

Problem 4. Consider the "competitive system" below. Here  $x(t)$  and  $y(t)$  are populations of two species which compete (so an interaction between the members of the species tend to slow the growth of both species).

$$\left| \begin{array}{l} x=0 \\ 2-\frac{2x}{3}-y=0 \end{array} \right.$$

$$\frac{dx}{dt} = 2x(1-x/3) - xy = 2x - \frac{2x^2}{3} - xy = x \left( 2 - \frac{2x}{3} - y \right)$$

$$\frac{dy}{dt} = y - xy = y(1-x) \quad x=1, y=0$$

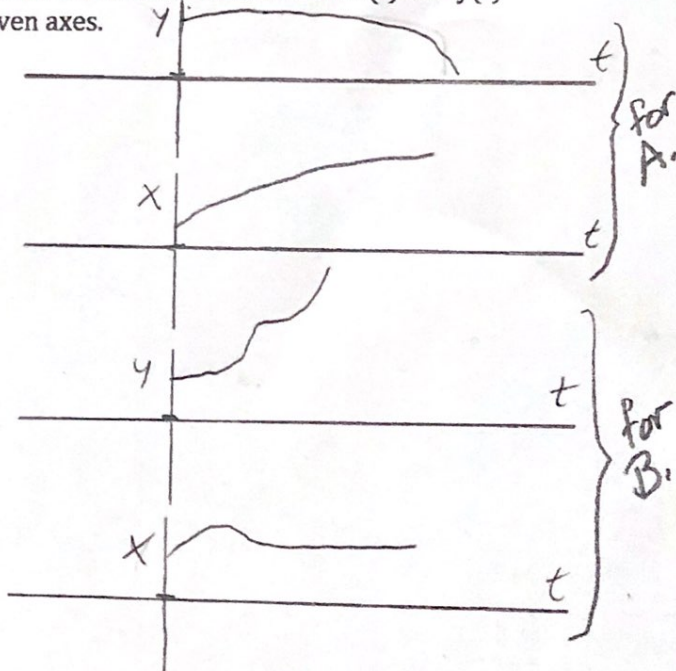
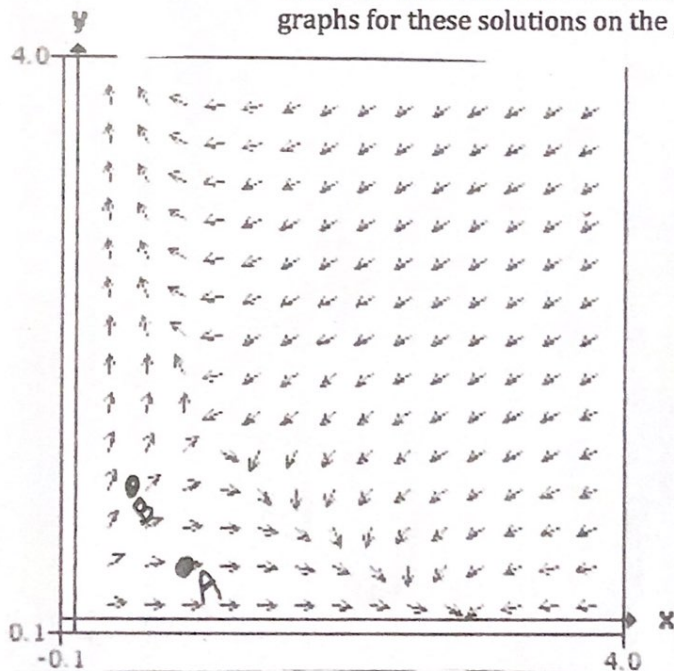
a.) Find the equilibrium points for this system.

$(0,0)$  - zero population

$(1, 4/3)$  - stable population

$(3,0)$  - stable  $x$  population

b.) The direction field is given below for this system. Sketch carefully the solutions with initial conditions A and B as marked and draw the  $x(t)$  and  $y(t)$  graphs for these solutions on the given axes.



c.) Does this model satisfy the "competitive exclusion principle" of ecology (check out what this is on the internet). Answer in ONE OR TWO BRIEF sentences.

Yes because the  $\frac{dx}{dt}$  population stabilizes at the equilibrium point  $(3,0)$ , where the  $y$  population goes extinct  $\left(\frac{dy}{dt}\right)$ .