

	Claude 3 Opus	Claude 3 Sonnet	Claude 3 Haiku	GPT-4	GPT-3.5	Gemini 1.0 Ultra	Gemini 1.0 Pro
Undergraduate level knowledge <i>MMLU</i>	<b>86.8%</b> 5-shot	<b>79.0%</b> 5-shot	<b>75.2%</b> 5-shot	<b>86.4%</b> 5-shot	<b>70.0%</b> 5-shot	<b>83.7%</b> 5-shot	<b>71.8%</b> 5-shot
Graduate level reasoning <i>GPQA, Diamond</i>	<b>50.4%</b> 0-shot CoT	<b>40.4%</b> 0-shot CoT	<b>33.3%</b> 0-shot CoT	<b>35.7%</b> 0-shot CoT	<b>28.1%</b> 0-shot CoT	—	—
Grade school math <i>GSM8K</i>	<b>95.0%</b> 0-shot CoT	<b>92.3%</b> 0-shot CoT	<b>88.9%</b> 0-shot CoT	<b>92.0%</b> 5-shot CoT	<b>57.1%</b> 5-shot	<b>94.4%</b> Maj1@32	<b>86.5%</b> Maj1@32
Math problem-solving <i>MATH</i>	<b>60.1%</b> 0-shot CoT	<b>43.1%</b> 0-shot CoT	<b>38.9%</b> 0-shot CoT	<b>52.9%</b> 4-shot	<b>34.1%</b> 4-shot	<b>53.2%</b> 4-shot	<b>32.6%</b> 4-shot
Multilingual math <i>MGSM</i>	<b>90.7%</b> 0-shot	<b>83.5%</b> 0-shot	<b>75.1%</b> 0-shot	<b>74.5%</b> 8-shot	—	<b>79.0%</b> 8-shot	<b>63.5%</b> 8-shot
Code <i>HumanEval</i>	<b>84.9%</b> 0-shot	<b>73.0%</b> 0-shot	<b>75.9%</b> 0-shot	<b>67.0%</b> 0-shot	<b>48.1%</b> 0-shot	<b>74.4%</b> 0-shot	<b>67.7%</b> 0-shot
Reasoning over text <i>DROP, F1score</i>	<b>83.1</b> 3-shot	<b>78.9</b> 3-shot	<b>78.4</b> 3-shot	<b>80.9</b> 3-shot	<b>64.1</b> 3-shot	<b>82.4</b> Variable shots	<b>74.1</b> Variable shots
Mixed evaluations <i>BIG-Bench-Hard</i>	<b>86.8%</b> 3-shot CoT	<b>82.9%</b> 3-shot CoT	<b>73.7%</b> 3-shot CoT	<b>83.1%</b> 3-shot CoT	<b>66.6%</b> 3-shot CoT	<b>83.6%</b> 3-shot CoT	<b>75.0%</b> 3-shot CoT
Knowledge Q&A <i>ARC-Challenge</i>	<b>96.4%</b> 25-shot	<b>93.2%</b> 25-shot	<b>89.2%</b> 25-shot	<b>96.3%</b> 25-shot	<b>85.2%</b> 25-shot	—	—
Common Knowledge <i>HellaSwag</i>	<b>95.4%</b> 10-shot	<b>89.0%</b> 10-shot	<b>85.9%</b> 10-shot	<b>95.3%</b> 10-shot	<b>85.5%</b> 10-shot	<b>87.8%</b> 10-shot	<b>84.7%</b> 10-shot

	Claude 3 Opus	Claude 3 Sonnet	Claude 3 Haiku	GPT-4V	Gemini 1.0 Ultra	Gemini 1.0 Pro
Math & reasoning <i>MMMU (val)</i>	<b>59.4%</b>	<b>53.1%</b>	<b>50.2%</b>	<b>56.8%</b>	<b>59.4%</b>	<b>47.9%</b>
Document visual Q&A <i>ANLS score, test</i>	<b>89.3%</b>	<b>89.5%</b>	<b>88.8%</b>	<b>88.4%</b>	<b>90.9%</b>	<b>88.1%</b>
Math <i>MathVista (testmini)</i>	<b>50.5%</b> CoT	<b>47.9%</b> CoT	<b>46.4%</b> CoT	<b>49.9%</b>	<b>53.0%</b>	<b>45.2%</b>
Science diagrams <i>AI2D, test</i>	<b>88.1%</b>	<b>88.7%</b>	<b>86.7%</b>	<b>78.2%</b>	<b>79.5%</b>	<b>73.9%</b>
Chart Q&A <i>Relaxed accuracy (test)</i>	<b>80.8%</b> 0-shot CoT	<b>81.1%</b> 0-shot CoT	<b>81.7%</b> 0-shot CoT	<b>78.5%</b> 4-shot CoT	<b>80.8%</b>	<b>74.1%</b>

## Claude 3 Opus

- 200k context window
- Great for needle in a haystack
- Different way of prompting (**go into more detail**)
- \$15/million input tokens, \$75/million output tokens
  - Sonnet is worse than GPT-4 but 5x cheaper than Opus
- Slowest to use

*Example responses at the bottom*

The responses are the best at generating specific, high-quality questions

## GPT-4 Turbo

- Currently using
- \$10/1M token input, \$30/1M token output

[Example Generate Questions Prompt Response 1](#)

[Example Generate Questions Prompt Response 2](#)

[Example Generate Questions Prompt Response 3](#)

Free responses not as specific as we'd like

## Gemini

- By far the fastest in our testing
- Likely integration with Apple products
- Does not even follow the specified formatting instructions

[Example Generate Questions Prompt Response 1](#)

[Example Generate Questions Prompt Response 2](#)

[Example Generate Questions Prompt Response 3](#)

## Takeaways:

- Claude 3 Opus is SOTA for text-based tasks
- GPT 4V still better for image-based tasks

- So if we choose to integrate something where we take our knowledge graph and input it into an LLM as a visual, we'll want to use GPT 4V ideally
- Might be valuable to incorporate different LLMs based on the task at hand
- Using Claude may give us better results but requires some tweaking of our prompts to be even better, particularly by incorporating more test cases and making them longer to take advantage of the context window and its Needle in the Haystack performance
  - For example, providing an entire example section of notes, tests, etc for another subject that we hand-write and use as a baseline for performance measurements
- **Key Insight:**
  - If this will be used in school systems then each system will likely have its own AI model of choice, just like how they use Google, Microsoft, Slack, etc.
  - We're cutting off  $\frac{1}{3}$  of our user group by only using OpenAI
  - Solution: add more composability and modularity to ChatEDU by migrating to LangChain and allowing for easy switching between different model families
    - Still gives us the composability to say that GPT-3.5 = Haiku, GPT-4 = Opus, etc.

### ***Prompt used for testing:***

Given an array of notes, generate an assignment that includes 5 questions. These questions can be either multiple choice or free response.

Free response questions should be either understanding questions, which challenge students to demonstrate their understanding of a particular concept, or application questions, which challenge students to apply their understanding to a particular problem.

The questions should flow logically, where completion of each question leads to the sentiment of the next question.

The questions must be gradeable by AI later in the pipeline.

Multiple choice questions should be challenging and force the user to demonstrate understanding of the topic. They SHOULD NOT simply be a definition or a fact. They should be a question that requires the user to think about the topic and apply their knowledge. Feel free to use examples of scenarios or practice examples to help the user understand the topic better. Ensure there are no ambiguities in the answers, meaning there is ONLY ONE correct answer to the problem. DO NOT include any explanation of the correct answer.

Multiple choice questions must take the following JSON format:

```
{
  tag: "${QuestionTypes.MultipleChoice}",
  question: {
    question: 'string: <question>?',
    options: `object: {
      "A": string,
      "B": string,
      "C": string,
      "D": string
    }`,
    answer: "string: <A/B/C/D>"
  }
}
```

Text-based understanding questions should ask the user to demonstrate their understanding of the topics covered in their notes. They should be able to explain the concepts and why they are relevant.

Understanding questions must take the following JSON format:

```
{
  tag: "${QuestionTypes.FreeResponse}",
  question: 'string: <question>?'
}
```

Text-based application questions should ask the user to apply the concepts covered in their notes. They should be able to demonstrate their understanding of the concepts by applying them to a new situation. Create examples or practice problems based on the concepts covered in the notes. Do NOT include any indication of the answer.

Application questions must take the following JSON format:

```
{
  tag: "${QuestionTypes.FreeResponse}",
  question: 'string: <question>?'
}
```

Notes:

```
[{"id":19,"name":"Rational Approximations","content":"# Rational Approximations of Irrational Numbers\n\n## Introduction\n\nIrrational numbers are numbers that cannot be expressed as a simple fraction of two integers, unlike rational numbers which can be expressed in the form of `a/b` where `a` and `b` are integers and `b` is not zero. However, we can find rational numbers that are very close to an irrational number, and these are known as rational approximations.\n\n## Importance\n\nRational approximations are important because they allow us to work with irrational numbers in a practical way in various fields such as engineering, physics, and computer science. Since we often rely on numerical methods to solve problems, having a rational approximation enables us to perform calculations that would otherwise be impossible with pure irrational numbers.\n\n## Methods\n\nThere are several methods to find rational approximations:\n\n* **Continued Fractions**: Each irrational number can be represented as an infinite continued fraction. Truncating this continued fraction at a certain point gives a rational approximation of the irrational number.\n\n* **Pell's Equation**: Pell's equation is an indeterminate quadratic equation that can be used to find approximations to square roots of numbers that are not perfect squares.\n\n* **Diophantine Approximation**: This area of number theory deals with how well irrational numbers can be approximated by rational numbers. The Dirichlet's approximation theorem is a result that guarantees very good rational approximations for real numbers.\n\n## The Golden Ratio Example\n\nAn example of an irrational number is the golden ratio, often denoted by the Greek letter phi ( $\phi$ ), which is approximately 1.61803. It can be represented by the continued fraction  $[1;1,1,1,...]$ . A rational approximation of the golden ratio can be obtained by truncating the continued fraction, such as  $1, 1/2, 3/2, 5/3, 8/5, ...$  with each fraction representing a better approximation.\n\n## Pi Example\n\nPi ( $\pi$ ) is another famous irrational number, approximately equal to 3.14159. Since ancient times, mathematicians have sought to approximate Pi. An early approximation was  $22/7$ , but with modern computational methods, we can generate rational approximations with very high precision.\n\n## Conclusion\n\nRational approximations provide a pragmatic means to deal with the complexities of irrational numbers. While they are not exact, they can be accurate enough for practical purposes depending on how precise the approximation needs to be
```

for a given application. Thus, understanding rational approximations is essential for anyone working in fields that require numerical accuracy and measurement."

,"notebook\_id":4,"topic\_id":51,"order\_position":0},{ "id":17,"name":"Properties of integer exponents","content":"#### Introduction\n\nThis lesson covers the properties of integer exponents, aligning with standard 8.EE.A.1. Students will learn to simplify expressions using exponent rules.\n\n#### Objectives\n\n\* Understand zero and negative exponents.\n\* Apply exponent rules: product, quotient, power of a power, and power of a product.\n\n#### Zero Exponent Rule\n\n\* \*\*Definition\*\*: Any non-zero base raised to the zero power equals one.\n\* \*\*Example\*\*:  $3^0 = 1$ \n\n#### Negative Exponents\n\n\* \*\*Definition\*\*: A negative exponent represents the reciprocal of the base raised to the positive exponent.\n\* \*\*Example\*\*:  $2^{-3} = 1/(2^3) = 1/8$ \n\n#### Properties of Exponents\n\n#### Product Rule\n\n\* \*\*Rule\*\*: When multiplying with the same base, add the exponents.\n\* \*\*Example\*\*:  $x^a \cdot x^b = x^{(a+b)}$ \n\n#### Quotient Rule\n\n\* \*\*Rule\*\*: When dividing with the same base, subtract the exponents.\n\* \*\*Example\*\*:  $x^a / x^b = x^{(a-b)}$ \n\n#### Power of a Power\n\n\* \*\*Rule\*\*: When raising an exponent to another exponent, multiply the exponents.\n\* \*\*Example\*\*:  $(x^a)^b = x^{(a \cdot b)}$ \n\n#### Power of a Product\n\n\* \*\*Rule\*\*: Distribute the exponent to each base in a multiplication inside a parenthesis.\n\* \*\*Example\*\*:  $(xy)^a = x^a \cdot y^a$ "

,"notebook\_id":4,"topic\_id":42,"order\_position":0},{ "id":20,"name":"Square Root and Cube Root Solutions","content":"# Square Root and Cube Root Solutions\n\nUnderstanding square roots and cube roots is crucial in mathematics, as they allow us to solve a variety of problems including those in algebra, geometry, and real-world applications. Square roots and cube roots are the inverse operations of squaring and cubing a number, respectively.\n\n## Square Root\n\nThe square root of a number is a value that, when multiplied by itself, gives the original number. The square root of  $x$  is written as  $\sqrt{x}$ . For example, the square root of 25 is 5, because  $5^2 = 25$ . \n\n#### Properties of Square Roots\n\n\* The square root of a perfect square is an integer.\n\* The square roots of non-perfect squares are irrational numbers.\n\* Every positive number has two square roots: a positive and a negative root.\n\* The square root of 0 is 0.\n\* The square root of a negative number is not a real number (it is an imaginary number). \n\n#### Finding Square Roots\n\n1. \*\*Estimation\*\*: Use logic to estimate the square root of a number.\n2. \*\*Prime Factorization\*\*: Break the number down into its prime factors and pair the factors to find the square root.\n3. \*\*Using a Calculator\*\*: For non-perfect squares or for convenience, a calculator can provide the square root. \n\n## Cube Root\n\nThe cube root of a number is a value that, when multiplied by itself three times, gives the original number. The cube root of  $x$  is written as  $\sqrt[3]{x}$ . For example, the cube root of 27 is 3, because  $3^3 = 27$ . \n\n#### Properties of Cube Roots\n\n\* The cube root of a perfect cube is an integer.\n\* Cube roots can be found for negative numbers (e.g.,  $\sqrt[3]{(-27)} = -3$ ).\n\* Unlike square roots, every number has one real cube root. \n\n#### Finding Cube Roots\n\n1. \*\*Estimation\*\*: Roughly estimate

the cube root by finding two perfect cubes it lies between.  
2. **Prime Factorization:**

Decompose the number into prime factors and group them in triples to find the cube root.

3. **Using a Calculator:** For more complex numbers, the cube root can often be found using a scientific calculator.

### Solving Equations Involving Roots

In solving equations where roots are involved, it is common to isolate the root on one side of the equation and then raise both sides of the equation to the power that corresponds with the root (square or cube) to eliminate the root.

For example, to solve  $\sqrt{x} = 5$ , square both sides to get  $x = 25$ .

Similarly, to solve  $\sqrt[3]{x} = 4$ , cube both sides to get  $x = 64$ .

### Applications

Square roots and cube roots appear in various areas such as geometry (e.g., calculating the side of a square given its area), physics (e.g., determining the intensity of an earthquake with the Richter scale), and engineering (e.g., designing containers of a certain volume).

### Practice Problems

- Find the square root of 144.
- What is the cube root of 125?
- If  $\sqrt{x} = 12$ , what is  $x$ ?
- Solve for  $y$ :  $\sqrt[3]{(2y + 1)} = 3$

### Conclusion

Square roots and cube roots are fundamental in mathematics. By understanding how to find and manipulate them, students can solve a wide range of problems and better comprehend the world of

numbers.

Metadata: {"notebook\_id":4,"topic\_id":58,"order\_position":0}, {"id":12,"name":"Understanding Rational and Irrational Numbers","content":

### Understanding Rational and Irrational Numbers

Rational and irrational numbers are classifications of real numbers. They are important concepts in mathematics, and understanding the differences between them is crucial for students as they progress in math.

### Rational Numbers

Rational numbers are numbers that can be expressed as the quotient or fraction of two integers, with the denominator being a non-zero number. They can also be represented as either terminating or repeating decimals.

#### Characteristics of Rational Numbers:

- They can be written in the form  $a/b$ , where  $a$  and  $b$  are integers and  $b$  is not zero.
- They have a decimal representation that either ends after a finite number of digits or begins to repeat a sequence.
- Examples of rational numbers include  $1/2$ ,  $-3$ ,  $0.75$ , and  $6.333...$

### Irrational Numbers

Irrational numbers, on the other hand, cannot be expressed as a simple fraction. They have non-terminating and non-repeating decimal expansions, which means that the decimal goes on forever without repeating a pattern.

#### Characteristics of Irrational Numbers:

- They cannot be written as a simple fraction  $a/b$  where  $a$  and  $b$  are integers and  $b$  is not zero.
- Their decimal expansions neither terminate nor repeat.
- Examples include  $\pi$  (pi),  $\sqrt{2}$  (the square root of 2), and  $e$  (the base of natural logarithms).

### Comparison

Here's a quick comparison between rational and irrational numbers:

Characteristic	Rational Numbers	Irrational Numbers
Representation	Can be expressed as $a/b$	Cannot be expressed as $a/b$
Decimal Expansion	Terminating or Repeating	Non-terminating and Non-repeating
Examples	$3/4$ , $-5$ , $0.333...$ , $7$	$\pi$ , $\sqrt{3}$ , $e$ , $\sqrt{1/2}$

### Adding

and Multiplying\n\n\* When you add or multiply two rational numbers, the result is also a rational number.\n\* When you add or multiply a rational number with an irrational number, the result is an irrational number (except in some special cases, like multiplying by zero).\n\* Multiplying two irrational numbers might result in either a rational or an irrational number, depending on the numbers.\n\n## Conclusion\n\nUnderstanding the difference between rational and irrational numbers is key to working with different types of problems in algebra, trigonometry, and calculus. Recognizing whether a number is rational or irrational can help students to identify the appropriate methods to solve mathematical problems involving these types of numbers.", "notebook\_id": 4, "topic\_id": 37, "order\_position": 0}, {"id": 21, "name": "Operations with Scientific Notation", "content": "", "notebook\_id": 4, "topic\_id": 60, "order\_position": 0}]

The whole response should take the following format:

```
{
  name: <Assignment Name>,
  questions: [<Question 1>, <Question 2>, ...]
}
```

## MODEL A

*Example Generate Questions Prompt Response 1:*

Here is the assignment with 5 questions based on the provided notes:

```
{
  name: "Exploring Numbers: Rational, Irrational, and More",
  questions: [
    {
      tag: "${QuestionTypes.MultipleChoice}",
      question: {
        question: "Which of the following numbers is an example of an irrational number?",
        options: {
          "A": "22/7",
          "B": "0.3333...",
          "C": " $\sqrt{5}$ ",
          "D": "-4"
        },
        answer: "C"
      }
    }
  ]
}
```



```

    },
    {
      tag: "${QuestionTypes.FreeResponse}",
      question: "Explain in your own words why rational approximations of irrational
numbers are important in fields like engineering and computer science."
    },
    {
      tag: "${QuestionTypes.MultipleChoice}",
      question: {
        question: "What is the result when you raise any non-zero number to the
power of zero?",
        options: {
          "A": "0",
          "B": "1",
          "C": "The original number",
          "D": "Undefined"
        },
        answer: "B"
      }
    },
    {
      tag: "${QuestionTypes.FreeResponse}",
      question: "A cube has a volume of 512 cubic units. What is the length of each
side of the cube? Show your work and explain your reasoning."
    },
    {
      tag: "${QuestionTypes.MultipleChoice}",
      question: {
        question: "If  $x^a * x^b = x^{10}$ , and  $a + b = 10$ , which of the following could be
the values of a and b?",
        options: {
          "A": "a = 3, b = 7",
          "B": "a = 5, b = 10",
          "C": "a = -3, b = 13",
          "D": "a = 0, b = 10"
        },
        answer: "A"
      }
    }
  ]

```

```
}
```

### *Example Generate Questions Prompt Response 2*

Here is the generated assignment with 5 questions based on the provided notes:

```
{
  name: "Rational and Irrational Numbers Assignment",
  questions: [
    {
      tag: "${QuestionTypes.MultipleChoice}",
      question: {
        question: "What is the result when you multiply a rational number by an irrational number?",
        options: {
          "A": "Always a rational number",
          "B": "Always an irrational number",
          "C": "Could be either rational or irrational",
          "D": "Always zero"
        },
        answer: "B"
      }
    },
    {
      tag: "${QuestionTypes.FreeResponse}",
      question: "Explain why the cube root of a negative perfect cube is a real number, while the square root of a negative number is not a real number."
    },
    {
      tag: "${QuestionTypes.MultipleChoice}",
      question: {
        question: "Which of the following is the correct way to solve the equation  $\sqrt[3]{x + 5} = 2$ ?",
        options: {
          "A": "Square both sides",
          "B": "Cube both sides",
          "C": "Divide both sides by 3",
          "D": "Subtract 5 from both sides"
        },
        answer: "B"
      }
    }
  ]
}
```

```

    },
    {
      tag: "${QuestionTypes.FreeResponse}",
      question: "Give an example of a real-world situation where rational approximations
of irrational numbers would be useful, and explain why."
    },
    {
      tag: "${QuestionTypes.MultipleChoice}",
      question: {
        question: "What is the value of  $(-2)^0$  according to the zero exponent rule?",
        options: {
          "A": "0",
          "B": "-1",
          "C": "1",
          "D": "undefined"
        }
      },
      answer: "C"
    }
  ]
}

```

### *Example Generate Questions Prompt Response 3*

Here is the generated assignment with 5 questions based on the provided notes:

```

{
  name: "Fundamentals of Number Theory and Algebra",
  questions: [
    {
      tag: "FreeResponse",
      question: "Explain why rational approximations are important when working with
irrational numbers. Provide an example of a real-world application where rational
approximations would be used."
    },
    {
      tag: "MultipleChoice",
      question: {
        question: "What is the result of the following expression:  $(x^3)^4 * x^{-5}$ ",
        options: {
          "A": " $x^7$ ",

```

```

        "B": "x^12",
        "C": "x^-17",
        "D": "x^-2"
    },
    answer: "A"
}
},
{
    tag: "FreeResponse",
    question: "Solve the following equation and explain each step:  $\sqrt[3]{(5x - 2)} = 2$ "
},
{
    tag: "MultipleChoice",
    question: {
        question: "Which of the following is an irrational number?",
        options: {
            "A": "0.3333...",
            "B": "22/7",
            "C": " $\sqrt{5}$ ",
            "D": "-4/9"
        },
        answer: "C"
    }
},
{
    tag: "FreeResponse",
    question: "Give an example of two irrational numbers that, when multiplied together, result in a rational number. Prove that the product is rational."
}
]
}

```

## MODEL B

Example Generate Questions Prompt Response 1:

```

{
    "name": "Exploring Numbers and Their Properties",
    "questions": [

```

```

{
  "tag": "${QuestionTypes.MultipleChoice}",
  "question": {
    "question": "Which of the following best represents a rational approximation of Pi
(π)?",
    "options": {
      "A": "3.14159",
      "B": "22/7",
      "C": "1.61803",
      "D": "√2"
    },
    "answer": "B"
  }
},
{
  "tag": "${QuestionTypes.FreeResponse}",
  "question": "Explain how the properties of integer exponents can simplify the
expression (2^3 * 2^-1)^2."
},
{
  "tag": "${QuestionTypes.FreeResponse}",
  "question": "Apply the concept of square roots to solve for x in the equation x^2 =
49."
},
{
  "tag": "${QuestionTypes.MultipleChoice}",
  "question": {
    "question": "If a number is added to an irrational number, the result is always:",
    "options": {
      "A": "A rational number",
      "B": "An irrational number",
      "C": "An integer",
      "D": "Cannot be determined without additional information"
    },
    "answer": "B"
  }
},
{
  "tag": "${QuestionTypes.FreeResponse}",

```

```
    "question": "Given the expression  $5 \times 10^3 + 2 \times 10^2$ , express the sum in scientific notation."
```

```
  }  
]  
}
```

Example Generate Questions Prompt Response 2:

```
{  
  "name": "Exploring Number Concepts",  
  "questions": [  
    {  
      "tag": "${QuestionTypes.MultipleChoice}",  
      "question": {  
        "question": "If a number is added to its reciprocal, the sum is a rational number. Which of the following numbers would result in an irrational sum when added to its reciprocal?",  
        "options": {  
          "A": "2",  
          "B": "1/2",  
          "C": " $\sqrt{2}$ ",  
          "D": "3"  
        },  
        "answer": "C"  
      },  
    },  
    {  
      "tag": "${QuestionTypes.FreeResponse}",  
      "question": "Explain how the power of a product rule applies when simplifying expressions with integer exponents, using the expression  $(3x)^4$  as an example."  
    },  
    {  
      "tag": "${QuestionTypes.FreeResponse}",  
      "question": "Given the equation  $\sqrt{x} + 6 = 10$ , apply your knowledge of square roots to find the value of x."  
    },  
    {  
      "tag": "${QuestionTypes.MultipleChoice}",  
      "question": {  
        "question": "Which of the following is an example of a rational approximation for the irrational number  $\pi$ ?",
```

```

    "options": {
      "A": "22/7",
      "B": " $\sqrt{2}$ ",
      "C": "3.141",
      "D": "Both A and C"
    },
    "answer": "D"
  }
},
{
  "tag": "${QuestionTypes.FreeResponse}",
  "question": "If the golden ratio (approximately 1.61803) is expressed as a continued fraction, how would truncating this continued fraction at the fifth term compare in accuracy to using the fraction 8/5 as an approximation? Provide your reasoning."
}
]
}

```

Example Generate Questions Prompt Response 3:

```

{
  "name": "Understanding and Applying Mathematical Concepts",
  "questions": [
    {
      "tag": "${QuestionTypes.FreeResponse}",
      "question": "Explain the difference between rational and irrational numbers, providing examples for each. Why is this distinction important in mathematics?"
    },
    {
      "tag": "${QuestionTypes.MultipleChoice}",
      "question": {
        "question": "If  $x^2 = 49$  and  $x > 0$ , what is the value of  $x$  when expressed in scientific notation?",
        "options": {
          "A": "7",
          "B": " $7.0 \times 10^0$ ",
          "C": " $49.0 \times 10^0$ ",
          "D": " $7.0 \times 10^1$ "
        }
      },
    }
  ]
}

```

```

    "answer": "B"
  },
  {
    "tag": "${QuestionTypes.FreeResponse}",
    "question": "Describe the process of using prime factorization to find the square root of a number. Provide an example with a non-perfect square and explain why its square root is irrational."
  },
  {
    "tag": "${QuestionTypes.MultipleChoice}",
    "question": {
      "question": "Using Pell's equation for rational approximations, which of the following could be a rational approximation of the square root of 2?",
      "options": {
        "A": "1.41",
        "B": "1.5",
        "C": "1/√2",
        "D": "√2/2"
      }
    },
    "answer": "A"
  }
],
{
  "tag": "${QuestionTypes.FreeResponse}",
  "question": "Given that the golden ratio ( $\phi$ ) is an irrational number represented by the continued fraction  $[1; 1, 1, 1, \dots]$ , explain how truncating this continued fraction at different lengths results in varying degrees of rational approximations. Use this concept to approximate  $\phi$  to three different levels of accuracy."
}
]
}

```

## MODEL C

Example Generate Questions Prompt Response 1:

```

[1] {
  "tag": "${QuestionTypes.FreeResponse}",

```



"question": "What are rational numbers? How can you represent them in different ways (e.g., fraction, decimal)?"

}

[2] {

"tag": "\${QuestionTypes.FreeResponse}",

"question": "Define irrational numbers. Give two examples and explain why they cannot be expressed as a simple fraction."

}

[3] {

"tag": "\${QuestionTypes.MultipleChoice}",

"question": {

"question": "Which of the following numbers is an example of an irrational number?",

"options": {

"A": "1/2",

"B": " $\sqrt{2}$ ",

"C": "3.14",

"D": "-7"

},

"answer": "B"

}

}

[4] {

"tag": "\${QuestionTypes.FreeResponse}",

"question": "How are rational and irrational numbers added or multiplied? Can the result always be a rational number? Explain your answer."

}

[5] {

"tag": "\${QuestionTypes.MultipleChoice}",

"question": {

"question": "If you add a rational number and an irrational number, what is the result always guaranteed to be?",

"options": {

"A": "A rational number",

"B": "An irrational number",

"C": "An integer",

"D": "It depends on the specific numbers"

},

```
"answer": "B"
}

}
```

Example Generate Questions Prompt Response 2:

```
[1] {
  "tag": "${QuestionTypes.FreeResponse}",
  "question": "What are rational numbers? How can you represent them in different ways (e.g., fraction, decimal)?"
}
```

```
[2] {
  "tag": "${QuestionTypes.FreeResponse}",
  "question": "Define irrational numbers. Can you give some examples of irrational numbers and explain why they are considered irrational?"
}
```

```
[3] {
  "tag": "${QuestionTypes.FreeResponse}",
  "question": "Consider the numbers  $\frac{3}{4}$ ,  $\sqrt{2}$ , and 7. Classify each number as rational or irrational and justify your reasoning."
}
```

```
[4] {
  "tag": "${QuestionTypes.FreeResponse}",
  "question": "Explain how adding or multiplying rational and irrational numbers affects the result. Can the result be rational in all cases?"
}
```

```
[5] {
  "tag": "${QuestionTypes.MultipleChoice}",
  "question": {
    "question": "Which of the following statements about rational and irrational numbers is ALWAYS true?",
    "options": {
      "A": "The product of two rational numbers is always rational.",
      "B": "The sum of a rational number and an irrational number is always rational.",
      "C": "The quotient of two irrational numbers is always irrational.",
      "D": "All irrational numbers are negative."
    }
  },
}
```

```
"answer": "A"
}

}
```

Example Generate Questions Prompt Response 3:

**Questions:**

[1]{"tag": "\${QuestionTypes.FreeResponse}", "question": "What are rational numbers? Explain how they can be represented in different ways."}

[2]{"tag": "\${QuestionTypes.FreeResponse}", "question": "Define irrational numbers. How can you differentiate them from rational numbers based on their decimal representation?"}

[3]{"tag": "\${QuestionTypes.MultipleChoice}", "question": "Which of the following numbers is an example of an irrational number?\n A.  $\frac{3}{4}$ \n B.  $\sqrt{2}$ \n C. 7.5\n D. -22", "answer": "B"}

[4]{"tag": "\${QuestionTypes.FreeResponse}", "question": "Give an example of a real-world scenario where understanding rational and irrational numbers is important."}

[5]{"tag": "\${QuestionTypes.MultipleChoice}", "question": "If you add a rational number and an irrational number, what will the result always be?\n A. Rational number\n B. Irrational number\n C. It depends on the specific numbers\n D. Not possible to add", "answer": "B"}