Optimal Policies in (Antras et al., 2021)

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1 Roadmap

1. We re-did the optimal tariff policy exercises in the case where only tariffs by the home country vary
2. We re-did the optimal tariff policy exercises in the case where all four policy parameters can vary
3. We consider a tariff war game for our first counterfactual
4. We consider a second counterfactual where each country chooses an optimal tariff assuming tit-for-tat

2 Optimal Tariff Policy: Only Tariffs by Home Vary

We first quantify the model to see the effects of downstream and upstream tariffs on welfare. As in the paper[1], we list the model's parameters that we use in our estimation to check if we obtain the same results. In panel A. we write the four parameters that are obtained from previous work or to keep input versus final good production as symmetric as possible and these are fixed. Without loss of generality, elasticities of substitution across varieties in each sector are fixed to 4 and entry costs are symmetric across sectors and countries and fixed at 1. Panel B. includes the parameters we obtain from the data. $1-\alpha$ captures the share of inputs in production and is measured as the share of revenue used to pay for intermediate inputs in production. Data for 2007 for the United States from the World Input Output Database (WIOD) and from CEPII is used to obtain the input revenue share and labor endowment respectively. US productivity is normalized to 1 in both sectors, so we only need to estimate four parameters: trade costs in each sector $\{\tau^d, \tau^u\}$, and sectoral productivity $\{A^d, A^u\}$. To estimate the model, as in the paper, we use the vector of parameters $\left\{\tau^d, \tau^u, A^d, A^u\right\}$ that minimizes the sum of squares of the differences between model generated and empirical moments, subject to equilibrium constraints. Panel C. shows the estimated values of RoWs' productivities and iceberg trade costs in each sector.

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[1] Tables 1 and 2 ar verbatim from the paper. We did not re-estimate these quantities, but focused on replicating the structural model solving.

Table 1: Parameter Values Used in the Estimation

A. Fixed Values		
θ	Elasticity of substitution, input varieties	4
σ	Elasticity of substitution, final-good varieties	4
f^d	Entry costs, final-good sector	1
f^u	Entry costs, input sector	1
B. Values Measured From Data		
$1-\alpha$	Expenditure on inputs relative to total sales	0.4517
L^{us}	Scaled population in US	0.4531
L^{row}	Scaled population in RoW	9.5469
C. Estimated Values		
A^d_{row}	Productivity in final-good sector, RoW relative to US	0.2821
A^u_{row}	Productivity in input sector, RoW relative to US	0.1114
τ^d	Iceberg cost for final goods from US to RoW	3.0301
τ^u	Iceberg cost for inputs from US to RoW	2.6039

Notes: This table summarizes the values of the model parameters. $L^{\text{us}} = 10 \times \frac{Pop^{\text{us}}}{\text{Popus + Poprow}}$ and $L^{\text{row}} = 10 \times \frac{Pop^{\text{row}}}{\text{Popus + Poprow}}$.

Next, we present the set of moments that were targeted in the estimation. These are necessary to solve for changes in equilibrium outcomes in response to a counterfactual change in tariffs (hat algebra approach).

Table 2: Targeted Moments

Description	Data	Model
Sales share to US from US in final goods	0.9644	0.9591
Sales share to RoW from RoW in final goods	0.9767	0.9814
Sales share to US from US in intermediate good	0.9209	0.9053
Sales share to RoW from Row in intermediate good	0.9762	0.9744
Expenditure share in US for US in final good	0.9342	0.9459
Expenditure share in RoW for RoW in final good	0.9862	0.9844
Expenditure share in US for US in int. good	0.9044	0.9281
Expenditure share in RoW for RoW in int. good	0.9799	0.9644
Total sales (ups. sector) to total expenditure (downs. sector) in US	0.7679	0.4616
Total sales (ups. sector) to total expenditure (downs. sector) in RoW	1.1194	0.4466
Total sales (downs. sector) to total expenditure (downs. sector) in US	0.9973	0.9969
Total sales (downs. sector) to total expenditure (downs. sector) in RoW	0.9989	0.9991
Total expenditure in downstream good in the US relative to RoW	0.3733	0.3610
Value of the Objective	0.5491	

Notes: This table presents the targeted moments in the estimation. Column 1 presents moments from the data and column 2 presents their estimated counterparts. Note that in the model, total sales upstream to total expenditure downstream cannot be larger than 1 since the upstream sector is pure value added.

Now, we use the parameters we estimated above to compute the optimal tariff levels on final goods and intermediate inputs for the United States when the rest of the world sets a zero tariff on US goods (final goods or inputs), but the US charges positive tariffs on the RoW final goods and inputs. Maximum welfare is attained at:

$$t_H^d = 0.3860$$
$$t_H^u = 0.2008$$

so the optimal unilateral tariff on final goods is close to twice as large as the optimal unilateral tariff on inputs. These numbers are relatively similar to the ones obtained in the paper (i.e. $(0.3860, 0.2008)$ v.s. $(0.3947, 0.2074)$). We get the same magnitude and for the first case difference starts in the second decimal whereas for the second one only in the third decimal.

3 Optimal Tariff Policy: All Four Policy Parameter Vary

Now, we repeat the results from before for the case in which, apart from setting tariffs, the Home government sets a domestic subsidy on the purchases of intermediate inputs. We study the joint determination of four policy instruments: import tariffs on inputs, t_H^u and on final goods t_H^d, a domestic subsidy to input purchases s_H^u, and an export tax on final goods ν_H^d. This amounts to optimizing over s_H^u (instead of setting $s_H^U = 1/\theta$), and also optimizing over the export tax downstream. We find that the optimal vector of policies is given by

$$t_H^d = 0.3367$$
$$t_H^u = 0.0042$$
$$s_H^u = 0.2503$$
$$\nu_H^d = 0.0000$$

so, as in the paper, we still find a significant amount of tariff escalation as part of the optimal policy vector. We can see that our results $((t_H^d, t_H^u, s_H^u, \nu_H^d) = (0.3367, 0.0042, 0.2503, 0.0000)$, are very similar to the ones obtained in the paper $(\left(t_H^d, t_H^u, s_H^u, \nu_H^d\right) = (0.3391, 0.0046, 0.2500, -0.0007))$, with only very small differences around the third and fourth decimals. They also have the same sign, except for ν_H^d because we obtain zero.

Summarizing our findings, as in Table 5 of Antras et al., 2021, we have

Table 3: Optimal Tax Instruments

	A. Tax Instruments				B. Welfare	
	t_H^d	t_H^u	ν_H^d	s_H^u	U_{US}	U_{RoW}
Zero Tariff Equilibria					0.0315	0.1022
Just tariff (no subsidies)	0.3860	0.2008			0.0318	0.1016
All policies	0.3367	0.0042	0.0000	0.2503	0.0323	0.1016

An potential explanation on why we obtain some small differences in the previous sections can be found on the appendix.

4 Side Point

One question[2] that remains open is whether the optimal subsidy in the open economy remains $\frac{1}{\theta}$. We consider the cases $(\sigma = 4, \theta = 7)$ and $(\sigma = 7, \theta = 4)$. We do not find a subsidy of exactly $\frac{1}{\theta}$.

[2]Per Agus' request

- $(\sigma = 4, \theta = 7) : s^u = 0.1403$
- $(\sigma = 7, \theta = 4) : s^u = 0.2549$

5 Counterfactual I: Tariff War Game

For this counterfactual we consider that both Home and Foreign want to maximize their own welfare and use their own tariffs as actions. Formally, the equilibrium is the fixed point of the best responses. To find the equilibrium numerically, we make each country play their best response iteratively until we find the equilibrium tariffs.

We decided to consider this counterfactual because it is common in the literature and it seemed like an obvious extension to consider the retaliation implications of setting tariffs. For example, (Li, He, and Lin, 2018) simulate the effects of a trade war between China and the US. First they consider unilateral reactions from the US, but then consider retaliations from China in a non-cooperative and cooperative Nash bargaining equilibrium. They find that the US can gain if China does not retaliate, but it will lose if they do retaliate, and this matches our results below. Other papers, like (Ossa, 2014), (Bagwel, Staiger, and Yurukoglu, 2020), (Markusen and Wigle, 1989), and (Felbermayr, Jung, and Larch, 2013), also consider trade wars with Nash equilibrium. Then, it is clear that this extension is common in the literature.

Results can be found in the next section.

6 Counterfactual II: Tit-for-tat

For our second counterfactual, one country knows that its opponent is going to match the tariffs set by them for any choice of tariff. Note that this assumption means the opponent is not playing optimally.

We decided to consider this counterfactual because, even though it is not very common in the literature, it has been discussed in the media as the tactic that was being followed in the US-China trade war. For example, David Dollar, and Peter A. Petri stated for Bloomberg, on October 5$^{\text{th}}$, 2018, they refer to these escalations as tit-for-tat tariff escalation, which is in neither country's best interest[3]. BBC news, in December 2020, refer to escalations on the US-China trade war also as a tit-for-tat[4]. Other news, like the one written in Reuters by David Lawder, and Se Young Lee in August, 2019, also study the case of the US-China trade war latest tit-for-tat escalation. In this case, China put retaliatory tariffs on \$75 billion worth of US goods, and a few hours later, ex-President Trump increased tariffs on \$550 billion worth in Chinese goods[5]. Finally, the economist, on September 22$^{\text{nd}}$, 2018, they wrote about the tariffs the US put and how China retaliated[6]. Then, we thought that, even thought this is not commonly done in the literature, it would be interesting to check the case in which retaliations were done in a tit-for-tat manner, since that seems to be the

[3] This can be find in https://www.brookings.edu/blog/order-from-chaos/2018/10/05/why-its-time-to-end-the-tit-for-tat-tariffs-in-the-u-s-china-trade-war/

[4] News can be find at https://www.bbc.com/news/business-55132425

[5] News can be found in https://www.reuters.com/article/us-usa-trade-china/trump-heaps-another-5-tariff-on-chinese-goods-in-latest-tit-for-tat-escalation-idUSKCN1VD21E

[6] News can be found at https://www.economist.com/finance-and-economics/2018/09/20/america-and-china-are-in-a-proper-trade-war

case for the US-China trade war.

We code both counterfactuals for the case with no subsidies and we obtain, written as:

Table 4: Optimal Tax Instruments

	A. Tax Instruments				B. Welfare	
	t_H^d	t_H^u	t_F^d	t_F^u	U_{US}	U_{RoW}
Zero Tariff Equilibria					0.0315	0.1022
Nash	0.4077	0.2259	0.4144	0.2113	0.0313	0.1018
Tit-for-tat H (no restriction)	0.0239	−0.0811	0.0239	−0.0811	0.0316	0.1021
Tit-for-tat H (with restriction)	0.0206	−0.0003	0.0206	0	0.0315	0.1022
Tit-for-tat F (no restriction)	−0.0164	−0.0369	−0.0164	−0.0369	0.0316	0.1022
Tit-for-tat F (with restriction)	0	0	0	0	0.0315	0.1022

We start by discussing the Nash equilibrium case. We can see that in this case home downstream tariffs are not that different than the ones set in the optimal tariff with and without subsidies, but even higher (0.4077 v.s. 0.3860 and 0.3367). In the case of the upstream home tariff, we can see that it is very similar to the one obtain in the optimal policy with just tariffs (no subsidies) and higher than the one obtained considering all policies. Then, even in a trade war with Nash-tariff, there is still a great amount of tariff escalation as part of the optimal tariffs. How does Foreign respond to Home's best response? Foreign sets both upstream and downstream tariffs in a very similar manner as Home. This is very intuitive since we would expect countries to react in a similar manner if they are both considering each others best responses and similar to the tariffs set when not considering subsidies. As expected, the welfare for both the US and RoW are a bit smaller than the non-tariff equilibria, which implies that it is not worth it to enter a trade war with Nash equilibrium. When considering both countries best responses, RoW is damaged more than the US in terms of welfare compared to the zero tariff equilibria, in absolute terms, but less in percentage terms.

For the tit-for-tat case, rows 3 and 4 consider when Home chooses tariffs and Foreign retaliates by copying Home's decision, without and with sign restrictions[7] respectively, and then rows 5 and 6 consider when Foreign makes choices and Home retaliates by copying, without and with sign restrictions respectively. In the case of row 3, Home would still set a positive tariff downstream, but a negative one upstream. Also, the downstream tariff is very small, and less than 10% than the optimal downstream tariffs we obtained above, so it is actually very close to zero. It seems like if Foreign reacts with a tit-for-tat strategy it is optimal to not have tariff escalation as part of the optimal policy. The upstream tariff is a small negative tariff, which makes sense if the strategy is to make Foreign react by setting a negative upstream tariff too. The welfare obtained by this strategy is a bit higher for the US (making the tariff decision) and a bit smaller for RoW, compared to the zero tariff equilibria, but it is almost insignificant. Tariffs change a bit when we set sign restrictions (tariffs cannot be negative), then, since home cannot "play" with setting negative tariffs, they set the downstream tariff even lower and the upstream tariff is basically zero. In terms of welfare, this is equivalent to a zero tariff equilibria for both countries, which makes sense since the tariffs set are basically zero. In column 6, we obtain basically the same results. When Foreign is setting the tariffs and we add sign restrictions, we obtain the zero tariff equilibria. Finally, in column 5, when

[7]The sign restriction is that no country will pursue a negative tit-for-tat strategy. For example, if US subsidizes imports, RoW will not respond in kind if we consider the sign restriction.

Foreign is setting the tariff and there is no sign restriction, Foreign sets negative tariffs for both downstream and upstream sectors, making Home also set negative tariffs in both sectors. In this case the welfare is the same for Foreign and just a bit better for Home compared to the zero tariff case. Nevertheless, we do not believe that this case would actually happen in practice, so we advise to be cautious about this result. The sign restricted tit-for-tat scenario seems much more plausible, and, unsurprisingly, delivers optimal choices close to a zero tariff agreement in any case.

Summarizing, the main takeaway here would be that the tariff agreement (zero tariff) and tit-for-tat outcomes are pretty similar, except for the fact that whoever gets to optimize can become a tiny bit better off. The amount is small enough that it is lost to rounding in some cases. We can clearly see that the tariff war Nash Equilibrium is the worst outcome for everyone involved in the trade war. We think that the tariff war game is much closer to what actually happened in equilibrium, and even in the US-China. The idea is that at the beginning it may look like countries are acting like tit-for-tat but are actually probably just considering the best responses in the Nash equilibrium sense.

7 Note for Felix/Agus

We are happy to chat more about any of the results found herein. In terms of helping check the results currently in the paper, my (Chase) assessment is that everything seems to be correct, since nowhere do we deviate wildly from Agus' results. We also mention this below, but my suspicion is that KNITRO is helping you with solving the model more than you might think. Using `Ipopt` seems to work for equilibrium solving, but pure Newton-Raphson does not, indicating a smarter method must be occurring there, and our outer layer uses `Optim`, but it seems slower and suboptimal[8]. My feeling is that utility in the tariff/policy space is akin to the classic Rosenbrock function about its optimum. Of course, this issue bleeds into solving the tit-for-tat and tariff war games, so this explains why our results are not as tight.

[8]Full disclosure, I was having issues getting automatic differentiation to work with the struct I was using to store the model. Getting `ForwardDiff.jl` to work might also solve some issues, but I tried for a while to no avail.

References

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A Computational Details

Code for this project was written in `Julia` for the open economy, and `MATLAB` for the closed economy. Both will be available on the project's Github.

A.1 Solving Equilibrium

The details for solving for any given equilibrium, including the equilibrium conditions, are available in the companion note dedicated explicitly to this purpose.

A.2 Optimal Tariff

To find the the optimal tariff, we use a nested-nonlinear solution method. The general idea is that for any given set of tariffs and/or subsidies, we may solve for an equilibrium, so all we need to do is consider an outer optimizer which maximizes home utility over choices of policy variables. In particular

1. To solve for any given equilibrium we use the `Ipopt` wrapper inside `JuMP` for `Julia`.

2. To optimize over choices of policies, we use `Optim`.

A.3 Tariff Game

To solve the tariff game, we consider iterative best responses.

1. Set all policies to zero.

2. Home chooses their optimal policy, fixing Foreign's policy.

3. Foreign chooses their optimal policy, fixing Home's policy.

4. Repeat 2 and 3 until the policies do not change (change by a low tolerance)

A.4 Tit-for-Tat

Solving the tit-for-tat setup is most similar to either country simply setting optimal tariff without retaliation. The only difference is that we consider optimizing over policies, *taking into account* that any policy will imply an exact corresponding retaliation by the opponent. Strangeness can occur in this setting, however, since it may be optimal to set import *subsidies* precisely so that the relaliatory subsidy occurs. To potentially avoid this type of strategic behavior, we can also bound the retaliation below by zero, so that retaliation only occurs for import tariffs (not subsidies).

A.5 Precision

Our results do not exactly match those from the paper. However, in terms of the objective, each layer of the solver is quite close. For example, directly testing the paper's solution for optimal tariffs against ours in terms of utility shows essentially no change, so it is unsurprising our solver is not finding it exactly. My intuition is that this is because, near the optimum, the objective is quite "flat". In two dimensions, this can even be seen in Figure 2 in the paper, wherein the contour plot gets flat around the solution.

In terms of practically addressing the problems with a black box, using KNITRO or Gurobi or some other state-of-the-art solver will probably fix the issue. If we wanted to fix the issue ourselves (or do what I am guessing these commercial solvers do), we could dispatch a mini-batch of gradient optimizers, then choose the solution as the maximum of the solutions found. This technique is usually helpful for "rough" surfaces, but we are using it for the opposite purpose, namely that the surface is too flat, so gradient search can get stuck close, but not at, the optimum.