4.15

(a)

feasible set of LP relaxation

$\cup$

// of LP Boolean

(b) optimal solutions are same

4.60 log-optimal investment strategy

✓ $\lambda(t) = \frac{w(t)}{w(t-1)}$

✓growth rate $= \{\prod_{t=1}^{N} \lambda(t)\}^{\frac{1}{N}}$

$\xrightarrow{\log}$ $\frac{1}{N}\sum_{t=1}^{N} \log \lambda(t)$

✓ $\lambda(t)$ is random $\{p_1^T x, p_2^T x, \cdots p_m^T x\}$

✓ $\sum_{j=1}^{m} \pi_j =$, $\pi_j = prob(\lambda(t) = p_j^T x)$

✓ long term growth rate

$R_{lt} = \sum_{j=1}^{m} \pi_j \log(p_j^T x)$

Solution:

$$\begin{cases} \text{maximize} & \sum_{j=1}^{m} \pi_j \log(p_j^T x) \\ \text{s.t} & x \succeq 0,\ 1^T x = 1, \\ & 1^T \pi = 1 \end{cases}$$

$\log(p_j^T x)$: concave $\rightarrow$ $\sum \log$: concave

Constraints : affine.

5.13 (a)

$$L(x, \lambda, \nu) = c^T x + \lambda^T (Ax - b) - \nu^T x + x^T \mathrm{diag}(\nu) x$$

→ minimize in x!

$$\nabla_x L(x, \lambda, \nu) = 2\,\mathrm{diag}(\nu) x + (c + A^T \lambda - \nu) = 0$$

$$\Rightarrow g(\lambda, \nu) = \begin{cases} -b^T \lambda - \frac{1}{4} \sum_{i=1}^{n} \frac{(c_i + a_i^T \lambda - \nu_i)^2}{\nu_i} & \nu \succeq 0 \\ -\infty & \text{otherwise.} \end{cases}$$

⇒ resulting dual problem is

$$\begin{aligned} &\text{maximize} && -b^T \lambda - \frac{1}{4} \sum_{i=1}^{n} \frac{(c_i + a_i^T \lambda - \nu_i)^2}{\nu_i} \\ &\text{s.t} && \nu \succeq 0, \ \lambda \succeq 0 \end{aligned}$$

⇒ we can simplify this in ν

$$\begin{aligned} &\text{maximize} && -b^T \lambda + \sum_{i=1}^{n} \min\{0, c_i + a_i^T \lambda\} \\ &\text{s.t} && \lambda \succeq 0 \end{aligned}$$

(b)

① Lagrangian Relaxation

$$L(x, u, v, w) = c^T x + u^T(Ax - b) - v^T x + w^T(x - 1)$$

$$= (c + A^T u - v + w)^T x - b^T u - 1^T w$$

② LP Relaxation

$$g(u, v, w) = \begin{cases} -b^T u - 1^T w, & A^T u - v + w + c = 0 \\ -\infty & \text{otherwise.} \end{cases}$$

→ The Dual problem of ①, ② is

$$\begin{aligned} \text{maximize} \quad & -b^T u - 1^T w \\ \text{s.t} \quad & A^T u - v + w + c = 0, \\ & u \succeq 0, v \succeq 0, w \succeq 0 \end{aligned}$$

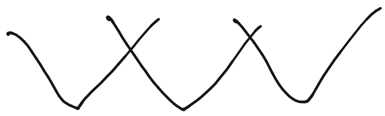
Conclusion: Same lower-bound.

6.2

Solution of $\left[\text{minimize} \quad \|x\mathbf{1} - b\|_p \right]$

in l_1, l_2, l_∞ norms.

① l_1 : $\sum_{i=1}^{m} |x - b_i|$



$\nabla f = \sum_i \text{sign}(x - b_i) \rightsquigarrow x =$ 중앙값 (median)

② l_2 : $\sum_{i=1}^{m} (x - b_i)^2 \xrightarrow{\frac{d}{dx}} 2 \sum_{1}^{m} x - b_i$

$x = \frac{1}{m} \sum_{1}^{m} b_i$ (average)

③ l_∞ : $\max |x - b_i|$

$\rightarrow \dfrac{\max b_i + \min b_i}{2}$