Precise Robust Positioning with Inertial/GPS RTK

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BIOGRAPHY

Dr. Bruno M. Scherzinger obtained the B.Eng. degree from McGill University in 1977 and the M.A.Sc. and Ph.D. degrees in system control engineering from the University of Toronto respectively in 1979 and 1983. He is a founding partner and Chief Scientist at Applanix Corporation.

ABSTRACT

This paper describes a Position and Orientation System (POS) with inertially aided RTK under development at Applanix, whose design objective is to achieve robust positioning with decimeter-level accuracy. This paper discusses the following integration strategies. A *loosely coupled integration* comprises the integration of inertial and GPS navigation solutions in a Kalman filter). A *tightly coupled integration* comprises the integration of GPS and inertially predicted observables in a Kalman filter).

This paper describes the design of the POS and its performance during preliminary tests. The POS implements tightly coupled inertial/GPS integration with floated ambiguity estimation and fixed integer search in a single Kalman filter. The POS is able to recover L1 integer ambiguities within seconds of a short-duration GPS outage while maintaining decimeter-level accuracy throughout the outage. This paper discusses the factors that control the maximum outage duration for rapid fixed integer ambiguity recovery and the position accuracy during the outage.

INTRODUCTION

This paper presents the design and performance of two inertially aided real-time kinematic (RTK) systems that Applanix has developed and tested. The design objective is to achieve robust real-time positioning with decimeter-level accuracy and accurate orientation (roll, pitch, heading). *Robust positioning* describes a positioning system's ability to maintain position data continuity and accuracy through most or all anticipated operational conditions. The operational condition of interest here is the loss of good GPS satellite visibility due to partial or total obstruction of the sky. The applications for a robust RTK positioning and orientation system are:

- Machine control is the use of a navigation system to provide positioning of and possibly participate in the guidance of mining, earth moving or farming machinery. A robust positioning system is needed to provide continuous data and accuracy through GPS satellite shading by other machines, fixed structures or the faces of an open-pit mine.

- Land survey includes land seismic surveying and property or cadastral surveying. A robust positioning system is needed in those land survey scenarios such as seismic surveying in dense forests or land surveying in dense urban canyons where GPS coverage and line-of-sights for laser theodolite surveying are either marginal or non-existent.

- Road survey includes road inspection, centerline surveying and roadside inventory survey using a vehicle moving at traffic speed. A robust positioning and orientation system is needed to handle sections of a road where GPS is obstructed by buildings or foliage.

- Intelligent transportation systems describes an evolving technology area that includes automatic guidance of vehicles on highways. A robust positioning and orientation system is needed to provide continuous and accurate vehicle position to the guidance subsystem.

The key performance attributes investigated here are the following:

- position accuracy during availability of GPS observables from 4 or more satellites (full coverage),
- position accuracy during availability of GPS observables from 2-4 satellites (partial coverage or partial outage),
- position accuracy during availability of GPS observables from 0 or 1 satellites (full outage),
- time to re-acquire integer RTK after full and partial GPS outages.

The time duration of an outage of observables from a particular satellite is the outage time of dual frequency data. Hence the outage time comprises the time of actual signal shading plus the receiver's time to reacquire full dual-frequency phase lock on the satellite signal.
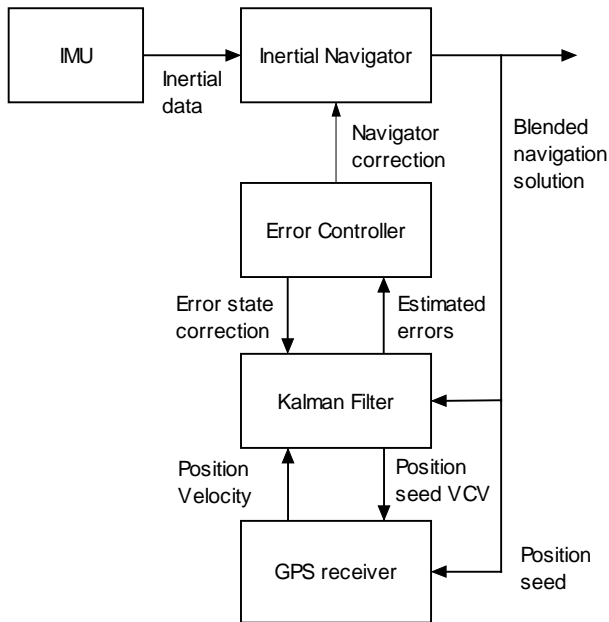
capability. The system that Applanix developed and tested uses a Trimble MS750 receiver which implements RTK position seeding.

The second system described here is a tightly coupled inertial/GPS integrated system with integrated RTK shown in Figure 2. *Tightly coupled inertial/GPS* integration implies the Kalman filter processes the GPS pseudorange, phase and Doppler observables. *Integrated RTK* implies the Kalman filter that estimates the inertial navigator errors also estimates the floated phase ambiguities, and an on-the-fly (OTF) ambiguity resolution algorithm operates on these to fix the integer ambiguities. In this case, the GPS receiver is strictly a sensor of the GPS observables. Any GPS receiver that outputs the observables and satellite ephemeredes can be used. The navigation functions in the GPS receiver, namely position and clock offset fixing and RTK, are not used.
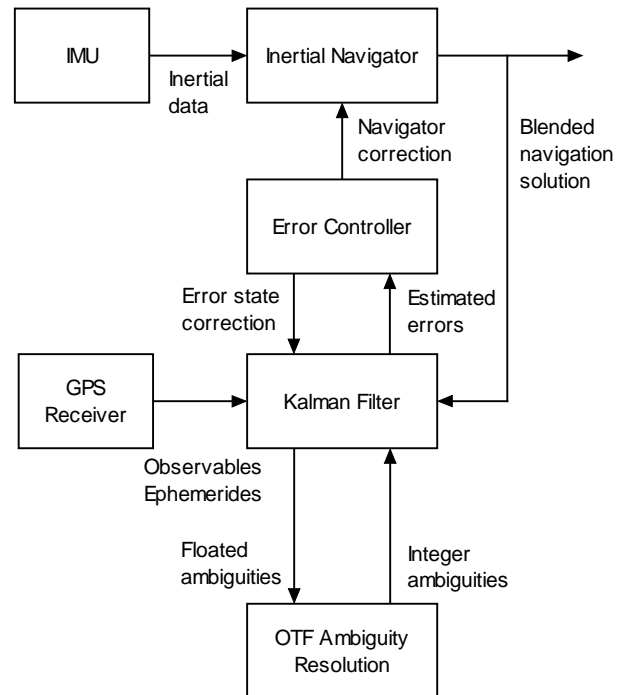

Figure 1: Loosely coupled inertial/GPS architecture

Figure 2: Tightly coupled inertial/GPS architecture

The first system to be considered is a loosely coupled inertial/GPS integrated system with loosely coupled inertial RTK aiding shown in Figure 1. *Loosely coupled inertial/GPS integration* implies the Kalman filter processes the GPS position and velocity solution to aid the inertial navigator. *Loosely coupled RTK aiding* here implies the GPS receiver imports an inertial position seed comprising the inertial navigator position and position variance-covariance (VCV) matrix to accelerate the time to a fixed integer RTK position. In this case, the GPS receiver is a self-contained navigation subsystem with RTK functionality that includes position-seeding

A third class of systems mentioned here for completeness is called deeply integrated inertial/GPS integrated systems. *Deep inertial/GPS integration* implies the navigation filter generates the pseudorange and phase tracking loop feedback signals. The GPS receiver and the aided inertial navigation system are no longer distinct subsystems. An example of current research work in deep integration is [6].

GPS manufacturers and other researchers with an interest in RTK positioning are putting a lot of effort into the minimization of time to integer RTK. Current claims of performance during good satellite coverage are as low as 5 seconds. Some claims of success with single-epoch integer RTK under restricted conditions have been reported. This interest in rapid integer RTK is motivated by the major weakness of stand-alone GPS, the lack of robustness due to susceptibility to signal shading. A receiver will provide no useful position data during an outage, and usually requires the same integer RTK recovery time regardless of the outage duration.

An inertial/GPS integrated system on the other hand provides continuous data and maintains decimeter-level accuracy during short GPS outages provided the inertial sensors are sufficiently accurate. This de-emphasizes the importance of the time to integer RTK recovery following short (1-10 second) outages. As the outage duration increases, the inertial navigator position drift contributes decreasingly to integer RTK recovery, with the limiting case being the time to an unaided integer RTK recovery. [5] presents a theoretical analysis of inertially aided RTK performance in terms of the reduction of search space volume that inertial aiding can provide.

LOOSELY COUPLED INTEGRATION

Applanix implemented the loosely coupled integration shown in Figure 1 as a modification of its Position and Orientation System for Land Vehicles (POS/LV) [8]. The current-generation POS/LV implements a loosely coupled inertial/GPS integration with a generic interface to the GPS receiver's navigation solution. The POS/LV uses a tactical-grade inertial measurement unit (IMU) in the 1-10 degree/hour performance category. The modifications comprise the integration of the Trimble MS750 receiver and the implementation of inertial position seeding. The POS provides the blended position solution and the position VCV sub-matrix from the Kalman filter to the GPS receiver's position seeding input. This is a simple extension of a standard inertial/GPS integration system, however its time to integer RTK performance depends in part on the performance of the GPS receiver's RTK position seeding function.

The integrated system was tested in a series of van tests during which GPS outages were induced either by driving under an obstruction or by powering down the GPS antenna for a given outage period. Table 1 lists the compiled statistics from the outage tests. Figure 3 shows the integrated RMS position error during a 10 second GPS outage when position seeding to the GPS receiver was disabled. The receiver's integer RTK recovery time was on the order of 30-40 seconds, a typical time for most receivers with RTK capability.

Figure 4 shows the integrated RMS position error during a similar 10 second GPS outage with position seeding enabled. The position seed standard deviation was in the range of 0.5-0.8 meters after a 15 second outage. The receiver's integer RTK recovery time is now on the order of 14 seconds.
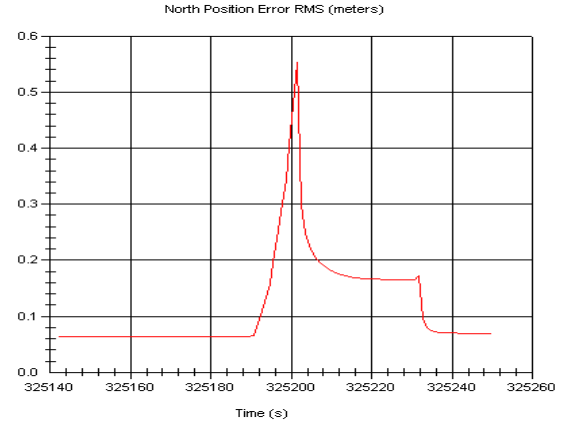

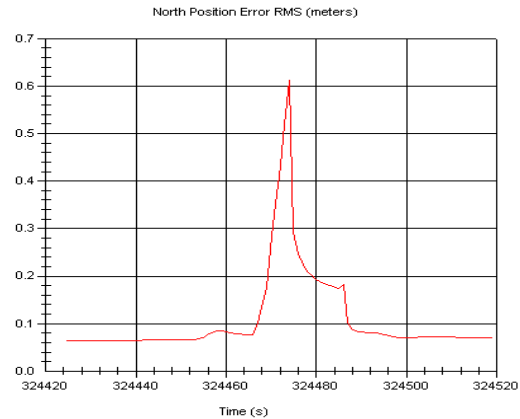Figure 3: RMS position error without position seeding

Figure 4: RMS position error with position seeding

Table 1: Integer RTK recovery time

GPS Outage	Recovery with Position Seed	Recovery without Position Seed
< 30 seconds	13 seconds	35 seconds
30-60 seconds	14 seconds	45 seconds
> 60 seconds	14 seconds	100 seconds

The RTK recovery performance is partially dependent on the represented accuracy of the position seed in the form of the position VCV matrix that is part of the position seeding record. This dependency can be manipulated through proper Kalman filter design and use of good inertial sensors. The performance is also dependent on the

GPS receiver's handling of the data and the degree of conservativeness that the receiver's RTK algorithm assigns to the accuracy of the incoming position seed. This dependency is beyond the control of the integrator.

TIGHTLY COUPLED INTEGRATION

Applanix has developed its next generation POS embedded software to implement the tightly coupled inertial/GPS integration with integrated RTK shown in Figure 2. Here the integration Kalman filter and the OTF ambiguity search modules are closely integrated. The Kalman filter estimates widelane and L1 floated ambiguities. The OTF search module attempts to fix first the wide lane ambiguities and then the L1 ambiguities. When the ambiguities are not fixed, the Kalman filter processes uncorrected widelane and L1 phase double differences. When either the widelane or L1 ambiguities are fixed, the Kalman filter validates the fixed ambiguities and then processes precise phase double differences to achieve full RTK accuracy.

Kalman Filter

The Kalman filter uses the modified ψ-angle inertial navigator error model described in [1]. This implementation permits "on-the-go" alignment and transition to full navigation accuracy. The Kalman filter implements the following basic states as well as other application-specific states:

- inertial navigator errors (10 states)
- gyro and accelerometer biases (6 states)
- floated phase DD ambiguities (12 states)

The Kalman filter constructs the following measurements:

- inertial-GPS pseudoranges
- inertial-GPS L1 phases
- inertial-GPS wide-lane (L1-L2) phases

For RTK operation, the Kalman filter constructs double-differences of these measurements using pseudorange and phase observables from a base receiver. The j-th double-differenced phase measurement for a single baseline is given as follows:

$$z_{DD_j} = \nabla \Delta r_j\left(\hat{\bar{r}}_{SNV}, \bar{r}_{base}\right) + \lambda\left(\nabla \Delta \phi_j - \nabla \Delta N_{j0}\right)$$

where:

λ is the widelane or L1 wavelength,
$\hat{\bar{r}}_{SNV}$ is the computed inertial navigator position,
$\bar{r}_{base}$ is the base receiver position,
$\nabla \Delta \phi_j$ is the double-differenced phase,

$\nabla \Delta N_{j0}$ is the initial integer ambiguity,

$$\nabla \Delta r_j\left(\hat{\bar{r}}_{SNV}, \bar{r}_{base}\right) = \Delta r_j\left(\hat{\bar{r}}_{SNV}, \bar{r}_{base}\right) - \Delta r_b\left(\hat{\bar{r}}_{SNV}, \bar{r}_{base}\right)$$

is the predicted range double difference to the j-th and base satellites in a base satellite double-differencing method,

$$\Delta r_j\left(\hat{\bar{r}}_{SNV}, \bar{r}_{base}\right) = r_j\left(\hat{\bar{r}}_{SNV}\right) - r_j\left(\bar{r}_{base}\right)$$

is the single difference between the inertial navigator position and the base receiver position for the j-th satellite, and

$$r_j\left(\hat{\bar{r}}_{SNV}\right) = \left|\bar{r}_{SV_j} - \hat{\bar{r}}_{SNV}\right|$$

is the range from the inertial navigator position to the j-th satellite.

The measurement model is given as follows:

$$z_{DD_j} = -\left(\bar{e}_j - \bar{e}_b\right) \cdot \delta \bar{r}_{SNV} + \lambda \nabla \Delta N_j + \varepsilon_{\nabla \Delta \phi_j}$$

where:

$\delta \bar{r}_{SNV}$ is the inertial navigator position error in the modified ψ-angle inertial navigator error model,

$\nabla \Delta N_j$ is the ambiguity double-difference,

$\varepsilon_{\nabla \Delta \phi_j}$ is the phase double-difference noise,

$\bar{e}_j$ is the unit line-of-sight (LOS) vector from the inertial position to the j-th satellite, given by:

$$\bar{e}_j \cong \frac{\partial r_j\left(\hat{\bar{r}}_{SNV}\right)}{\partial \hat{\bar{r}}_{SNV}} = \frac{1}{r_j\left(\hat{\bar{r}}_{SNV}\right)}\begin{bmatrix} x_{SV_j} - \hat{x}_{SNV} \\ y_{SV_j} - \hat{y}_{SNV} \\ z_{SV_j} - \hat{z}_{SNV} \end{bmatrix}$$

These observables measurements implement the "tightly coupled" inertial/GPS integration. It has notable advantages over a loosely coupled integration, in particular providing ongoing aiding to the inertial navigator when the number of visible satellites drops below the minimum 4 needed to compute a GPS position fix.

OTF Search Module

The OTF search module implements a modified Fast Ambiguity Search Filter (FASF) algorithm first proposed in [3]. This algorithm is particularly well suited for integration with the inertial/GPS Kalman filter. It

performs the integer search in ambiguity space as defined by the floated ambiguity states in the Kalman filter. The modification to the original algorithm reported in [3] is the inclusion of an ambiguity decorrelation algorithm similar to that reported in [4].

Cycle Slip Detection

The POS implements the following methods of cycle slip detection. In combination, these provide a highly reliable detection of cycle slips.

- receiver-reported cycle slips,
- predicted phase increment tests,
- Kalman filter measurement residual test.

The predicted phase increment tests compare phase increments over a prescribed test interval with corresponding predicted phase increments computed from integrated Doppler frequency and from inertial predicted phase increment. The Doppler predicted phase increment is computed as a trapezoidal integration of Doppler frequency:

$$\Delta\phi_{\Delta f_j}(t) = \int_{t-\Delta t_{test}}^{t} \Delta f_j(\tau)d\tau \cong \Delta t_{test} \frac{\Delta f_j(t_i) + \Delta f_j(t_{i-1})}{2(t_i - t_{i-1})}$$

where:

Δf is the Doppler frequency,
Δt_{test} is the test interval,
Δt_{test} is the observables sampling interval.

The inertial predicted phase increment is:

$$\Delta\phi_{SNV_j}(t) = \frac{1}{\lambda}\left(r_j\left(\hat{\bar{r}}_{SNV}(t)\right) - r_j\left(\hat{\bar{r}}_{SNV}(t - \Delta t_{test})\right)\right)$$

A cycle slip is declared when either of the following conditions holds:

$$\left|\phi_i(t) - \phi_i(t - \Delta t_{test}) - \Delta\phi_{\Delta f_i}\right| > \delta\phi_{\Delta f}$$

$$\left|\phi_i(t) - \phi_i(t - \Delta t_{test}) - \Delta\phi_{SNV_i}\right| > \delta\phi_{SNV}$$

where $\delta\phi_{\Delta f}$ and $\delta\phi_{SNV}$ are cycle slip detection thresholds that can be chosen empirically or from consideration of the statistics of the predicted phase increments. For example for the *j*-th satellite phase measurement:

$$\delta\phi_{SNV} = \gamma\sigma_{\Delta\dot{r}_{SNV}}\Delta t_{test}$$

where $\sigma_{\Delta\dot{r}_{SNV}}$ is the inertial predicted range rate error standard deviation given by:

$$\sigma_{\Delta\dot{r}_{SNV}} = \sqrt{\bar{e}_j^T P_{\delta V_{SNV}} \bar{e}_j}$$

$P_{\delta V_{SNV}}$ is the inertial navigator velocity error covariance sub-matrix from the Kalman filter, and γ is an empirically selected scale factor.

The Kalman filter measurement residual test is a χ^2 test on the phase measurement residuals against the innovations variances predicted by the Kalman filter. This is a standard procedure for rejecting occasional erroneous measurements or "wild points". A hitherto undetected cycle slip will result in a sudden failure of the residual test and subsequent continuous rejection of the measurement.

TEST RESULTS

This section describes the results of an investigation that explores the inertially aided RTK performance of the POS:

- position error during a full GPS outage,
- position error during a partial GPS outage,
- time to recover RTK accuracy after partial and full outages.

A full GPS outage is here defined to be the complete absence of any or all GPS observables data to the navigation processing algorithm. It includes the time that a GPS antenna is completely shaded plus the receiver re-acquisition time. A partial outage is defined to occur when observables from 2 or 3 satellites are available, so that a GPS receiver is unable to compute a position fix.

The POS being tested contained a dual frequency GPS receiver and a tactical-grade IMU having the following performance attributes:

- 3 degrees/hour gyro bias
- 0.1 degrees/√hour gyro random walk
- 500 μg accelerometer bias
- $(50 \mu g)^2$/Hz accelerometer random noise

A dual frequency GPS base receiver was located on the roof of the Applanix building. All inertial and GPS data were recorded during the test. The results presented here were computed with Applanix's POS Real-Time SIMulator (RTSIM), which reproduces the embedded software's real-time solution on a desktop or laptop computer. Disabling all roving receiver observables at the specified outage times simulated full outages. Retaining roving receiver observables for 2 or 3 satellites at the specified outage times simulated partial outages. The reference solution for error evaluation is a blended navigation solution computed by Applanix's POSPAC post-processing software [2].
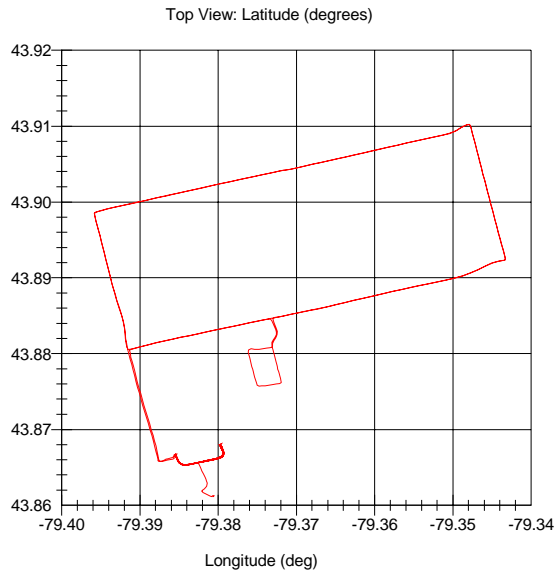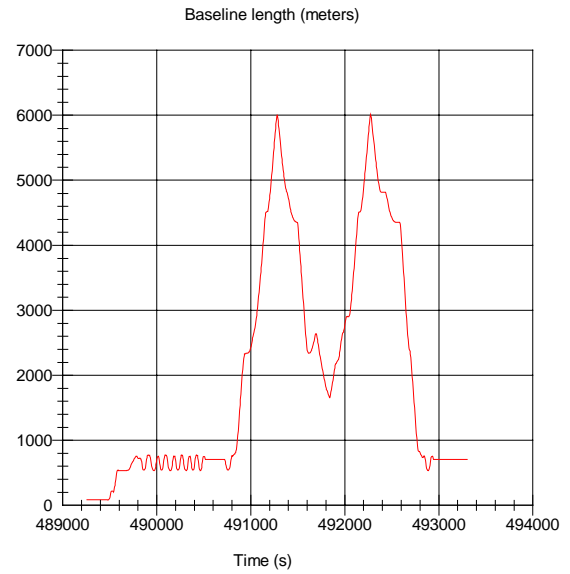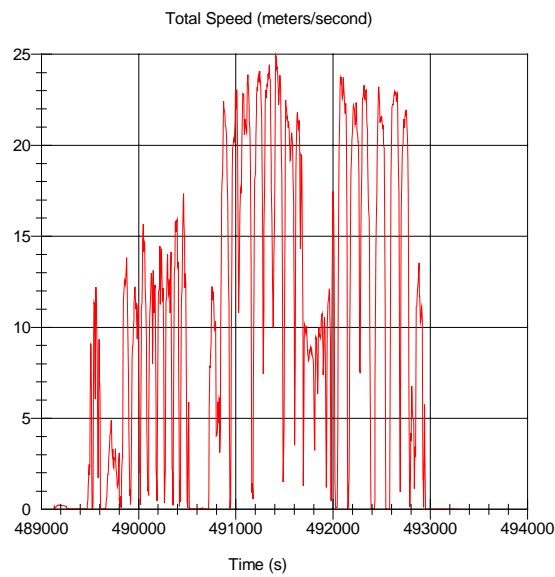
Figure 5: Van test trajectory

Figure 6: Van test speed profile

The test trajectory comprised a drive along the city streets near the Applanix building in Richmond Hill, Ontario. Figure 5 shows a plan view of the van test trajectory. Figure 6 shows the vehicle speed profile during the test. Figure 7 shows the baseline length between the POS and the reference receiver during the test. Full GPS coverage during the test varied between 7 and 9 satellites.

Figure 7: Baseline length (meters)

Full Outages

In this investigation, 9 full outages of duration 10, 30 and 60 seconds were simulated across the available data. Figure 8 and Figure 9 show horizontal position errors for 10- and 30-second full GPS outages. Table 2 lists the statistics for the simulated outages. The time to full integer recovery is on the order of 3-4 seconds. This is in part a consequence of the particular mechanization, which fixes and tests the L1 ambiguities at earliest one epoch after it fixes the widelane ambiguities.
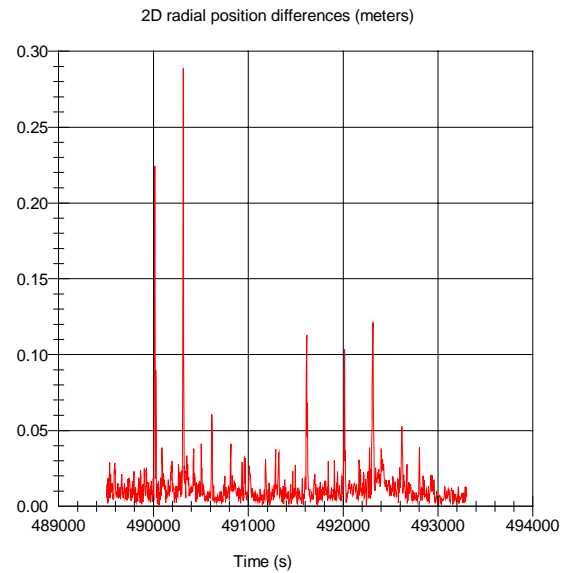

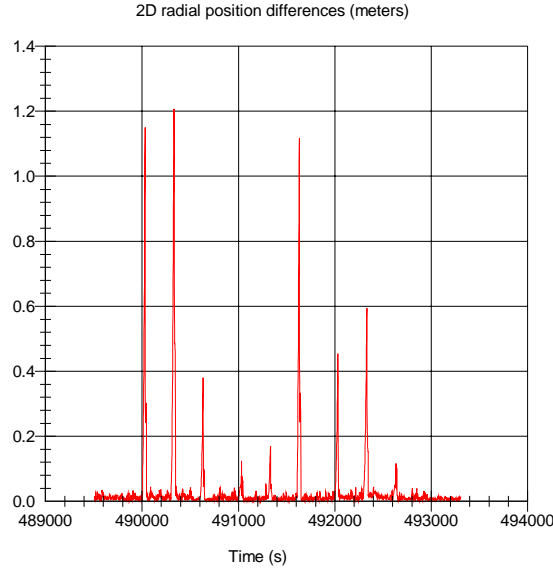Figure 8: Horizontal position error during 10 second full GPS outages

Figure 9: Horizontal position error during 30 second full GPS outages

Table 2: Full GPS outage statistics

Full Outage Duration (seconds)	Horizontal Position Error Standard Deviation (meters)	Average Time to Full RTK Recovery (seconds)
10	0.1	3
30	0.5	3
60	1.8	4

Partial Outages

In this investigation, 9 partial outages with 3 retained satellites of duration 10, 30 and 60 seconds were simulated across the available data. Figure 10 shows the horizontal position error for 30-second partial GPS coverage with 3 satellites. Table 3 lists the statistics for position error during the partial outages for several combinations of 3 satellites from among the visible satellites. The average time for integer RTK recovery is 1 second, which implies that the POS was either able to repair all integer ambiguities at the end of a partial outage or resolve the ambiguities within one epoch.

Table 3: Statistics for 3 retained satellite partial outages

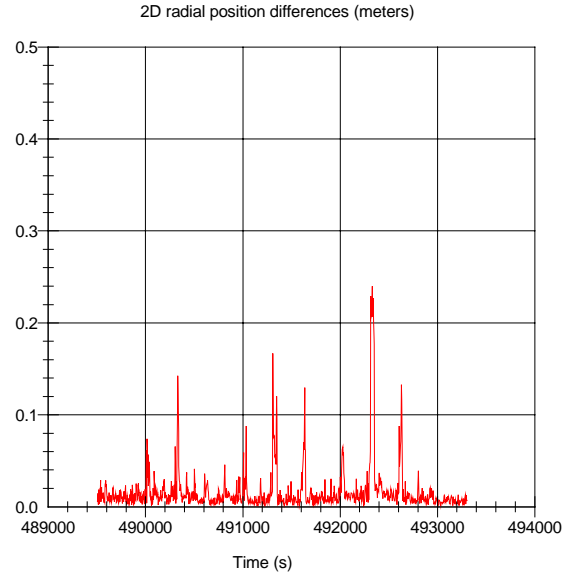| Partial Outage Duration (seconds)	Horizontal Position Error Standard Deviation (meters)	Average Time to Full RTK Recovery (seconds)
10	0.1	1
30	0.25	1
60	0.3	1

Figure 10: Horizontal position error during 30 second coverages with 3 satellites

Ambiguity Dilution of Precision

The ambiguity dilution of precision (ADOP) was introduced in [7] and is given as follows:

$$ADOP = \left(\sqrt{\det\left(P_a \right)} \right)^{\frac{1}{n}} = \left(\det\left(P_a \right) \right)^{\frac{1}{2n}}$$

where P_a is the VCV matrix of the floated ambiguities. ADOP is given in units of cycles, and is a measure of the precision of the floated ambiguities. Its value is invariant under volume-preserving transformations of the ambiguities such as ambiguity decorrelation transformations. It is examined here to investigate the retention of ambiguity precision during GPS outages provided by inertial aiding.

Figure 11 shows the widelane ADOP for the case of 10-second full outages. The initial ADOP indicates the floated widelane ambiguity precision with a priori baseline position accuracy determined by the differential pseudorange position solution. Thereafter the maximum ADOP's after the end of the 10-second outages indicate significantly more precise floated widelane ambiguity precision due to the position accuracy maintained by the inertial solution during the outages.

Figure 12 shows the widelane ADOP for the case of 60-second full outages. The initial ADOP is the same as that from the 10-second outage case, however the subsequent maximum ADOP's are larger, as would be expected due to the larger inertial position error growth during the 60-second outages. The consequence of less precise ambiguities is a potentially longer search time.
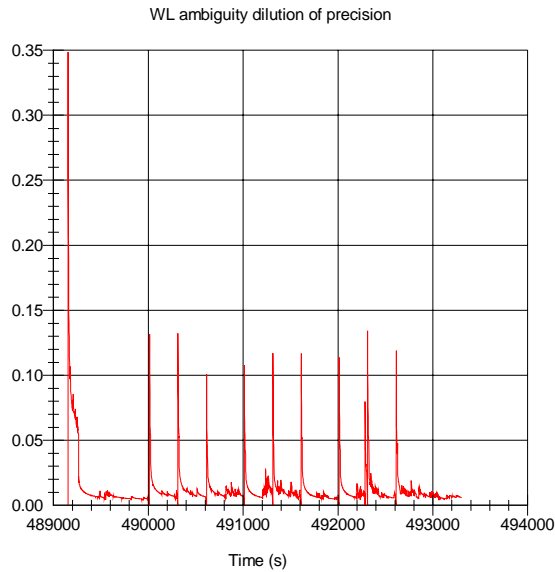
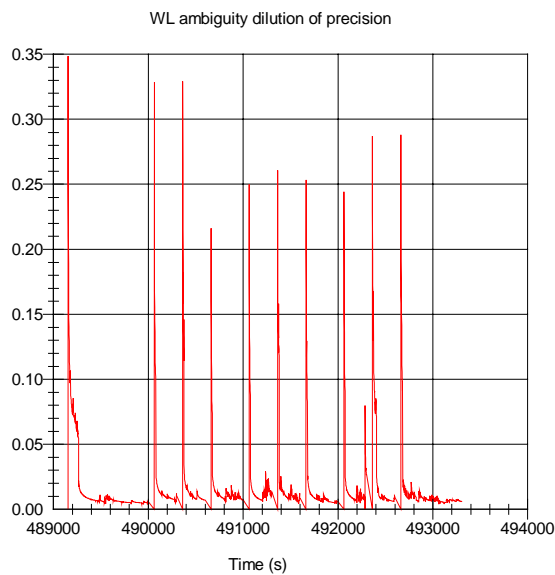Figure 11: Widelane ADOP during 10-second full outages

Figure 12: Widelane ADOP during 60-second full outages

CONCLUSIONS

This paper has described two levels of inertial-GPS integration for the purpose of obtaining inertially aided RTK. A loosely coupled integration relies on the performance and interface characteristics of the GPS receiver's navigation filter and RTK position seeding algorithm. Applanix's version of this form of integration has demonstrated integer RTK recovery times of 10-15 seconds after full GPS outages lasting up to 60 seconds. The advantage of such an integration method is that the integration software is generic and simple to implement

because it relies on the GPS receiver to perform the RTK function and provide a position and velocity solution. The disadvantages are twofold. The user has no visibility into these functions beyond the interface specification and is required to make conservative assumptions about their operation and error characteristics in a loosely coupled integration. Also the GPS navigation solution and hence aiding to the inertial navigator drop out when fewer than 4 satellites are visible.

In a tightly coupled integration, the GPS receiver is used as a source of observables and satellite orbital and clock parameters. The integer ambiguity search function is combined with the integration Kalman filter, so that i) there is no limit on visibility of data/information between these modules, and ii) the integer ambiguity search is by construction inertially aided. This form of integration demonstrated integer RTK recovery times of 1-4 seconds after partial and full GPS outages lasting up to 60 seconds and position error standard deviations up to several meters. Furthermore the benefit of tightly coupled inertial-GPS integration, i.e. uses observables data when fewer than 4 satellites are visible, is realized.

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