9.8 Steepest descent Method in l_∞-norm

$$\Delta x = \{ \underset{d}{\operatorname{argmin}} \quad \nabla f(x)^T d \ , \ \|d\|_\infty \leq 1 \}$$

$$\rightarrow \Delta x = \quad \text{sign}\,(\nabla f(x))$$

10.1 Nonsingularity of the KKT matrix

(a)

✓ ① $\longleftrightarrow$ ②

$N(P) \cap N(A) = \{0\}$ ⇒ $N(A)$ 에서 $xPx > 0$

$\Longleftrightarrow$ $Ax = 0$ 인 x 에서, $xPx > 0$

✓ ② $\longleftrightarrow$ ③

F = basis of $N(A)$ → $x = Fz$, $z \in R^{n \times (n-p)}$

$$x^T P x = z^T \underbrace{F^T P F}_{P^* > 0} z > 0$$

✓ ② $\longleftrightarrow$ ④

$P + A^T Q A > 0$ for some $Q \succeq 0$

$$x^T P x + x^T A^T Q A x = x^T P x + (Ax)^T Q (Ax) = x^T P x > 0$$

$(Ax = 0 \rightarrow x^T P x > 0) \Longleftrightarrow P + A^T Q A > 0$ for some $Q \succeq 0$

→

(b) $\begin{bmatrix} P & A^T \\ A & 0 \end{bmatrix}$ has exactly n positive, p negative eigenV

(if (a) holds)

$P + A^TA > 0$. $R^T(P+A^TA)R = I$.

$AR = U\Sigma V_1^T$, $V = [\overset{p}{V_1} \;\; \overset{n-p}{V_2}]$, $S = \underbrace{[\underbrace{\Sigma}_{p} \;\; 0]}_{n}\,\}p$

$V^TR^T(P+A^TA)RV = V^TR^TPRV + S^TS = I$

$V^TR^TPRV = I - S^TS = \mathrm{diag}(\overbrace{1-\sigma_1^2, 1-\sigma_2^2, \ldots}^{p} 1, \cdots, 1) = \Lambda$

$$\begin{bmatrix} V^TR^T & 0 \\ 0 & U^T \end{bmatrix}\begin{bmatrix} P & A^T \\ A & 0 \end{bmatrix}\begin{bmatrix} RV & 0 \\ 0 & U \end{bmatrix} = \begin{array}{c} \begin{bmatrix} \Lambda & S^T \\ S & 0 \end{bmatrix}\begin{array}{l} \}n \\ \}p \end{array} \\ \begin{array}{cc} \underbrace{}_{n} & \underbrace{}_{p} \end{array} \end{array}$$

$$\begin{array}{cccc} \overbrace{}^{p} & \overbrace{}^{n-p} & & \\ \end{array}$$
$$\begin{bmatrix} \Lambda & 0 & \Sigma \\ 0 & I & 0 \\ \Sigma & 0 & 0 \end{bmatrix}\begin{array}{l} \}p \\ \}n-p \\ \}p \end{array}$$

$\hookrightarrow M' = P^TMP$, ($P$: permutation matrix)

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$$\begin{pmatrix} \boxed{\begin{matrix} 1-\sigma_1^2 & \sigma_1 \\ \sigma_1 & 0 \end{matrix}} & & & \\ & \square & & \\ & & \square & \\ & & & \square \;\; ,,,, \end{pmatrix}$$

11.13 Self-concordance and negative entropy

(a) $f'(x) = 1 + \log x$, $f''(x) = \frac{1}{x}$, $f'''(x) = -\frac{1}{x^2}$

$\left| -\frac{1}{x^2} \right| \leq 2\left(\frac{1}{x}\right)^{\frac{3}{2}}$ ← False

(b) $f'(x) = t\log x + t - \frac{1}{x}$

$f''(x) = \frac{t}{x} + \frac{1}{x^2}$, $f'''(x) = -\frac{t}{x^2} - \frac{2}{x^3}$

$\left| \frac{-t}{x^2} - \frac{2}{x^3} \right| \leq 2\left(\frac{t}{x} + \frac{1}{x^2}\right)^{\frac{3}{2}}$

implementing simple calculas ...

True!