

Networks and Random Processes Assignment 2

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1 Kingman's Coalescent

1.1 A

N_t is the number of particles at time t with $N_0 = L$. The process $(N_t : t \geq 0)$ has the state space $\{1, \dots, L\}$

1.1.1 Transition Rate of the process

$$r(n, n-1) = \binom{L}{2}, n \geq 2$$

QUESTION - WHAT ABOUT SAME STATE? $r(n, n) =$

QUESTION - WHAT ABOUT OTHER STATES - HOW TO WRITE IT?

$$r(n, y) = , y \neq n, n-1$$

1.1.2 Generator

This is a jump process, so the generator is

$$(\mathcal{L}f)(x) = \int_{\mathbb{R}} r(x, y)[f(y) - f(x)]dy$$

For this process

$$(\mathcal{L}f)(n) = r(n, n-1)(f(n-1) - f(n))$$

$$(\mathcal{L}f)(n) = \binom{n}{2}(f(n-1) - f(n))$$

1.1.3 Master Equation

The master equation is

$$\frac{d}{dt}\pi_t(n) = \pi_t(n+1)r(n+1, n) - \pi_t(n)r(n, n-1)$$

$$\frac{d}{dt}\pi_t(n) = \pi_t(n+1)\binom{n+1}{2} - \pi_t(n)\binom{n}{2}$$

QUESTION - IS THIS RIGHT? QUESTION - IS THE NOTATION OKAY?

QUESTION - WHAT ABOUT EDGES?

1.1.4 Ergodicity

The process is ergodic.

1.1.5 Absorbing States

The unique absorbing state is $N = 1$.

1.1.6 Stationary Distributions

Let a distribution $\pi = [N = 1, N = 2, \dots, N = L]$

The unique stationary distribution is

$$\pi_0 = [1, 0, \dots, 0]$$

1.2 B - Mean Time to Absorption

Show it is equal to $E(T) = 2(1 - \frac{1}{L})$

Stefan said - sum expectation time across all steps (time to move to next state). Should be okay.

Waiting time for each state is

$$\beta_n = \frac{1}{\binom{n}{2}}$$

The waiting times of different states are independent, so the mean time to absorption is

$$E(T) = \sum_{i=1}^L \beta_i$$

$$E(T) = \sum_{i=1}^L \frac{1}{\binom{i}{2}}$$

$$E(T) = \sum_{i=1}^L \frac{2}{i!}$$

$$E(T) = \frac{2L}{\sum_{i=1}^L i!}$$

QUESTION - WTF? SUM OF FACTORIAL SEEMS WRONG

1.3 C - Rescaled process

Rescale the process to N_t/L

$$(\mathcal{L}^L f)(n/L) = \frac{1}{L} \binom{n}{2} (f(\frac{n-1}{L}) - f(\frac{n}{L}))$$

1.4 D

2 Ornstein-Uhlenbeck process

2.1 A

2.2 B

2.3 C

3 Moran Model and Wright-Fisher diffusion

3.1 A

3.2 B

3.3 C

3.4 D

3.5 E

3.6 F