

Supplementary data :

Movies showing the temporal evolution of the spatial structures.

Each video is divided in three panels. On the left is a 3D representation of the system with fibers colored according to their alignment indicator (see colorbar on the right) and edges of the spatial domain drawn in black, in the middle the stereographic projection of the fibers directional vectors and on the right the trajectory of the simulation in the plane A_{\max} - A_{sim} . The current time (in U_t) is displayed at the top.

Movie1.mp4 : Simulation of a dense system ($L_{\text{fib}} = 3000$) with fast remodeling dynamics ($\nu_{\text{link}} = 0.1$) and low equilibrium linked fiber fraction ($\chi_{\text{link}} = 0.2$).

This video shows a system quickly organizing : at $t = 1000U_t$, the system has already transitioned from its initial unorganized state to a curved state. At $t = 3000U_t$, the main direction can be seen emerging in the form of a large cluster of points in the stereographic projection. At $t = 10\,000U_t$, the stereographic projection displays a planar distribution of the directional vectors, with extra accretion of points in the main direction and total depletion in the perpendicular direction. The system has already nearly reached its maximal alignment indicator and, from that point onward, it will mainly undergo small local adjustments of the fibers position and orientation (see the 3D representation on the left panel). The alignment indicators of individual fibers harmonise, the mean alignment indicator increases slightly and the point cloud of the stereographic projection condensates in a clear straight band. During the entire simulation, the semi-major axis length A_{\max} of the stereographic projection covariance ellipse stays nearly constant.

Movie2.mp4 : Simulation of a dense system ($L_{\text{fib}} = 3000$) with slow remodeling dynamics ($\nu_{\text{link}} = 0.001$) and low equilibrium linked fiber fraction ($\chi_{\text{link}} = 0.2$).

This video shows a system organizing more slowly than the previous one (approximately twice slower) but achieving a more aligned final state. The system reaches a curved state at $t = 1900U_t$. The main direction can be seen emerging on the stereographic projection around time $t \approx 5000U_t$. The point cloud of the stereographic projection then begins to condensate around this main direction in a nearly symmetric manner while the various local structures rotate to align together (see left panel), reaching an aligned state at $t = 23\,000U_t$ and continuing to align.

Movie3.mp4 : Simulation of a dense system ($L_{\text{fib}} = 3000$) with fast remodeling dynamics ($\nu_{\text{link}} = 0.1$) and high equilibrium linked fiber fraction ($\chi_{\text{link}} = 0.8$).

This video shows a system organizing very quickly, with a stereographic projection adopting as early as $t = 4000U_t$ a band-like pattern which quickly gets thinner. At $t = 6000U_t$, the 3D representation shows a clear wavy pattern with very uniform local alignment indicators. At that time the mean alignment indicator is already high (> 0.9). The stereographic projection then begins to contract while the wavy pattern flatten, and the simulation ends in an aligned state.

Movie4.mp4 : Simulation of a dense system ($L_{\text{fib}} = 3000$) with slow remodeling dynamics ($\nu_{\text{link}} = 0.001$) and high equilibrium linked fiber fraction ($\chi_{\text{link}} = 0.8$).

This video shows a system evolving very slowly. The mean alignment indicator reaches the 0.5 threshold around $t = 11\,000U_t$. At that time, the local alignment

indicator of individual fibers displays wide discrepancies and the stereographic projection point cloud has not visibly changed. A main direction can be seen emerging at approximately $t = 20\,000U_t$, but the central cluster of points is very large and does not contract over time, as can be seen by the fact that the quantifier A_{\max} does nearly not decrease. The system ends in a curved state with heterogeneous local structures.

Movie5.mp4 : Comparison of two simulations with different fiber densities, both ending in an aligned state. The top row shows a dense system ($L_{\text{fib}} = 3000$) with intermediate remodeling dynamic ($\nu_{\text{link}} = 0.01$) and moderate equilibrium linked fiber fraction ($\chi_{\text{link}} = 0.3$). The bottom row displays a sparse system ($L_{\text{fib}} = 1500$) with intermediate remodeling dynamic ($\nu_{\text{link}} = 0.01$) and very high equilibrium linked fiber fraction ($\chi_{\text{link}} = 0.9$).

It is noteworthy that the two systems display a very similar temporal evolution. This comes from the fact that they have the same remodeling speed ν_{link} and a comparable number of links per fiber $N_{\text{linkperfib}}$. The latter is achieved by giving the sparse system a higher equilibrium linked fiber fraction χ_{link} , which compensate for its lesser number of linkable configurations (i.e. overlapping fiber pairs).