Measuring and Predicting Running Time

Victor Milenkovic

Department of Computer Science
University of Miami

CSC220 Programming II – Spring 2018

Outline

Running times of different implementations

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- We have two implementations of PhoneDirectory: ArrayBasedPD and SortedPD.

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Running times of different implementations

- We have two implementations of PhoneDirectory: ArrayBasedPD and SortedPD.
- Each has implementations of find, addOrChangeEntry, and removeEntry.
- Can we compare their speeds?

find

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- ArrayBasedPD.find

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- SortedPD.find
  - Only really helpful when n (size) is large.

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  - Look for Vic?
  - Have to compare Vic with n entries, where $n =$ size, which is 6.
- SortedPD.find
  - Only really helpful when n (size) is large.
  - Requires $\log_2 n$ comparisons

addOrChangeEntry

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  - n array accesses. Actually $n - 1$ reads and n writes, where n is 7. So $2n - 1$.

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  - find uses $\log_2 n$ comparisons
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  - Let's add Abe.
  - Abe, Ann, Bob, Eve, Ian, Jay, Zoe
  - n array accesses. Actually $n-1$ reads and n writes, where n is 7. So $2n-1$.
  - Total time is $\log_2 n$ comparisons plus $2n-1$ array accesses.

removeEntry

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  - Eve, Bob, Zoe, Ian, Ann
  - Time for 1 comparison and 2 array accesses.

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  - Eve, Bob, Zoe, Ian, Ann
  - Time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)

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  - The program still uses 2 array accesses to “remove” Eve (but it could be smarter).

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  - So Eve is worst case, requiring time for n comparisons and 2 array accesses.

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  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?

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  - removeEntry calls find.
  - find takes 1 comparison to find Jay.
  - Eve, Bob, Zoe, Ian, Ann
  - Time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
  - The program still uses 2 array accesses to “remove” Eve (but it could be smarter).
  - So Eve is worst case, requiring time for n comparisons and 2 array accesses.
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  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?

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  - Who takes longest to remove? Jay?
  - removeEntry calls find.
  - find takes 1 comparison to find Jay.
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  - Time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
  - The program still uses 2 array accesses to “remove” Eve (but it could be smarter).
  - So Eve is worst case, requiring time for n comparisons and 2 array accesses.
- SortedPD.removeEntry
  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?
  - find takes $\log_2 n$ comparisons to locate Ann.

removeEntry

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  - Jay, Bob, Zoe, Ian, Ann, Eve
  - Who takes longest to remove? Jay?
  - removeEntry calls find.
  - find takes 1 comparison to find Jay.
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  - Time for 1 comparison and 2 array accesses.
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  - Bob, Eve, Ian, Jay, Zoe

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  - find takes 1 comparison to find Jay.
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  - Time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
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  - So Eve is worst case, requiring time for n comparisons and 2 array accesses.
- SortedPD.removeEntry
  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?
  - find takes $\log_2 n$ comparisons to locate Ann.
  - Bob, Eve, Ian, Jay, Zoe
  - n array reads and writes to move everyone else back.

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- ArrayBasedPD.removeEntry
  - Jay, Bob, Zoe, Ian, Ann, Eve
  - Who takes longest to remove? Jay?
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- SortedPD.removeEntry
  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?
  - find takes $\log_2 n$ comparisons to locate Ann.
  - Bob, Eve, Ian, Jay, Zoe
  - n array reads and writes to move everyone else back.
  - Total is $\log_2 n$ comparisons and $2n$ array accesses. Actually the first n should be $n - 1$.

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  - find: $\log_2 n$ comparisons
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- Constants don't matter.
- $\log_2 n = 3.3219 \log_{10} n$, so we just say $\mathrm{O}(\log n)$
- Only the dominant term matters.
- Accurate, up to a constant factor, for large n.

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  - find: $\log_2 n$ comparisons – $\mathrm{O}(\log n)$

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- So the answer is 100 microseconds.

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- For $n = 1000$,
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- So 75 microseconds.

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  - $t = 25 \cdot 3$
  - $t = 75$
- So 75 microseconds.
- Notice that I used the same log base 10. You can't switch log bases in the middle, or you will get a different (and wrong) answer.

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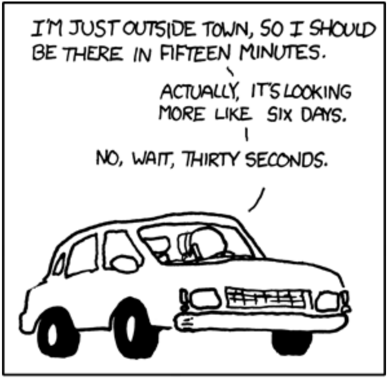
- Different log. Same answer!



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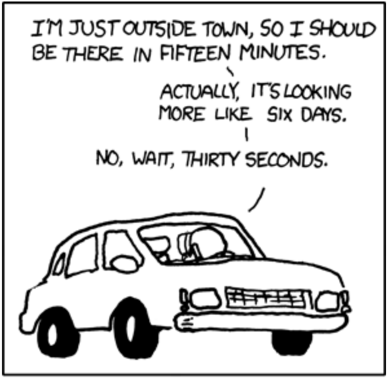

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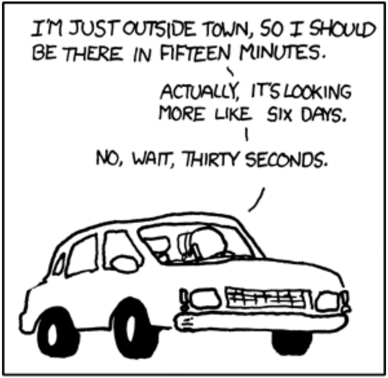

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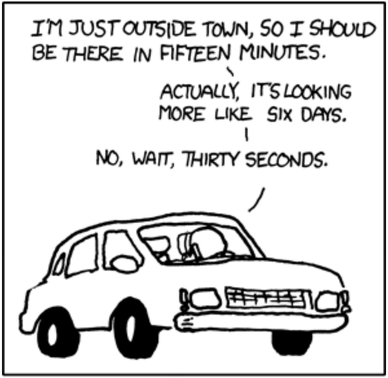

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- Answer: repeat the experiment many times and take the average.

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- For example, run it once to get an approximate time. Figure out how many times you can run it in one second. Run it that many times and take the average running time.