HW2

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EC330 A1

1a) $\lim_{n\to\infty} \frac{n-1}{n-330} \Rightarrow \lim_{n\to\infty} \frac{f'(n)}{g'(n)} \to \frac{1}{1}$, $\boxed{f=\Theta(g)}$

b) $\lim_{n\to\infty} \frac{n^{2/3}}{n^{1/2}} \Rightarrow \frac{n^a}{n^b} \to n^{a-b} = n^{2/3-1/2} \to \lim_{n\to\infty} n^{1/6} = 0$
$\to \boxed{f=\Omega(g)}$

c) $\lim_{n\to\infty} \frac{330n+\log n}{n+(\log n)^2} = \frac{330+\frac{\log n}{n}}{1+\frac{(\log n)^2}{n}}$

$\to \lim_{n\to\infty} 330+\frac{\log n}{n} = 330$

$\to \lim_{n\to\infty} 1+\frac{(\log n)^2}{n} \Rightarrow 1+\frac{d/dn\,(\log n)^2}{d/dn\, n}$

$\to \lim_{n\to\infty} 1+\cancel{\frac{2}{n}}^{0}$

$\boxed{f=\Theta(g)} \leftarrow \frac{330}{1}$

d) $\lim_{n\to\infty} \frac{n\log n}{\log(330n)} = \frac{\log n}{330\log(330n)} \Rightarrow \frac{f'n}{g'n} = \frac{1/n}{330/n}$, $\boxed{f=\Theta(g)}$

e) $\lim_{n\to\infty} \frac{330\log(n)}{\log(n^3)} \Rightarrow \frac{f'n}{g'n} = \frac{330/n}{3/n} = \frac{330}{3}\lim_{n\to\infty}\frac{n}{n} = 110$
$\boxed{f=\Theta(g)}$

f) $\lim_{n\to\infty} \frac{\log(330n)}{\log(n)} \Rightarrow \frac{f'n}{g'n} = \frac{1/n}{1/n} \Rightarrow \lim \frac{n}{n} \to 1$, $\boxed{f=\Theta(g)}$

g) $\lim_{n\to\infty} \frac{n^{1.01}}{n\log^2 n} \to \frac{d/dn\, n^{1.01}}{d/dn\, n\log^2 n} = \frac{1.01n^{.01}}{\log n} \Rightarrow \frac{f''n}{g''n} = \frac{n^{\frac{.00005}{.99}}}{1/n}$

$\lim_{n\to\infty} = 0$

$\boxed{f=\Omega(g)}$

h) $\lim_{n\to\infty} \frac{\frac{n^2}{\log n}}{n\log^2 n} = \frac{n^2}{\log n\, n\log^2 n} = \frac{n}{\log^3 n} \Rightarrow \frac{f'n}{g'n} = \frac{n}{3\log^2 n}$

$\frac{f''n}{g''n} = \frac{n}{6} \Rightarrow \frac{1}{6}\lim_{n\to\infty} n = 0$, $\boxed{f=\Omega(g)}$

i) $\lim_{n\to\infty} \frac{\log n^{\log n}}{n/\log n} \Rightarrow \frac{e^{\log n \log\log n}\log n}{n}$

$= \frac{n^{\log\log n}\log n}{n}$

$= n^{\log(\log(n))-1}\log(n)$

$\lim_{n\to\infty} n^{\log(\log(n))-1}\log n$

$\to 0$, $\boxed{f=\Omega(g)}$

j) $\lim_{n\to\infty} \frac{\sqrt{n}}{(\log(n))^3} = \frac{n^{1/2}}{3\log n} \Rightarrow \frac{f'n}{g'n} = \frac{n}{6n^{1/2}} = \lim_{n\to\infty} \frac{n^{1/2}}{6} = 0$

$\boxed{f = \Omega(g)}$

k) $\lim_{n\to\infty} \frac{n^{1/2}}{5^{\log_2 n}} = \frac{n^{1/2}}{5^{\frac{\log n}{\log 2}}} = \frac{n^{1/2}}{e^{\log\left(5^{\frac{\log n}{\log 2}}\right)}} = \frac{n^{1/2}}{n^{\frac{\log 5}{\log 2} - 1/2}}$

$\lim_{n\to\infty} n^{\frac{-\log(2) + \log(5)}{2\log(2)}} = 0$, $\boxed{f = \Omega(g)}$

l) $\lim_{n\to\infty} \frac{3^n}{n 2^n} = \frac{1}{n}\left(\frac{3}{2}\right)^n \Rightarrow \frac{f'n}{g'n} = \frac{n}{n}\left(\frac{3}{2}\right)^{n-1}$ $\lim_{n\to\infty}\left(\frac{3}{2}\right)^{n-1} = 0$

$\boxed{f = \Omega(g)}$

m) $\lim_{n\to\infty} \frac{n!}{2^n}$ 2^n grows slower then $n!$, so $\boxed{f = \Omega(g)}$

n) $\lim_{n\to\infty} \frac{\frac{d}{dx}(\log n)^{10}}{\frac{d}{dx} n^{0.1}} \to \lim_{n\to\infty} \frac{10/n}{0.1 n^{-0.2}} = \frac{10 n^{0.9}}{0.1 n}$ $\lim_{n\to\infty} \frac{100}{n^{0.1}} = 0$

$\boxed{f = O(g)}$

o) $\lim_{n\to\infty} \frac{\sum_{i=1}^{n} i^k}{n^{k+1}}$

$1^k + 2^k + \dots \infty^k < \infty^{k+1}$

$1^k + 2^k + \dots \infty^k + \infty^{k+1} < \infty^{k+2}$

$\therefore \lim_{n\to\infty} \frac{\sum_{i=1}^{n} i^k}{n^{k+1}} = 0$, $\boxed{f = O(g)}$

2) $(\log n)^{\log n} = O(2^{(\log n)^2})$

$\lim_{n\to\infty} \frac{(2^{(\log n)^2}}{(\log n)^{\log n}} = \lim_{n\to\infty} e^{(\log n)^2 \log(2) - \log n(\log(\log(n)))}$

$= e^{\lim_{n\to\infty} (\log n)^2 \log 2 - \log n (\log(\log n))}$

$u = \log n$

$du = \frac{1}{n} dn$

$= e^{\lim_{n\to\infty}\left(1 - \frac{\log \log n}{\log(2) \log n}\right)(\log n)^2 \log(2)}$

$\frac{d}{du} \log u = \frac{1}{u} \cdot \frac{1}{n} = \frac{1}{n \log n}$

$= e^{(1) \lim_{n\to\infty} (\log n)^2 \cdot \log(2)} = \infty$,

thus $(\log(n))^{\log n} = O(2^{(\log n)^2})$.