Networks and Random Processes Assignment 2

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1 Kingman's Coalescent

1.1 A

N_t is the number of particles at time t with $N_0 = L$. The process $(N_t : t \geq 0)$ has the state space $\{1, ..., L\}$

1.1.1 Transition Rate of the process

$r(n, n-1) = \binom{L}{2}$, $n \geq 2$
QUESTION - WHAT ABOUT SAME STATE? $r(n, n) =$
QUESTION - WHAT ABOUT OTHER STATES - HOW TO WRITE IT? $r(n, y) =$, $y \neq n, n-1$

1.1.2 Generator

This is a jump process, so the generator is
$(\mathcal{L}f)(x) = \int_{\Re} r(x, y)[f(y) - f(x)]dy$

For this process
$(\mathcal{L}f)(n) = r(n, n-1)(f(n-1) - f(n))$
$(\mathcal{L}f)(n) = \binom{n}{2}(f(n-1) - f(n))$

1.1.3 Master Equation

The master equation is
$\frac{d}{dt}\pi_t(n) = \pi_t(n+1)r(n+1, n) - \pi_t(n)r(n, n-1)$
$\frac{d}{dt}\pi_t(n) = \pi_t(n+1)\binom{n+1}{2} - \pi_t(n)\binom{n}{2}$
QUESTION - IS THIS RIGHT? QUESTION - IS THE NOTATION OKAY? QUESTION - WHAT ABOUT EDGES?

1.1.4 Ergodicity

The process is ergodic.

1.1.5 Absorbing States

The unique absorbing state is $N = 1$.

1.1.6 Stationary Distributions

Let a distribution $\pi = [N = 1, N = 2, ..., N = L]$

The unique stationary distribution is

$\pi_0 = [1, 0, ..., 0]$

1.2 B

1.3 C

1.4 D

2 Ornstein-Uhlenbeck process

2.1 A

2.2 B

2.3 C

3 Moran Model and Wright-Fisher diffusion

3.1 A

3.2 B

3.3 C

3.4 D

3.5 E

3.6 F