

9.8 Steepest descent Method in  $\ell_\infty$ -norm

$$\Delta x = \left\{ \underset{d}{\operatorname{argmin}} \quad \nabla f(x)^T d, \quad \|d\|_\infty \leq 1 \right\}$$

$$\rightarrow \Delta x = \operatorname{sign}(\nabla f(x))$$

## 10.1 Nonsingularity of the KKT matrix

(a)

$$\checkmark \quad ① \leftrightarrow ②$$

$$N(P) \cap N(A) = \{0\} \Rightarrow N(A) \text{ orthogonal } x^T P x > 0$$

$$\Leftrightarrow Ax=0 \text{ only solution, } x^T P x > 0$$

$$\checkmark \quad ② \leftrightarrow ③$$

$$F = \text{basis of } N(A) \rightarrow \lambda = F_I, I \in \mathbb{R}^{n \times (n-p)}$$

$$\lambda^T P \lambda = \underbrace{z^T F^T P F}_P z > 0$$

$$P^* > 0$$

$$\checkmark \quad ② \leftrightarrow ④$$

$$P + A^T Q A > 0 \text{ for some } Q \succeq 0$$

$$\lambda^T P \lambda + \lambda^T A^T Q A \lambda = \lambda^T P \lambda + (A \lambda)^T Q (A \lambda) = \lambda^T P \lambda > 0$$

$$(Ax=0 \rightarrow x^T P x > 0) \Leftrightarrow P + A^T Q A > 0 \text{ for some } Q \succeq 0$$



(b)  $\begin{bmatrix} P & A^T \\ A & 0 \end{bmatrix}$  has exactly  $N$  positive,  $P$  negative eigenvalues  
(if (a) holds)

$$P + A^T A > 0 . \quad R^T (P + A^T A) R = I .$$

$$AR = U \sum V_i^T \quad , \quad V = \begin{bmatrix} P & n-p \\ V_1 & V_2 \end{bmatrix} , \quad S = \begin{bmatrix} n \\ \sum \end{bmatrix} \overset{P}{\underset{p}{\sim}} P$$

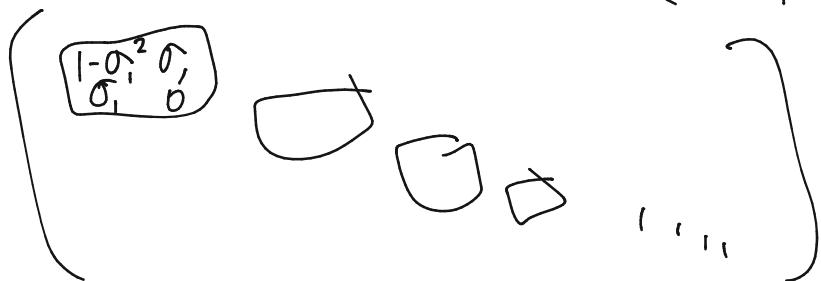
$$V^T R^T (P + A^T A) R V = V^T R^T P R V + S^T S = I$$

$$V^T R^T P R V = I - S^T S = \text{diag} \left( \underbrace{1 - \sigma_1^2, 1 - \sigma_2^2, \dots, 1, \dots, 1}_{n-p} \right) = \Lambda$$

$$\begin{bmatrix} V^T R^T & 0 \\ 0 & U^T \end{bmatrix} \begin{bmatrix} P & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} RV & 0 \\ 0 & U \end{bmatrix} = \begin{bmatrix} \Lambda & S^T \\ S & 0 \end{bmatrix} \overset{\substack{\uparrow \uparrow \\ n-p}}{=} P$$

$$\begin{bmatrix} P & n-p \\ \Lambda & 0 \\ 0 & 1 \\ \sum & 0 \end{bmatrix} \overset{\substack{\uparrow \\ p}}{=} \begin{bmatrix} \Sigma \\ 0 \\ 0 \end{bmatrix} ) P$$

$$\hookrightarrow M' = P^T M P , \quad (P : \text{permutation matrix})$$



↪ 합동변환

11.13) self-concordance and negative entropy

(a)  $f'(x) = 1 + \log x$ ,  $f''(x) = \frac{1}{x}$ ,  $f'''(x) = -\frac{1}{x^2}$

$$\left| -\frac{1}{x^2} \right| \leq 2 \left( \frac{1}{x} \right)^{\frac{3}{2}} \leftarrow \text{False}$$

(b)  $f'(x) = t \log x + t - \frac{1}{x}$

$$f''(x) = \frac{t}{x} + \frac{1}{x^2}, f'''(x) = -\frac{t}{x^2} - \frac{2}{x^3}$$

$$\left| -\frac{t}{x^2} - \frac{2}{x^3} \right| \leq 2 \left( \frac{t}{x} + \frac{1}{x^2} \right)^{\frac{3}{2}}$$

implementing simple calculus ...

True!