ENG EK381 Probability, Statistics and Data Science for Engineers — Spring 2020
Castañón — April 2, 2020

Exam 2

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Honor Code: This exam represents only my own work. I did not give or receive help.

Signature: Chase Maivald

General Note: In my view, the most important issue is to know how to approach a particular problem. Therefore, there will be partial credit for good solution outlines even if not all the mathematical manipulations are completed correctly. Be sure to attempt all problems!

- You have exactly **1 hour and 55 minutes** to complete this exam.
- The exam is Open Book. All work you want graded must be uploaded at the end.
- We will have a Zoom meeting window available. The link is on the Blackboard page, in the Tools/Zoom Meetings page. If you have any questions or need clarification, ask in the Chat window, or send e-mail, or ask by voice. I should be able to hear you.
- Keep track of time. **You must upload your solution before the upload link closes!!!**
- If there are urgent issues and you need to contact me, I can be reached at 1-781-258-3510.

*** Good Luck! ***

Problem	Points earned	out of	Problem	Points earned	out of
Problem 1		24	Problem 5		14
Problem 2		18	Problem 6		16
Problem 3		12	Problem 7		3
Problem 4		10	Exam 2 Quiz		3
			Total		100

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Exam 2

1a) property of PDF is $f(x) > 0$

$F_x(x) = \int_{-\infty}^{\infty} f(x) = 1$

so false

b) $f_x(x)$ is PDF of x

if x increases, PDF $G_x(x)$ can decrease

so true

c) $Var(x) = E(x)^2 - (E(x))^2$

$E(x) = 1$ $\quad Var(x) = 9$

$9 = E(x)^2 - 1^2$

$E(x)^2 = 10$

c is true

d) $E(x) = 1$

$y = 3x$

$E(y) = E(3x)$ $\quad$ d is true

$= 3E(x)$

$= 3x$

$E(y) = 3$

e) true

f) true

g) true

h) true

i) true

j) false

k) true $\quad Var(x+y) = Var(x) + Var(y)$

l) false

2) $E(X) = 2$

$X \to E(1/2)$

$Y = -2X$

$E(Y) = E(-2X)$

$= -2E(X)$

$= 4$

$Var(Y) = var(-2X)$

$= -2^2 \, var(X)$

$Var(Y) = 4 \, Var(X)$

$Ex^2 = \int_0^\infty x^2 f(x)\,dx$

$= \int_0^\infty x^2 \cdot \frac{1}{2} e^{-x/2}\,dx$

$= 1/2 \cdot \Gamma 3 / (1/2)^3$

$Ex^2 = 2/(1/2)^2$

$= 8$

$Var(X) = Ex^2 - (E(X))^2$

$= 8 - (2)^2$

$= 4$

$Var(Y) = 4 \cdot 4$

$= 16$

b) $X \to U[1,5]$

$f(x) = 1/5-1 \quad 1 \leq x \leq 5$

$= 1/4$

$Ex^2 = \int_1^5 x^2 f(x)\,dx = \int_1^5 x^2 \, 1/4 \, dx$

$= 1/4 \, (x^3/3) \Big|_1^5$

$= \frac{125-1}{12} = 124/12$

c) $X \to N(1,1)$

$Y = 2(x-1)$

$Y = 2x - 2$

$E(Y) = 2E(X) - 2$

$= 2x - 2$

$= 2 - 2$

$= 0$

$(E(Y))^2 = 0^2$

$= 0$

d) $E(x) = 2$ $Ex^2 = 8$

$Var(x) = E(x)^2 - (E(x))^2$

$= 8 - (2)^2$

$= 4$

$S.D(x) = \sqrt{var(x)}$

$= 2$

$\mu = 2$, $\sigma = 2$ $\quad X \to N(2,2)$

$P(2 \leq x \leq 7) = P\left(\frac{2-2}{2} \leq \frac{x-4}{\sigma} \leq \frac{7-2}{2}\right)$

$= P(0 \leq z \leq 2.5)$

$= P(z \leq 2.5) - P(z \leq 0)$

$\underline{P(2 \leq x \leq 7) = \phi(2.5) - \phi(0)}$ where $\phi(z) = P(z \leq z)$

e) $U[0,1]$ $\quad Y$ Gaussian $(x, 1)$

$E(x) = 1/2$ $\quad \underline{E(Y) = 1/2}$

f) $E((x+2y)^2) = Var((x+2y)) + (E(x) + E(2y))^2$

$= 1 + 2 + .4$

$= 3.4$

	$y=-1$	$y=0$	$y=2$
3a) marginal $p_X(x)$	0.3	0.55	0.15
marginal $p_Y(y)$	$x=-1$	$x=0$	$x=2$
	0.3	0.55	0.15

b) $E(X) = -1(.3) + 0(.55) + 2(.15)$
$= -.3 + .3 = 0$
$E(Y) = -1(.3) + 2(.15)$
$= -.3 + .3 = 0$

c) $\text{cov}(X,Y) = E(XY) - E(X)E(Y)$
$= 1(.2) + -2(.1) + -2(.1) + 4(.05)$
$= 0$

d) because $\text{cov}(X,Y) = 0$, X & Y are independent

e) $\frac{.2}{.2+.1} = 2/3$
0
$\frac{0.1}{0.2+0.1} = 1/3$

$$P_{X|Y}(x|y=-1) = \begin{cases} 2/3 & x=1 \\ 0 & x=0 \\ 1/3 & x=2 \end{cases}$$

f) $E(X|Y=-1) = 2/3(-1) + 1/3(2) = 0$

4a) $\int_{-\infty}^{\infty} f_X(x)\,dx = \int_0^5 c(5-x)\,dx + \int_{-5}^0 c(5+x)\,dx$
$= c(5x - \frac{x^2}{2})\Big|_0^5 + c(5x + \frac{x^2}{2})\Big|_{-5}^0$
$= c(25 - \frac{25}{2}) - c(-25 + \frac{25}{2})$
$= 12.5c + 12.5c = 25c$

$\underline{c = \frac{1}{25}}$

b) $\int_0^5 x \frac{1}{25}(5-x)\,dx + \int_{-5}^0 x \frac{1}{25}(5+x)\,dx$

c) $\int_0^5 x^2 \frac{1}{25}(5-x)\,dx + \int_{-5}^0 x^2 \frac{1}{25}(5+x)\,dx$
$- \left[\int_0^5 x \frac{1}{25}(5-x)\,dx + \int_{-5}^0 x \frac{1}{25}(5-x)\,dx\right]$

d) $P(X>1) = \int_1^5 \frac{1}{25}(5-x)\,dx$

e) $\dfrac{\int_4^5 \frac{1}{25}(5-x)\,dx}{\int_1^5 \frac{1}{25}(5-x)\,dx}$

5a) $E(A) = E(4X-1)$

$= 4 \cdot E(X) - 1$

$= 3$

$E(B) = E(X+2Y-5)$

$= E(X) + 2E(Y) - 5$

$= 1 + 2 - 5$

$= 3 - 5 = -2$

b) $Var(A) = Var(4X-1) = 4^2 \cdot Var(X)$

$= 16$

c) $Var(B) = Var(X+2Y-5)$

$= V(X+2Y)$

$= V(X) + 2^2 \cdot V(Y) + 2 \cdot Cov(X,Y)$

$= 1 + 4 \cdot 1 + 2(-.25)$

$= 5 - .5 = 4.5$

d) $Cov(A,B) = Cov(4X-1, X+2Y-5)$

$= 4 \cdot \underline{Cov(X, X+2Y)}$

$Cov(X, X+2Y) = 1 \cdot Cov(X,X) + 2 \cdot Cov(X,Y)$

$= Var(X) + 2 \cdot Cov(X,Y)$

$= 1 + 2(-.25)$

$= .5$

$\Rightarrow Cov(A,B) = 4 \cdot Cov(X, X+2Y) = 4(.5) = 2$

e) X & Y are not independent because $Cov(X,Y) \neq 0$

f)

6a) $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dy$ $\quad f_X(x) = \begin{cases} \int_0^{\sqrt{1-x^2}} \frac{2}{\pi}\, dy & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

b) No, X, Y is not a rectangle

c) $E(x) = 0$ by symm

d) $E(x^2) = \int_{-1}^{1} x^2 \frac{2\sqrt{1-x^2}}{\pi}\, dx$

$Var(x) = E(x^2) - (E(x))^2$

$= \int_{-1}^{1} x^2 \frac{2\sqrt{1-x^2}}{\pi}\, dx$

e) $cov(x, y) = E(xy) = \int_{-1}^{1} \int_0^{\sqrt{1-x^2}} 2/\pi\, uv\, du\, dv$

f) $\dfrac{\int_0^{\sqrt{1-0.8^2}} 2/\pi\, dy}{2/\pi}$

g) $\dfrac{\int_0^{\sqrt{1-0.8^2}} y \cdot 2/\pi\, dy}{2/\pi}$

h) $\int_{2x}^{\sqrt{1-x^2}} \int_{-1}^{1} 2/\pi\, u v\, du\, dv$