	3.5152

Table 11

Comparison of APE of British Pound for twelve months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	8.0588	1.9402	3.6593	4.0382
20	12.5042	1.9985	3.8363	3.1641
30	16.2601	2.0099	4.2255	4.4436
40	5.0380	2.4705	5.9351	3.2159
50	29.4346	2.1032	3.7526	3.4980
–1 to 1				
10	26.0955	1.9559	5.0032	3.7399
20	37.7726	2.0316	4.7303	2.8586
30	40.2769	1.9735	5.0996	2.8910
40	2.6845	2.4206	10.4419	3.4894
50	51.9842	1.9174	4.3366	2.8050
–2 to 2				
10	76.3967	1.9620	9.0523	3.3490
20	70.6855	2.0831	7.1331	3.9892
30	67.6186	1.9888	7.6525	3.8343
40	43.0578	2.4075	20.7845	3.4335
50	74.0742	2.1611	5.8022	2.8066

Table 12

Comparison of APE of Japanese Yen for one month ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	2.4603	2.0593	14.5323	2.9520
20	3.5354	1.5800	8.3428	2.5033
30	28.8166	1.4183	10.0313	1.5991
40	12.9180	1.6141	21.8654	1.7214
50	52.0512	1.4857	9.3081	1.5623
–1 to 1				
10	23.0217	2.0779	22.3990	2.2332
20	67.4813	2.1598	9.4217	1.4589
30	119.3974	1.4176	19.0184	1.5385
40	17.4428	1.4630	38.0472	1.6931
50	83.7948	1.4740	10.8545	2.3908
–2 to 2				
10	202.8847	1.7567	37.9534	2.8546
20	188.4589	1.5670	12.7716	2.9515
30	226.1240	1.4207	49.5637	1.4840
40	26.7137	1.4555	70.8575	1.8199
50	113.4854	1.4622	14.7751	2.5514

Table 13

Comparison of APE of Japanese Yen for three months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	12.7471	3.2007	17.2747	2.5240
20	12.5207	3.0346	8.4356	2.8962
30	35.0912	3.7952	8.1126	2.7652
40	10.3233	2.8224	23.7487	2.9512
50	54.0354	2.6445	8.5016	3.6199
–1 to 1				
10	25.1539	3.1999	5.2273	2.5509
20	64.7492	3.6750	8.1888	3.0100
30	123.3145	3.8267	16.2993	2.8311
40	35.7931	2.9054	39.7647	2.8337
50	84.7677	2.6505	9.6836	2.9892
–2 to 2				
10	200.7430	3.2480	40.650	2.5317
20	189.6279	3.7347	10.9791	2.5289
30	226.6121	3.8294	47.4554	2.9573
40	38.0082	2.8173	72.5810	3.0929
50	07.4131	2.6559	13.4082	2.7003

Table 14

Comparison of APE of Japanese Yen for six months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	23.4526	5.4760	23.5914	5.3263
20	34.5527	5.4434	12.4332	5.0251
30	32.4074	5.3767	8.2640	4.1842
40	9.4702	5.0412	28.0847	5.5925
50	56.8586	5.2203	10.2042	5.2122
–1 to 1				
10	28.8200	5.4703	30.7190	4.5181
20	59.5855	5.4499	10.1687	4.7996
30	122.1834	5.4098	12.8787	5.1595
40	8.1975	4.8937	43.0003	4.8299
50	87.8505	5.1407	10.796	5.2862
–2 to 2				
10	201.1631	5.4619	43.8154	4.8247
20	190.6564	5.3390	8.9726	4.7525
30	226.0968	5.4562	42.4814	3.8465
40	44.7602	4.8381	75.1507	5.0636
50	36.9626	5.0518	13.5687	4.7792

Table 15

Comparison of APE of Japanese Yen for nine months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	13.3322	5.2278	21.9618	5.1443
20	15.7415	4.9671	11.0099	4.9671
30	28.4333	4.9905	8.5482	4.9905
40	14.0613	4.9371	25.6813	4.9371
50	53.2105	5.1411	9.4762	5.1411
–1 to 1				
10	21.8024	5.4728	28.4610	4.6562
20	52.8302	5.0905	8.3618	5.5683
30	111.8736	4.9917	15.2231	4.5216
40	10.0947	4.6814	39.5155	5.9350
50	83.1180	5.1804	11.0187	6.9321
–2 to 2				
10	195.8382	5.4800	40.4303	4.2444
20	186.4510	5.0819	9.4050	4.8998
30	220.8475	5.0721	44.1892	5.1997
40	36.9069	4.5181	70.6502	4.8713
50	56.1166	5.1306	15.6154	5.0463

Table 16

Comparison of APE of Japanese Yen for 12 months ahead prediction using different models.

% of outliers	MLANN	WANN	FLANN	WFLANN
–0.5 to 0.5				
10	3.5238	2.8480	16.9868	2.0559
20	5.7145	2.8074	6.4137	2.5984
30	27.4684	3.6764	8.7557	4.5774
40	8.3592	2.5490	21.4660	4.0231
50	57.7866	3.0221	7.3333	2.2711
–1 to 1				
10	18.9156	2.9154	22.4935	2.5008
20	51.7963	2.9678	3.3557	2.5738
30	107.1198	4.3849	18.2862	2.8335
40	9.2015	2.3227	34.1281	2.3963
50	88.2302	2.8210	11.7065	3.5824
–2 to 2				
10	190.8357	3.1104	33.0675	3.4270
20	181.6273	2.9308	8.0471	2.3094
30	212.8022	4.6458	46.4282	3.2258
40	17.6306	2.1155	63.5981	3.1916
50	70.5324	2.7448	18.6970	5.5036

of convergence characteristics and the desired and predicted value during training for three different conversions are given in 3(a)–3(d), 4(a)–4(d) and 5(a)–5(d), respectively for Rupees, Pound and Yen. Table 1 shows the comparison of computational complexity involved in WANN and WFLANN forecasting models for computing one predicted output.

The comparison of convergence characteristics reveals that the square error based MLANN and FLANN model practically do not converge when the training samples are densely contaminated with outliers. As a result the estimated exchange rates in all cases do not match with the true ones.

5.3. Testing of the models

Once the training is completed, the weights of the WANN and WFLANN are frozen at the converged values and then testing or validation of the models are carried out. The test patterns are applied to the WANN/WFLANN model and the predicted outputs are obtained. The actual predicted output is obtained after adding the bias (which is the median of the errors) to the model output as the Wilcoxon norm used is not the actual norm rather it is a pseudo norm. The average percentage of error of these models computed using (25) are compared in Tables 2–16.

$$APE(k) = \text{mean}(PE(m)) \quad (25)$$

where k represents the number of months ahead prediction is made and $PE(m)$ is defined in (26).

$$PE(m) = \left(\frac{acr(m) - pcr(m)}{acr(m)} \right) \times 100 \quad (26)$$

where $acr(m)$ = actual conversion rate of m th month, $pcr(m)$ = predicted conversion rate of m th month, $PE(m)$ = the percentage of error corresponding to m th month.

6. Discussion on simulation results

The present investigation reveals some interesting observations.

- (i) When the training samples are contaminated with outliers the convergence as well as the prediction performance of squared error based FLANN and MLANN models drastically deteriorates and when dense outliers are present the convergence practically fails.
- (ii) However the Wilcoxon norm based WANN and WFLANN prediction models show consistent performance during training and testing phases both in presence of low and high density outliers.
- (iii) As the percentage of outliers in the training samples increases correspondingly the prediction potentiality of both the robust models decreases.
- (iv) Both the WANN and WFLANN models show identical performance for all conditions but the later model involves very low computational complexity.

7. Conclusion

The paper has proposed two efficient robust neural network based forecasting models for exchange rate prediction in presence of outliers in training data. In the first model multilayer neural network is trained by minimizing the Wilcoxon norm instead of conventional error square norm and the final output is obtained by adding the bias value to the model output. The second model consists of a low complexity single layer Wilcoxon Functional link

neural network model which maps the input data into nonlinear form using trigonometric expansion scheme and also Wilcoxon norm as the cost function. Computer simulation study of both the models reveal that both offer consistent prediction performance compared to the MLANN and FLANN model in presence of 10–50% outliers in the training data of different magnitude. However, out of the WANN and WFLANN models, the later offers low computational complexity and hence preferable as a robust prediction model for currency exchange rates.

References

- Anastasakis, Leonidas., & Mort, Neil. (2009). Exchange rate forecasting using a combined parametric and nonparametric self-organising modelling approach. *Expert Systems with Applications*, 36, 12001–12011.
- Box, G. E. P., & Jenkins, G. M. (1976). *Time series analysis: forecasting and control*. Holden-day.
- da Silva, C. G. (2008). Time series forecasting with a non-linear model and the scatter search meta heuristic. *Information Sciences*, 178, 3288–3299.
- Harvey, A. C. (1989). *Forecasting structural time series models and the Kalman filter*. Cambridge University Press.
- Hsieh, J.-G., Lin, Y.-L., & Jeng, J.-H. (2008). Preliminary study on Wilcoxon learning machines. *IEEE Transactions on Neural Networks*, 19(2), 201–211.
- Ince, H., & Trafalis, T. B. (2006). A hybrid model for exchange rate prediction. *Decision Support Systems*, 42, 1054–1062.
- Khashei, M., & Bijari, M. (2011). A novel hybridization of artificial neural networks and ARIMA models for time series forecasting. *Applied Soft Computing*, 11, 2664–2675.
- Khashei, M., & Bijari, M. (2012). A new class of hybrid models for time series forecasting. *Expert Systems with Applications*, 39, 4344–4357.
- Khashei, M., Bijari, M., & Ardali, G. A. R. (2009). Improvement of auto-regressive integrated moving average models using fuzzy logic and artificial neural networks (ANNs). *Neurocomputing*, 72, 956–967.
- Khashei, M., Hejazi, S. R., & Bijari, M. (2008). A new hybrid artificial neural networks and fuzzy regression model for time series forecasting. *Fuzzy Sets and Systems*, 159, 769–786.
- Leu, Y., Lee, C.-P., & Jou, Y.-Z. (2009). A distance-based fuzzy time series model for exchange rates forecasting. *Expert Systems with Applications*, 36, 8107–8114.
- Lubecke, T. H., Nam, K. D., Markland, R. E., & Kwok, C. C. Y. (1998). Combining foreign exchange rate forecasts using neural networks. *Global Finance Journal*, 9(1), 5–27.
- Majhi, R., Panda, G., & Sahoo, G. (2009). Efficient prediction of exchange rates with low complexity artificial neural network models. *Expert Systems with Applications*, 36, 181–189.
- McKean, J. W. (2004). Robust analysis of linear models. *Statistical Science*, 19(4), 562–570.
- Pai, P.-F., Chen, S.-Y., Huang, C.-W., & Chang, Y.-H. (2010). Analyzing foreign exchange rates by rough set theory and directed acyclic graph support vector machines. *Expert Systems with Applications*, 37, 5993–5998.
- Patra, J. C., Pal, R. N., Baliarsingh, R., & Panda, G. (1999). Nonlinear channel equalization for QAM signal constellation using artificial neural networks. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 29(2), 262–271.
- Patra, J. C., Pal, R. N., Chatterji, B. N., & Panda, G. (1999). Identification of nonlinear dynamic systems using functional link artificial neural networks. *IEEE Transactions on Systems, Man and Cybernetics, Part B: Cybernetics*, 29(2), 254–262.
- Patra, J. C., Panda, G., & Baliarsingh, R. (1994). Artificial neural network based nonlinearity estimation of pressure sensors. *IEEE Transactions on Instrumentation and Measurement*, 43(6), 874–881.
- Shady, M. R. El., & Shazly, H. E. El. (1997). Comparing the forecasting performance of neural networks and forward exchange rates. *Journal of Multinational Financial Management*, 7, 345–356.
- Shazly, M. R. El., & Shazly, H. E. El. (1999). Forecasting currency prices using a genetically evolved neural network architecture. *International Review of Financial Analysis*, 81, 67–82.
- Shin, T., & Han, I. (2000). Optimal signal multi-resolution by genetic algorithms to support artificial neural networks for exchange-rate forecasting. *Expert Systems with Applications*, 18, 257–269.
- Yu, L., Wanga, S., & Laic, K. K. (2005). A novel nonlinear ensemble forecasting model incorporating GLAR and ANN for foreign exchange rates. *Computers & Operations Research*, 32, 2523–2541.
- Yu, L., Wang, S., & Lai, K. K. (2009). A neural-network-based nonlinear metamodelling approach to financial time series forecasting. *Applied Soft Computing*, 9, 563–574.
- Zhang, Y.-Q., & Wan, X. (2007). Statistical fuzzy interval neural networks for currency exchange rate time series prediction. *Applied Soft Computing*, 7, 1149–1156.