

STA325 Case Study

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Packages/Data

```
library(tidyverse) # datawrangling
library(tidymodels) # modeling steps
library(knitr) # nice output
library(boot)
library(patchwork)
```

Investigating Relationship Between Mean and Froude's Number

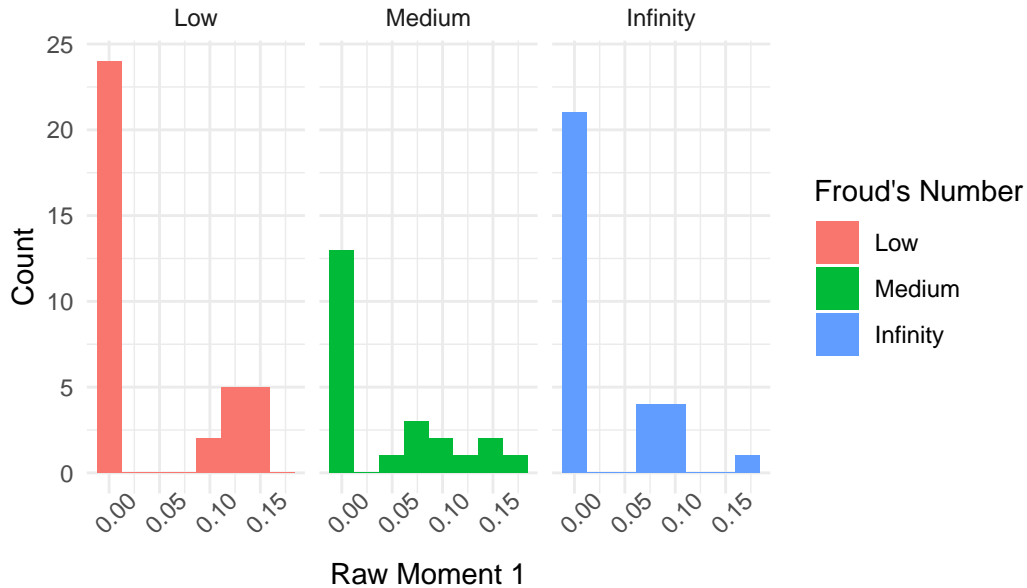
Table 1: Distribution of Froude's Number

Froude's Number	Count
0.052	36
0.300	23
Inf	30

Similar to Reynolds number, this number is somewhat non-continuous as there is only three buckets for the number. Therefore I am going to make it a categorical variable to help interpretability. For instance, how do we interpret Infinity? Instead I will make

- 0.052 == Low
- 0.300 == Medium
- Inf == Infinity

Relationship between Froud's Number and the Mean



It seems hard to distinguish what affect Froud's number has on the mean from this plot.

Inference

Stokes Number St

Stoke's Number quantifies the particle's characteristics i.e. it's size and density. Let's see how this parameter effects the distribution of the final turbulent space.

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.02727949	0.008642157	3.156561	0.002193806
St	0.01525690	0.007438449	2.051086	0.043266692

Using a linear model with only Stoke's predictor, we see that the predictor is statistically significant for an alpha level of 0.01. We can interpret it's coefficient as for every one point increase in Stoke's number, the **mean** of the final turbulent distribution increases by about 0.015 .

[1] "Variance Output"

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	50.13945	41.02933	1.222039	0.2249935
St	49.26535	35.31463	1.395041	0.1665549

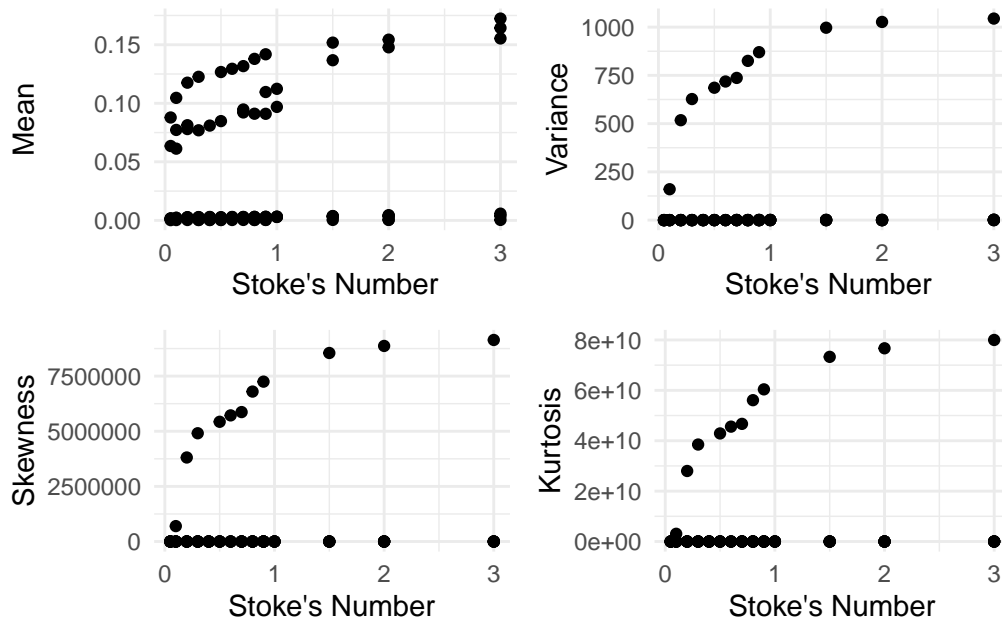
[1] "Skewness Output"

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	360097.4	341169.4	1.055480	0.2941282
St	457487.7	293650.2	1.557934	0.1228792

[1] "Kurtosis Output"

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2594695036	2849353546	0.9106259	0.36500867
St	4186989858	2452486167	1.7072430	0.09134354

Looking at the summary for a Linear Model predicting the variance of the final distribution with Stoke's Number, we can see that it has essentially no affect on the **variance**, **skewness**, or **kurtosis** of the final system. Overall, this parameter does not seem to have a great impact on the final resulting distribution in general. This can be made clear by visualizing it's relationship:



Reynold's Number Re

Reynold's Number **Re** in our data is a number that quantifies the initial fluid turbulence of the system.

Mean Model

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.1120959	0.003342233	33.53924	3.623490e-51
ReMedium	-0.1091886	0.004477683	-24.38507	2.139226e-40
ReHigh	-0.1117449	0.005421817	-20.61024	5.032396e-35

Looking at the output, it is clear that **Re** has a statistically significant relationship with the outputting mean of the distribution for each of its levels. However, the true effect is not very large. However, we can say that when **Re** == **Low** , the resulting mean is 0.11 points higher on average than **Medium** or **High**. When **Re** == **Medium** and **Re** == **High** the resulting mean is around 11 points **lower** than the baseline when **Re** == **Low**. Therefore, it is clear to see that **Re** has an **inverse** relationship with the mean of the resulting distribution.

Variance Model

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	265.3472	41.58665	6.380587	8.558301e-09
ReMedium	-265.2103	55.71481	-4.760139	7.751474e-06
ReHigh	-265.3418	67.46246	-3.933178	1.693051e-04

Similar to the **Mean** model, if we glance at the output, it is clear that **Re** has a statistically significant relationship with the outputting mean of the distribution for each of its levels. The true effect in this case is relatively large. When **Re** == **Low**, the resulting variance is 265.45 times higher on average than **Medium** or **High**. When **Re** == **Medium** or **Re** == **High** the resulting variance is around 265 times **lower** than the baseline when **Re** == **Low**. Therefore, it is clear to see that **Re** has an **inverse** relationship with the **variance** of the resulting distribution. In other words, **a higher Reynold's number results in a lower varied distribution.**

This follows the intuition that when a gravitational force is higher, clusters will form at a higher rate resulting in a lower variance number.

Prediction

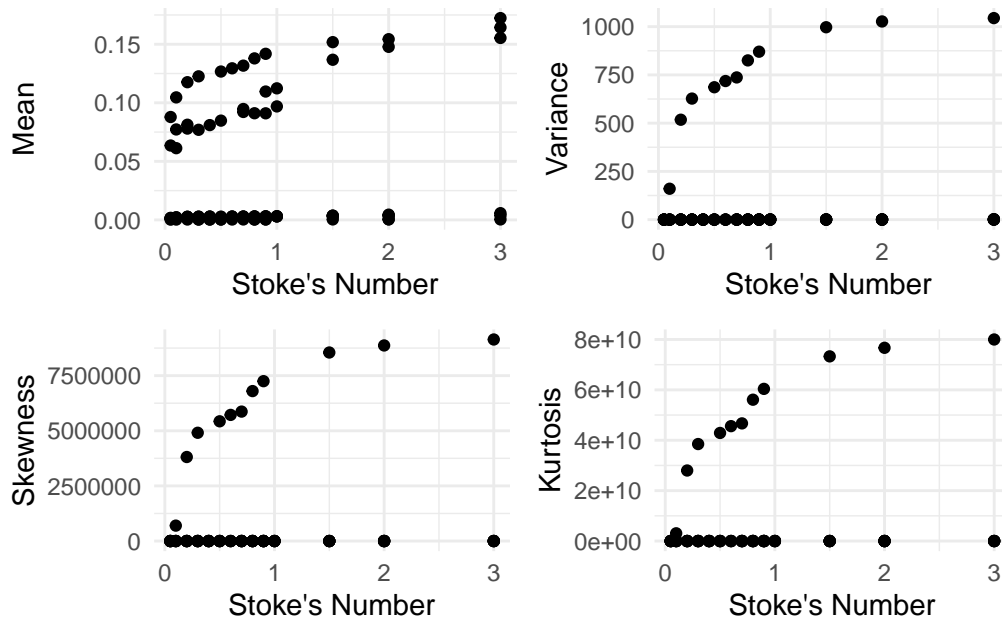
Simple Linear Regression

Table 2: Mean Squared Error using Simple Linear Regression

Mean	Variance	Skewness	Kurtosis
0.000218	37239.52	2.655954e+12	1.899136e+20

It is clear that the using a Simple Linear Regression model to predict anything but the **mean** is not a good idea as the performance is not good. Nonetheless it serves as a good baseline for predicting further complex models.

Polynomials



The optimal degree polynomials for predicting mean, variance, skew, and kurtosis are all 2 for Stokes Number.

Using 2nd degree polynomial models while also predicting the moments with Froude's and Reynolds' Numbers, we get lower MSE's for all four moments, however these MSEs are still outrageously high for skew and kurtosis, indicating that using polynomials would also not be the best way of predicting the moments.

Interaction Effects (to-do)

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.108580531	0.002377907	45.662236	5.265113e-57
ReMedium	-0.103060131	0.002987389	-34.498397	3.546585e-48
ReHigh	-0.106709512	0.003603124	-29.615831	1.743279e-43
FrMedium	-0.035778301	0.003540488	-10.105471	1.048667e-15
FrInfinity	-0.038678957	0.003219341	-12.014556	3.031953e-19
St	0.024312829	0.001744268	13.938697	1.233637e-22
ReMedium:FrMedium	0.028831079	0.003657032	7.883737	1.837273e-11
ReMedium:FrInfinity	0.033657145	0.003705031	9.084172	9.212342e-14
ReHigh:FrInfinity	0.034293911	0.004051186	8.465154	1.413955e-12
ReMedium:St	-0.027399550	0.001938283	-14.135990	5.685515e-23
ReHigh:St	-0.025833618	0.002505996	-10.308724	4.336296e-16
FrMedium:St	0.008944036	0.002379658	3.758539	3.333752e-04
FrInfinity:St	0.005955800	0.001995202	2.985061	3.812474e-03

Splines

Trees

Random Forests

Evaluating Our Models

TODO:

Splines, (Natural, Cubic...), Tree based model