Memoryless Property

Chase Joyner

Mathematical Sciences

May 23, 2013

Memoryless Property:

Let X be an exponentially or geometrically distributed random variable with parameter λ, then

$$P(X \geq s + t | X \geq t) = P(X \geq s).$$

That means the conditional probability of X exceeding $s + t$, given X exceeds t, is equal to the probability of X exceeding s regardless of t.

Example: Consider a two-server system in which a customer is served first by server 1, then by server 2, and then departs. The service times at server i are exponential random variables with rates $\mu_i, i = 1, 2$. When you arrive, you find server 1 free and two customers at server 2 — customer A in service and customer B waiting in line. What is the probability that customer A is still in service when you move over to server 2? (*Introduction to Probability Models, 9th edition by Sheldon M. Ross*)

Solution: Once you arrive, you see customer A is already being served. Since the exponential distribution is memoryless, then the probability is the same as if customer A's service started at the same time your service started. Let X_i denote the wait time for the current service at server i. So we have,

$$P(X_1 = \min\{X_1, X_2\}) = \frac{\mu_1}{\mu_1 + \mu_2}$$

This result is derived from a property of the exponential distribution when dealing with the minimum of exponential random variables.

Example: Suppose that you pull up to a car wash that has two available car washes. Also suppose that both car washes are currently occupied, but you are the only person in line for the next available car wash. If the amount of time to wash a car is exponentially distributed with mean β, what is the probability that you are the last of the three cars to leave the car wash?

Solution: Consider the time that you are able to pull up to a car wash. At this point, you are in a car wash and the other car wash is still occupied by the same car. Since the time to wash a car is exponentially distributed, and because the exponential distribution lacks memory, the probability that you are the last to leave the car wash is $\frac{1}{2}$.