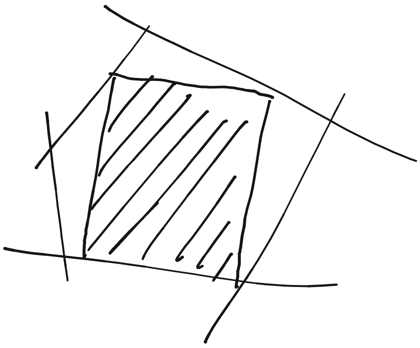
8.16 Maximum volume rectangle inside a polyhedron



$Ax \preceq b$

① in a silly way ...

$R^n \to$ Rectangle has 2^n vertices

$\to$ minimize $\prod_{i=1}^{n} (u_i - l_i)$

subject to $Al \le b$, $Au \le b$, $u \ge l$

② Smart way

$\to$ $a_{ij} > 0$: take u, $a_{ij} < 0$: take l $\Rightarrow$ maximum

$$a_{ij} = \underbrace{\max(a_{ij}, 0)}_{= a_{ij}^+} - \underbrace{\max(-a_{ij}, 0)}_{= a_{ij}^-} = a_{ij}^+ - a_{ij}^-$$

minimize $\prod_{i=1}^{n} (u_i - l_i)$ $\leftarrow$ It'll also be great to take $(\ )^{\frac{1}{n}}$

subject to $\sum_{j=1}^{n} (a_{ij}^+ u_j - a_{ij}^- l_j) \le b_i$

9.30 Gradient and Newton method

(a) G

minimize $f(x) = -\sum_{i=1}^{m} \log(1-a_i^T x) - \sum_{i=1}^{n} \log(1-x_i^2)$.

$$\nabla f(x) = a^T \cdot \frac{1}{1-a^T x} - \left(\frac{1}{1+x} - \frac{1}{1-x}\right) \quad \text{(vector)}$$

(b) Newton method

$$\nabla^2 f(x) = \overset{D}{\begin{bmatrix} - & & & 0 \\ & - & & \\ & & - & \\ 0 & & & - \end{bmatrix}} - \overset{D}{\left(\begin{matrix} - & & & 0 \\ & - & & \\ & & - & \\ 0 & & & - \end{matrix}\right)}$$

$Hv = -g$

instead of finding $v = H^{-1}g$,

solve $Hv = -g$