Active/Passive Policies

Chase Abram[*]

May 20, 2021

1 Setup

1.1 Standard New Keynesian Model

We begin with the standard New Keynesian system, a detailed derivation of which can be found in Ben Moll's note, where π is inflation, x is the output gap, ρ is the household discount rate, κ is parameterized by the Frisch labor elasticity, the degree of monopolistic competition, and the price adjustment cost, and i is the nominal interest rate.

$$\dot{\pi} = \rho\pi - \kappa x$$
$$\dot{x} = i - \pi - \rho$$

If the fiscal theory is truly important for understanding inflation (amongst other macroeconomic variables), why is there nothing fiscal in the above system? The answer is just modeling choice, thus far. To set this model up we assumed households are optimally consuming and working, and that firms are maximizing profits dynamically, facing a quadratic price adjustment cost. The typical next step is to impose an interest rate rule a la (Taylor, 1993), with the idea that the central bank can control i, and therefore discipline the model. We then often impose that the Taylor rule is sufficiently "active" so that only only one time path of inflation and output gap does not correspond to explosion. Mathematically, this "active" Taylor rule guarantees all the eigenvalues of the system are positive, and we *select* an equilibrium by selecting the steady-state values as a boundary condition in the infinite horizon.

1.2 Issues

A few potential issues arise. First, it is not clear why either of the boundaries are selected, in economic terms, since the dynamic system already takes into account optimizing behavior. This points to the underlying strangeness, which is that we have two jump variables, but no clear choice for boundaries. Contrast this with a simple neoclassical growth model, where capital is a state (not jump) variable, and there is an initial capital stock available, giving a boundary. The standard response to this problem is to simply say that we demand that this system return to its steady state.

Even if we take this as given, however, the path is not unique if one of the eigenvalues is negative. It should be noted that this is a peculiarity of the fact that our boundary is on the infinite horizon.

[*]Send corrections or comments to abram@uchicago.edu.

For any finite T, a boundary condition at T would uniquely pin down the system path, and the eigenvalues would merely determine the shape. In the infinite horizon, we get a new degree of freedom in that the *rate* of convergence is not determined up to scale (still assuming at least one negative eigenvalue)[1].

1.3 Adding Debt

Now let's allow for government debt in the model. This will change neither of the above equations, but will create a new state variable. Let b be the real value of government debt, and s be real surpluses. The evolution of real debt simply says that it accumulates based on the real interest rate, and surpluses devalue the debt (these could be real taxes used to pay down the debt, for example).

$$\dot{b} = (i - \pi)b - s$$

We immediately find the "hidden" passive fiscal policy in the standard model by noting two facts

(i) The No-Ponzi condition for government debt implies that the real value of debt must not explode too quickly, else households would not be willing to purchase the debt, per their transversality condition[2].

(ii) This equation is ignored in the standard two equation (plus policy) model above.

To reconcile the fact that a new condition must be met for this new equation, but at the same time this condition cannot generally be met by the solution to the above system, which is found independently of this equation, it *must* be that s is functioning to satisfy the condition for government debt. We will show below that, since the standard model restricts the policy space in this way, this fits within a more general definition of passive fiscal policy, and active monetary policy, provided we put standard additional structure on the policy space in terms of monetary policy.

2 Policy

2.1 Full System

Now let's consider the full system.

$$\begin{aligned}
\dot{\pi} &= \rho\pi - \kappa x \\
\dot{x} &= i - \pi - \rho \\
\dot{b} &= (i - \pi)b - s
\end{aligned}$$

With general policy rules

[1] This is akin to trying to solve $\dot{z} = az$, where $a < 0$ and our "boundary" is $\lim_{T\to\infty} z(T) = 0$. The boundary does not help with pinning the constant C in solution $z(t) = Ce^{at}$.

[2] Note that the No-Ponzi condition is imposed on the government, whereas the transversality condition for households comes from optimizing.

$$i = M(\pi, x, b)$$
$$s = F(\pi, x, b)$$

This system is generally nonlinear due to the fact that i and π pre-multiply b in the debt equation, and we allow for nonlinear policy rules. For boundary conditions, the real level of debt cannot jump, since its nominal level cannot jump and prices cannot jump in the sticky case. So the initial and terminal values of real debt are boundary conditions. We are still left wanting for a fully microfounded final boundary, but for now we will assume that the output gap converges to its steady-state, as the model should then return to its full steady state (in certain cases we can show this implies $\pi \to \bar{\pi}$).

It is worth emphasizing that we are now using two boundary conditions for a single equation (debt), and therefore we can heuristically think of one of them as pinning another variable, inflation being the most intuitive, since we do not have a boundary for it.

2.2 Active/Passive Classification

A policy (M, F) is a combination of a monetary policy M and a fiscal policy F. We may restrict the policy space so that only certain (M, F) are under consideration, but we may wish to restrict the policies to jointly depend on one another. So a subset of policy space will not in general be equal to a product of a subset of policies for M and F individually.

Definition 2.1 (Active and Passive Policies[3])**.** Let $\mathcal{P}$ be a subset of policy space.

First, let $(M, F) \in \mathcal{P}$ be a policy combination that yields a unique solution for (π, x, b).

(i) If there exists $(M', F') \in \mathcal{P}$, only different from (M, F) by changing parameters within monetary policy's control, such that the solution under (M', F') is not unique, but there does not exist $(M'', F'') \in \mathcal{P}$, only different from (M, F) by changing parameters within monetary policy's control, such that the there is no solution, then we say fiscal policy F is passive.

(ii) If there exists $(M', F') \in \mathcal{P}$, only different from (M, F) by changing parameters within monetary policy's control, such that there is no solution under (M', F'), but there does not exist $(M'', F'') \in \mathcal{P}$, only different from (M, F) by changing parameters within monetary policy's control, such that the solution is not unique, then we say fiscal policy F is active.

(iii) If there exists $(M', F') \in \mathcal{P}$, only different from (M, F) by changing parameters within fiscal policy's control, such that the solution under (M', F') is not unique, but there does not exist $(M'', F'') \in \mathcal{P}$, only different from (M, F) by changing parameters within fiscal policy's control, such that the there is no solution, then we say monetary policy M is passive.

(iv) If there exists $(M', F') \in \mathcal{P}$, only different from (M, F) by changing parameters within fiscal policy's control, such that there is no solution under (M', F'), but there does not exist $(M'', F'') \in \mathcal{P}$, only different from (M, F) by changing parameters within fiscal policy's control, such that the solution is not unique, then we say monetary policy M is active.

If none of the above statements hold, we cannot make statements about policy being active or passive within $\mathcal{P}$.

[3] This terminology dates back to (Leeper, 1991)

Note that we vary one policy type within the policy space to classify the other policy type. The motivation is that at a unique equilibrium, the system is jointly determined, so we cannot attach policies to eigenvalues, but if we are able to vary one policy and move to exactly one of either a passive-passive or active-active equilibrium, then we can say that the other policy must be, respectively, either active or passive.

2.3 Standard AM/PF

Consider the policy space below, where monetary policy uses a Taylor rule[4], and fiscal policy holds the real value of debt constant.

$$\mathcal{P} = \{(M, F) \mid M(\pi, x, b) = \bar{\rho} + \phi\pi, F(\pi, x, b) = (M(\pi, x, b) - \pi)b, \phi \in \mathbb{R}\}$$

This policy space is one-dimensional, and can be parameterized by ϕ, under monetary policy's control. We now show the intuitive outcome that fiscal policy is passive when $\phi > 1$, though we will need to amend the policy space to show monetary policy is active. The system becomes

$$\begin{aligned}
\dot{\pi} &= \rho\pi - \kappa x \\
\dot{x} &= (\phi - 1)\pi + \bar{\rho} - \rho \\
\dot{b} &= 0 \\
b(0) &= \lim_{T\to\infty} b(T) = \bar{b} \\
\lim_{T\to\infty} x(T) &= \bar{x} = 0
\end{aligned}$$

When $\phi > 1$, we recover the standard result that the equilibrium is unique (note that we are still selecting it using the boundary on x). If $\phi \leq 1$, the equilibrium is not unique. Therefore we have shown that fiscal policy F is passive for any (M, F) such that $\phi > 1$.

Note that in this incredibly narrow policy space we cannot say monetary policy is active, because we have no means to vary fiscal policy, since it controls no parameters. If we instead consider

$$\mathcal{P} = \{(M, F) \mid M(\pi, x, b) = \bar{\rho} + \phi\pi, F(\pi, x, b) = (M(\pi, x, b) - \pi)b - \gamma b, \phi, \gamma \in \mathbb{R}\}$$

we have system

$$\begin{aligned}
\dot{\pi} &= \rho\pi - \kappa x \\
\dot{x} &= (\phi - 1)\pi + \bar{\rho} - \rho \\
\dot{b} &= -\gamma b \\
b(0) &= \lim_{T\to\infty} b(T) = \bar{b} = 0 \\
\lim_{T\to\infty} x(T) &= \bar{x} = 0
\end{aligned}$$

Now we can consider the policy $\phi = 1.5$, $\gamma = 1$, and repeat the same exercise to show that fiscal policy is passive, but we can also try $\gamma < 0$, see that we have no solution (and we do have unique solutions otherwise), and conclude that monetary policy is active.

[4] The value $\bar{\rho}$ is the steady-state value of ρ, since we will later consider shocks to the discount rate, but assume the policy authorities cannot take these shocks into account directly.

This point should make clear that being active or passive is a property of a specific policy *within* an underlying policy space. If this is confusing, consider the singleton policy space, where the one policy delivers a unique solution. It would be strange to call either policy active or passive, since neither is able to vary within the policy space, so we cannot answer how the other would or would not need to respond. On the other extreme if we allow policies to simply be all measurable functions, for example, it seems unlikely we will be able to show anything with regards to active or passive policies, since we may be able to vary parameters under one policy type's control and move to either a passive-passive or active-active outcome. Hence we typically restrict to policies of interest, or feasibility.

2.4 Dual Taylor

Consider the policy space that allows for Taylor-type rules for both surpluses and the interest rate

$$\mathcal{P} = \{(M, F) \mid M(\pi, x, b) = \bar{\rho} + \phi\pi, F(\pi, x, b) = \bar{s} + \gamma b, \phi, \gamma \in \mathbb{R}, \bar{s} = (\bar{\rho} - \gamma)\bar{b}\}$$

The system is

$$\begin{aligned}
\dot{\pi} &= \rho\pi - \kappa x \\
\dot{x} &= (\phi - 1)\pi + \bar{\rho} - \rho \\
\dot{b} &= (\rho + (\phi - 1)\pi)b - \bar{s} - \gamma b \\
&= (\bar{\rho} - \gamma)(b - \bar{b}) + (\phi - 1)\pi b \\
b(0) &= \lim_{T\to\infty} b(T) = \bar{b} \\
\lim_{T\to\infty} x(T) &= \bar{x}
\end{aligned}$$

I omit the proof of the following theorem because proving it analytically may be tedious and/or quite difficult, so I leave it as task to be done if I later turn this into a formal paper. The main ideas to complete the proof may lie in here, though. It is not hard to convince yourself that following is true, however, by mentally tracing what will happen to the system in each case.

Theorem 1.

(i) When $\phi > 1$, monetary policy is active, otherwise monetary policy is passive

(ii) When $\gamma < \rho$, fiscal policy is active, otherwise fiscal policy is passive

Theorem 2. Consider the dual Taylor policy space

$$\mathcal{P} = \{(M, F) \mid M(\pi, x, b) = \bar{\rho} + \phi\pi, F(\pi, x, b) = \bar{s} + \gamma b, \phi, \gamma \in \mathbb{R}, \bar{s} = (\bar{\rho} - \gamma)\bar{b}\}$$

Fix $\phi > 1$, and consider a shock to the discount rate such that $\rho(t) \to \rho$. Then for any choice of $\gamma > \rho$, the impulse responses of x, π, i, and r are the same.

Proof. Let $\gamma_1 > \rho$, and (π_1, x_1, b_1) solve the system. This solution is unique because one policy is active and the other is passive. Now consider $\gamma_2 > \rho$. Nothing has changed in the $\dot{\pi}$ equation, and nothing has changed in the policy rule for i, therefore nothing has changed in the $\dot{x}$ equation, so

π_1 and x_1 will satisfy the first two equations, and since $x_1 \to \bar{x}$, it will still satisfy the boundary with the new fiscal policy. In the $\dot{b}$ equation the boundaries are unchanged, and the only effect of γ_2 will be that the rate at which b returns to $\bar{b}$ will be changed, since surplus reactivity to debt has changed. Therefore π_1 and x_1 and their implied i and r, combined with a surplus process which is still active, will solve the system under γ_2. □

2.5 Existence of Non-Classifiable Policies

We above argued that a monetary policy could not be classified because we had no way to vary fiscal policy. A natural question is whether we can ever have a non-classifiable policy in the setting where the other policy has freedom to move. We show the affirmative answer by an example.

Let $\mathcal{P}^*$ be defined

$$\mathcal{P}^* = \mathcal{P} \cup \{(M, F) \mid M(\pi, x, b) = \frac{F(\pi, x, b) + 1}{b} + \pi, F(\pi, x, b) = \bar{s} + \gamma b, \gamma \in \mathbb{R}, \bar{s} = (\rho - \gamma)\bar{b}\}$$

where $\mathcal{P}$ is the Dual Taylor policy space. Now consider the policy $\phi = 1.5$, $\gamma = 1.1 \cdot \rho$. From our above results we know this generates a unique determinate equilibrium, and we also know that if we instead consider $\phi = 0.9$, the equilibrium will not be unique. Finally, using the same $\gamma = 1.1 \cdot \rho$, consider the implied M from the part of the policy space we added. In this case we will mechanically have $\dot{b} = 1$, so cannot satisfy our boundaries, and therefore no equilibrium exists. So in this case our fiscal policy cannot be classified.

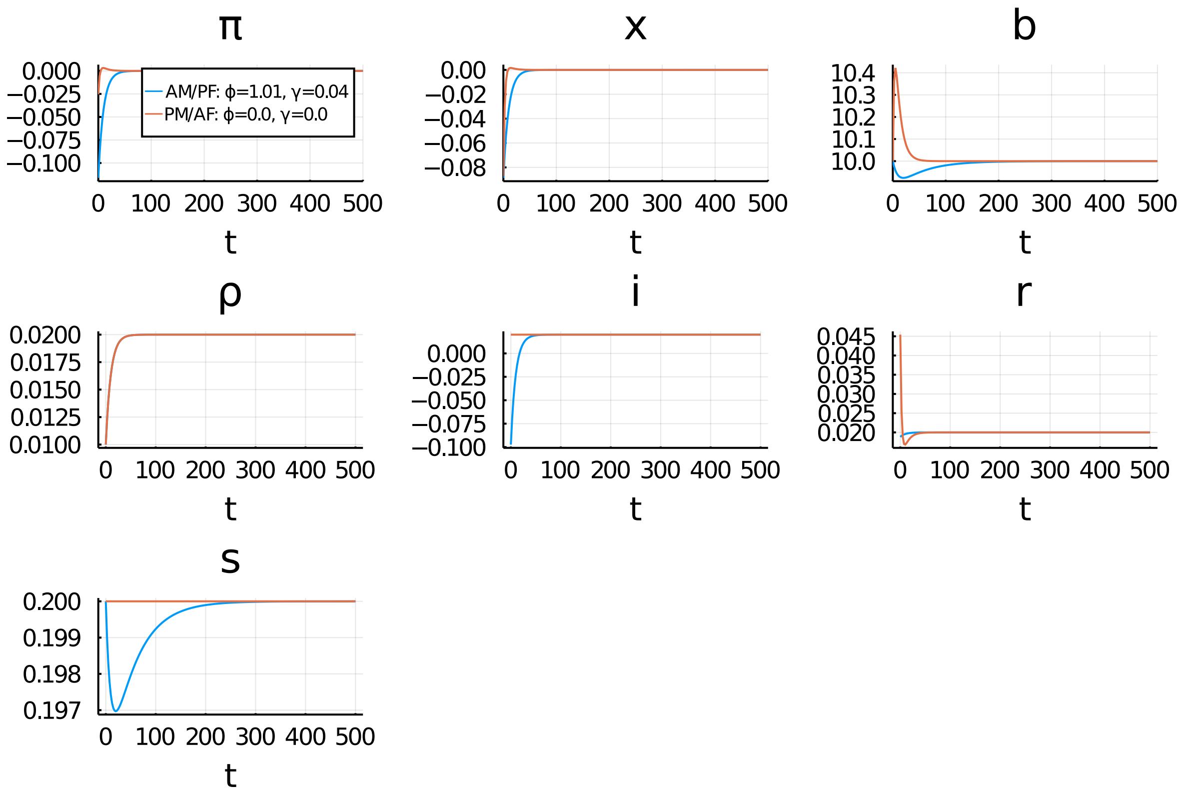
Does this example feel cooked up? Good. One takeaway could be that we likely want to be reasonable in defining our policy space so that classification is possible. Hence our focus on Dual Taylor.

3 IRFs

I include some experiments to illustrate the above results.

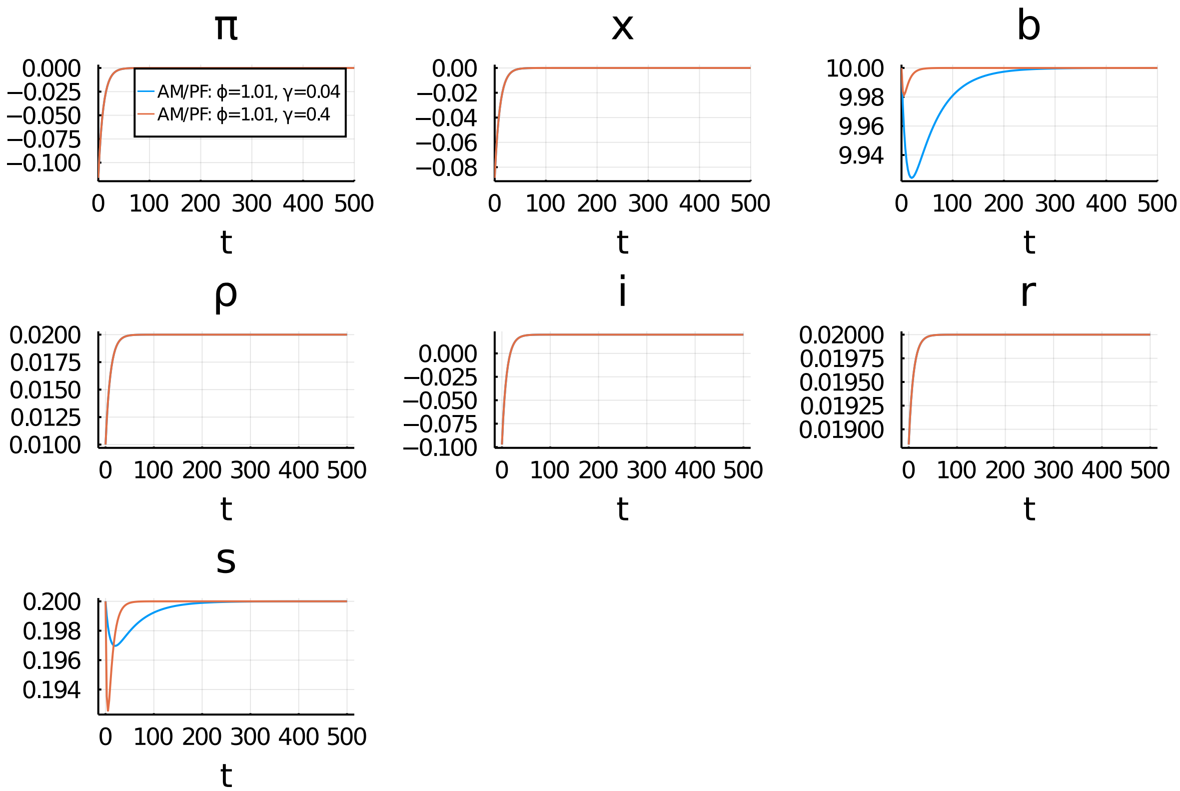
3.1 Basic Comparison

The blue line shows a standard AM/PF world, while the orange shows PM/AF, in particular the double peg.



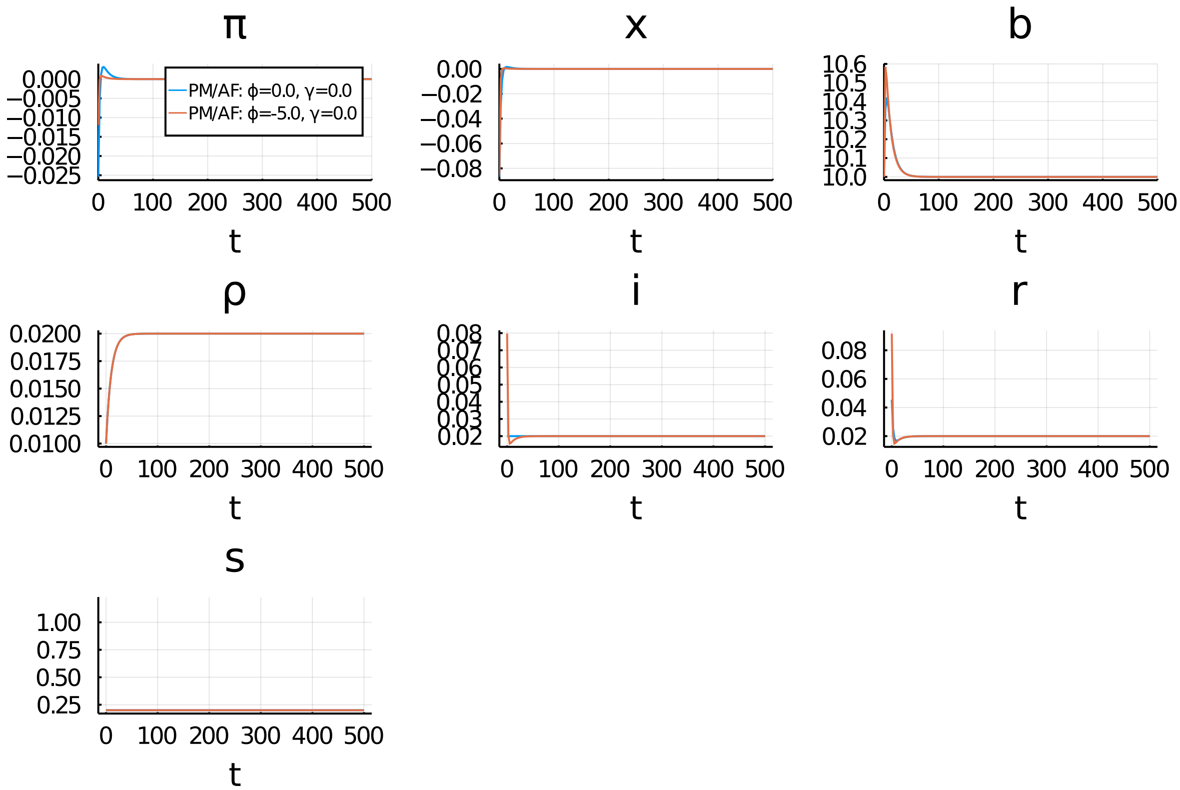
3.2 Invariance of FP under AM/PF

The irrelevance of $\gamma > \rho$, given $\phi > 1$, is demonstrated. Notice that a more reactive fiscal policy has surpluses dive more quickly, and therefore debt does not fall as far.



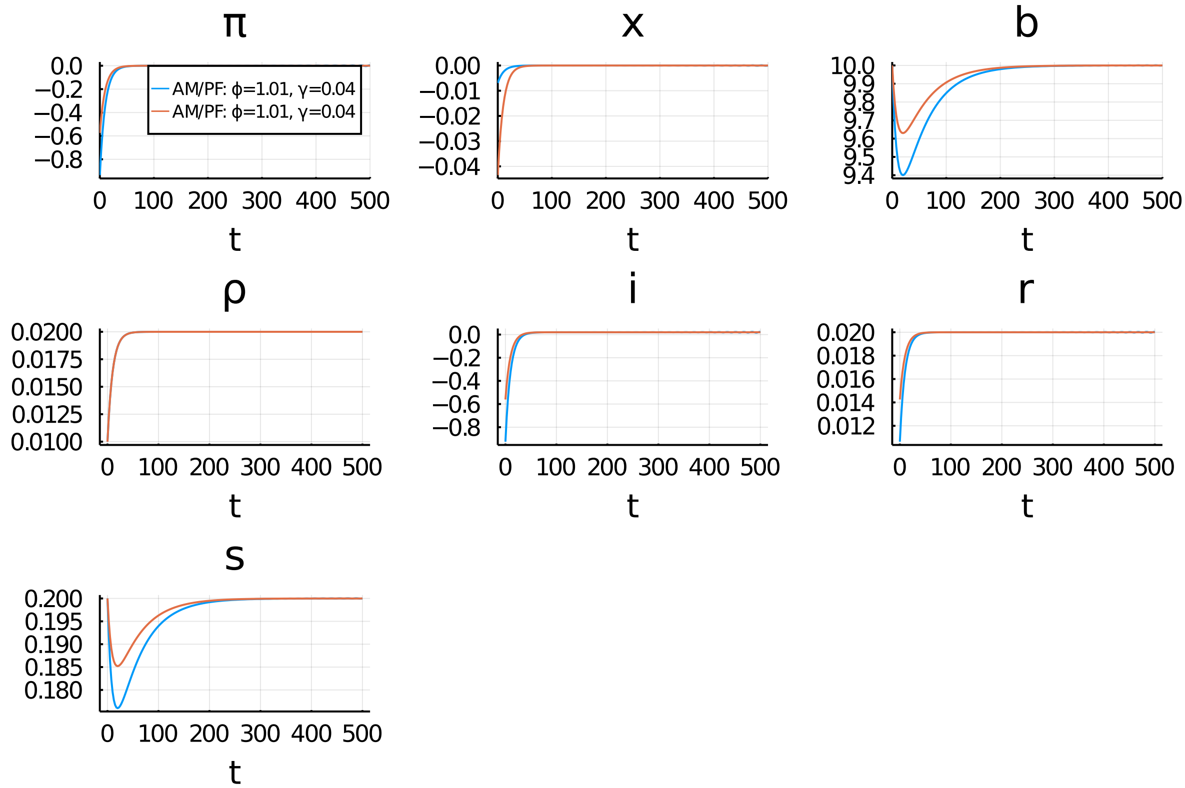
3.3 Non-Invariance in PM/AF

In the PM/AF regime, there is no converse result for timepaths being invariant to ϕ.



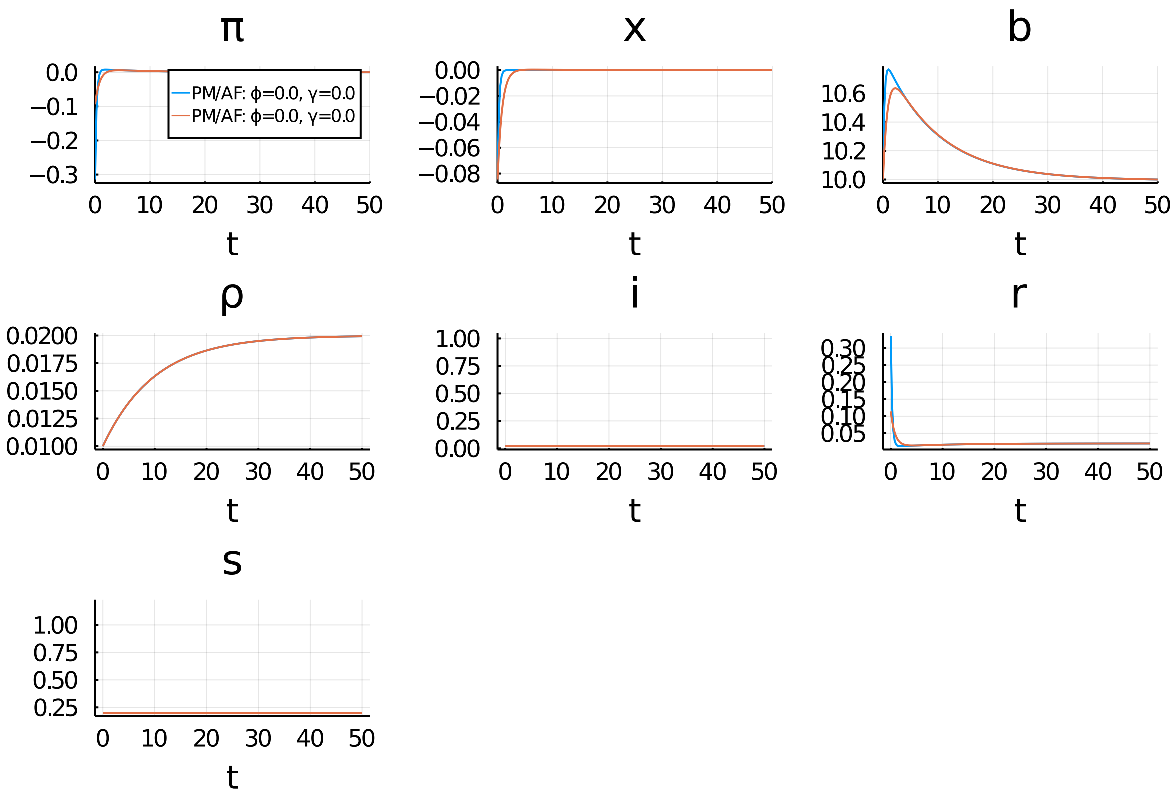
3.4 AM/PF varying stickiness

The blue plot has $\theta = 1$, and the orange has $\theta = 10$.



3.5 PM/AF varying stickiness

The blue plot has $\theta = 1$, and the orange has $\theta = 10$.



References

Leeper, Eric (1991). "Equilibria under 'active' and 'passive' monetary and fiscal policies". In: *Journal of Monetary Economics* 27.1, pp. 129–147.

Taylor, John B (1993). "Discretion versus policy rules in practice". In: *Carnegie-Rochester Conference Series on Public Policy* 39, pp. 195–214.