Additional exercises

(1) Minimax rational fit to the exponential

$$\left| \frac{p(t)}{q(t)} - y_i \right| \leq \gamma$$

$$| p(t_i) - y_i q(t_i) | \leq \gamma q(t_i)$$

With bisection ..

② Maximum likelihood prediction of team ability

(a)

game \ team	1	2	3	4	...	n
1	y_1	$-y_1$	0	0	0 ...	0
2	0	0			0 0	...
3						
4						
⋮						
m						

$= A$

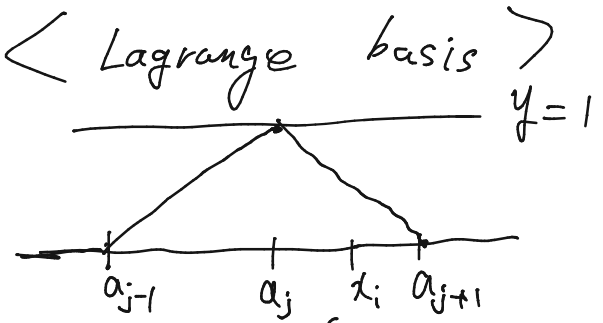
$$p(y \mid a) = \prod_{i=1}^{m} \Phi\left(\frac{y_i (a_{j(i)} - a_{k(i)})}{\sigma}\right)$$

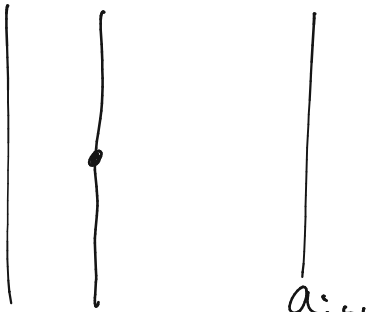
$$= \prod_{i=1}^{m} \Phi\left(\frac{(Aa)_i}{\sigma}\right)$$

$$\rightarrow \begin{cases} \text{maximize } l(a) = \sum_{i=1}^{m} \log \Phi\left(\frac{(Aa)_i}{\sigma}\right) & \text{(concave)} \\ \text{Subject to } 0 \le a \le 1 \end{cases}$$

③ piecewise-linear fitting

< Lagrange basis >





$$\Rightarrow f_j(x) = \min\left(\left(\frac{x-a_{j-1}}{a_j-a_{j-1}}\right)_+, \left(\frac{a_{j+1}-x}{a_{j+1}-a_j}\right)_+ \right)$$

$$f(x_i) = \frac{(x_i-a_j)f(a_{j+1}) + (a_{j+1}-x_i)f(a_j)}{a_{j+1}-a_j}$$

$$\begin{cases} \text{minimize} \quad \sum_{i=1}^{m} \left(f(x_i) - y_i\right)^2 \\ \text{s.t} \quad \frac{f(a_{j+1})-f(a_j)}{a_{j+1}-a_j} \geq \frac{f(a_j)-f(a_{j-1})}{a_j-a_{j-1}}, \\ \qquad a_0 < a_1 \cdots a_K, \; x_1 < \cdots x_m \end{cases}$$

$$\Downarrow \quad Z = \{f(a_i) \mid i=1 \cdots m\}, \quad f(x_i) = \sum_{j=0}^{K} f_j(x_i) Z_j$$

$$\hookrightarrow = f_j(x_i) f(a_j) + f_{j+1}(x_i) f(a_{j+1})$$

$$\begin{cases} \text{minimize} \; \| F_Z - y \|_2 \\ \text{s.t} \quad \frac{Z_{i+1}-Z_i}{a_{i+1}-a_i} \geq \frac{Z_i - Z_{i-1}}{a_i - a_{i-1}} \end{cases}$$

④ Robust least-squares with interval coefficient matrix

1-1

$$\|Ax-b\| = \|\bar{A}x-b+rx\|_2 \le \||Ax-b|+r|x|\|_2$$

$$\le \||Ax-b|+R|x|\|_2 \qquad (R: \text{non-negative})$$

$$\therefore \sup_{A\in\mathcal{A}} \|Ax-b\| = \|\bar{A}x-b\|_2 + R\|x\|_2$$

→ minimize $\|\bar{A}x-b\|_2 + R\|x\|_2$

1-2

minimize $\sup\|Ax-b\|_2 \iff \min \sup \|Ax-b\|_2^2$

Let $y = Ax-b$

$$\Rightarrow \begin{cases} \text{minimize} & y^Ty \\ \text{s.t} & -y \le Ax-b \le y \quad \text{for all } A\in\mathcal{A} \end{cases}$$

$$\bar{A}x+R|x|-b \le y \;,\; \bar{A}x-R|x|-b \ge -y$$

$$\Leftrightarrow \begin{cases} \text{minimize} & y^Ty \\ \text{s.t} & \bar{A}x+R|x|-b \le y \\ & \bar{A}x-R|x|-b \ge -y \end{cases}$$

$$\Leftrightarrow \bar{A}x+Rz-b \le y$$
$$\bar{A}x-Rz-b \ge -y$$
$$-z \le x \le z$$

(5) Total variation image interpolation

minimize

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(U_{ij}-U_{i-1,j}\right)^2+\left(U_{ij}-U_{ij-1}\right)^2$$

Subject to $\left(\text{known matrix} \times \left(U_{12}-U_{orig}\right)=0\right)$

(6) Relaxed and discrete A-optimal experiment design

minimize $\operatorname{tr}\left(\sum_1^p m_i v_i v_i^T\right)^{-1}$

subject to $m_1+\cdots m_p=m$, $m_i\in(0\cdots m)$, $i=1\cdots p$

$\Downarrow$ relaxing

minim $\left(\frac{1}{m}\right)\operatorname{tr}\left(\sum_1^p m_i v_i v_i^T\right)^{-1}$

subject to $1^T\lambda=1$, $\lambda\geq 0$