EK381 HW 11

Chase Manaco
U18719879

1a) Communicating classes ~ $\{0,1\}, \{2,3\}, \{4,5,6,7\}$

b) $\{0,1\}$ – period 2 $\{2,3\}$ – period 1
$\{4,5,6,7\}$ – period 3

c) $\{0,1\}$ – $\{4,5,6,7\}$ – recurrent
$\{2,3\}$ – transient

2a)
$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 1/3 & 2/3 & 0 \\ 1 & 1/2 & 0 & 1/2 \\ 2 & 1/4 & 1/4 & 1/2 \end{array}$$

b)
$$P^2 = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 4/9 & 2/9 & 1/3 \\ 1 & 7/24 & 11/24 & 1/4 \\ 2 & 1/3 & 7/24 & 3/8 \end{array}$$

c) $\vec{p}(0) = \left[\frac{1}{5}\ \frac{2}{5}\ \frac{2}{5}\right]$ $\vec{p}(1) = \vec{p}(0) \cdot P = \left[\frac{1}{5}\ \frac{2}{5}\ \frac{2}{5}\right] \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$
$= \left[\frac{11}{30}\ \frac{7}{30}\ \frac{2}{5}\right]$

d) $P[X_0 = 0, X_1 = 1, X_2 = 1] = p_0(0)\, P_{01} P_{11}$
$= 0$

$P[X_0 = 2, X_1 = 0, X_2 = 1] = p_2(0)\, P_{20} P_{01}$
$= \frac{2}{5} \cdot \frac{1}{4} \cdot \frac{2}{3} = 1/15$

e) $\vec{\pi} P = \vec{\pi}$ $\vec{\pi} = [9/25\ \ 8/25\ \ 8/25]$

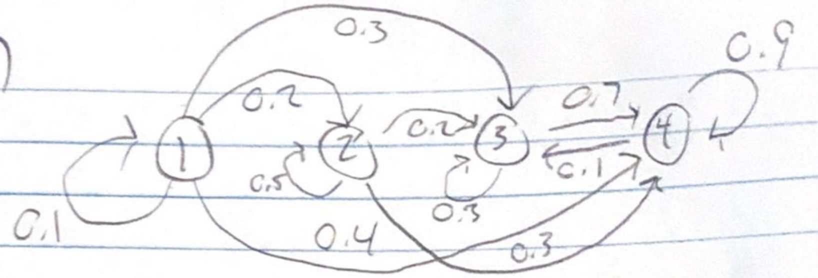
$$[\pi_1\ \pi_2\ \pi_3] \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix} = [\pi_1\ \pi_2\ \pi_3]$$

$\pi_2 = \pi_3$? $\frac{\pi_2}{2} + \frac{\pi_3}{2} = \pi_3$

$\frac{\pi_1}{3} + \frac{\pi_2}{2} + \frac{\pi_3}{4} = \pi_1 \Rightarrow \frac{3\pi_2}{4} = \frac{2\pi_1}{3}, \pi_1 = \frac{9\pi_2}{8}$

w/ normalization
property, $\pi_1 + \pi_2 + \pi_3 = 1$, $\frac{9\pi_2}{8} + \pi_2 + \pi_2 = 1$, $\pi_2 = \frac{8}{25}$

3a)



b) {1,2} - transient {3,4} - recurrent

c) A Markov chain w/ one communicating class is irreducible. This chain has two classes ∴ not irreducible.

d) $P_{11} > 0$, so the Markov chain is aperiodic i.e. it has states that can travel to itself.

e) $\vec{\pi} P = \vec{\pi}$

$[\pi_1 \ \pi_2 \ \pi_3 \ \pi_4] = \vec{\pi} P \Rightarrow$

$\pi_1 = 0.1\pi_1$

$\pi_2 = 0.2\pi_1 + 0.5\pi_2$

$\pi_3 = 0.3\pi_1 + 0.2\pi_2 + 0.3\pi_3 + 0.1\pi_4$

$\underline{\pi_1 = 0} \quad \underline{\pi_2 = 0}$

$0.7\pi_3 = 0.1\pi_4$

$\pi_4 = 0.7\pi_3 + 0.3\pi_4 \Rightarrow \pi_4 = \frac{0.7}{0.1}\pi_3$

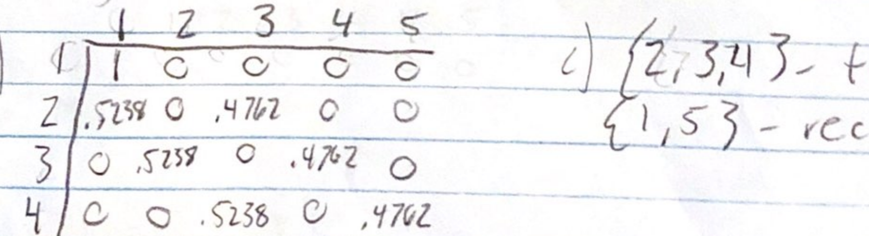
$\pi_4 = 0.875$

$\underline{\pi_3 = 0.125}$

$\underline{\vec{\pi} = [0 \ \ 0 \ \ 0.125 \ \ 0.875]}$

$P_{losing} = 0.5238$

4a)



$P_{winning} = 0.4762$

b)

	1	2	3	4	5
1	1	0	0	0	0
2	.5238	0	.4762	0	0
3	0	.5238	0	.4762	0
4	0	0	.5238	0	.4762
5	0	0	0	0	1

c) {2,3,4} - transient
{1,5} - recurrent

d) $P(20) = (P^t)^{20} P(0) = \begin{bmatrix} .45187 \\ 0 \\ .000954 \\ 0 \\ .547171 \end{bmatrix}$

$P_{\text{broke after 20 ATT}} = \underline{.451877}$

$P_{\$4 \text{ left after 20 ATT}} = \underline{.5472}$

e) $(P^t)^{100} P(0) = \left(\lim_{n \to \infty} P'(100)\right) P$

$P_{\text{leave broke after 100 ATT}} = \underline{.5475}$

$P_{\$4 \text{ left after 100 ATT}} = \underline{.4525}$

f)

g) $P_{32} \cdot P_{21} = 0.5238 \cdot 0.5238 = \underline{.2744}$