Problem 2: Suppose you borrow $10,000 at 4 percent per year compounded continuously. You pay back $500 dollars per year in many small payments.

a.) Write a differential equation model for the amount of money you owe at each time.

$$\frac{dy_1}{dt} = .04y - 500$$

b.) How much of the loan will you have paid off after 10 years?

$y' - .04y = -500$

$\mu(x) = e^{\int -.04\,dt} = e^{-.04t}$

$$y_1 e^{-.04t} = \int -500 e^{-.04t}$$

$$y_1 e^{-.04t} = 12500 e^{-.04t} + c_1$$

$$y_{1p}(t) = 12500 + c_1 e^{.04t}$$

$y_1(0) = \$10,000$ — $y_{1g}(t) = 12500 - 2500 e^{.04t}$

$$y_1(10) = 12500 - 2500 e^{.04(10)} \approx 8770.438$$

$10,000 - 8770.488 = \boxed{\$1229.57 \text{ paid}}$

c.) Suppose that after 10 years, you reduce your payments to $350 per year. Will you still pay off the loan? (If so, when, if not, how much will you owe in 50 years.) → This sol. is assuming $y(0) \to y(50)$ is the

$$\frac{dy_2}{dt} = .04y - 350$$

$y' - .04y = -350$

$\mu(x) = e^{\int -.04\,dt} = e^{-.04t}$

$$y_2 e^{-.04t} = \int -350 e^{-.04t}\,dt$$

$$y_2 e^{-.04t} = 8750 e^{-.04t} + c_1$$

$$y_{2p}(t) = 8750 + c_1 e^{.04t}$$

$$8770.488 \approx 8750 + c_1 e^{.04(0)}$$

$y_{2g}(t) = 8750 + 20.488 e^{.04t}$

$y_{2g}(t)$ cannot be paid to $y_2 = 0$, so after 50 yrs,

$y(40) = 8750 + 20.488 e^{(.04)(40)} \approx 8851.25$ owed.