AE 666: Adaptive and Learning Control Systems

Project Report

Simulation and Adaptive Control of a Hypersonic Glide Vehicle

Submitted By:

Group 21

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1 Problem Statement

1.1 Introduction

Hypersonic Glide Vehicles (HGVs) are strategic assets designed to fly at high altitudes and speeds, enhancing the performance of conventional ballistic missile systems. Their unique capability to perform evasive terminal maneuvers helps in evading anti-missile systems. The dynamics of these vehicles are modeled using 3 Degrees of Freedom (3DoF) point mass equations, incorporating atmospheric drag, lift, and gravitational forces.

1.2 3DoF Point Mass Equations

The following are the 3DoF point mass equations for a hypersonic vehicle:

$$\dot{H} = V \sin(\gamma) \tag{1}$$

$$\dot{R} = V \cos(\gamma) \frac{R_0}{R_0 + H} \tag{2}$$

$$\dot{V} = -\frac{D}{m} - g \sin(\gamma) \tag{3}$$

$$\dot{\gamma} = \frac{L}{mV} - \frac{g \cos(\gamma)}{V} + V \cos(\gamma) \frac{1}{R_0 + H} \tag{4}$$

Where:

- H is altitude (in meters).
- R is downrange (in meters).
- V is speed (in meters per second).
- γ is the path angle.
- m is constant mass (in kilograms).
- g is the acceleration due to gravity.
- R_0 is the average radius of earth and its value is 6356.766 km.
- L is lift and D is drag, which are functions of density of atmosphere, velocity, and aerodynamic coefficients.

Uncertainties like a variation in payload mass, moment of inertia, aerodynamic force may change vehicles' dynamic and unpredictable changes in the flying environment that significantly affect the stability and tracking performance and may lead to increment in the modelling error. The parameter velocity is measured by inertial measurement unit which is associated with many unknown bias and noises. The drag and lift models are assumed as the following equations. In the above set of equations lift force L is function aerodynamic parameters which in turn is function of angle of attack, this in turn is an input parameter in the above state equations (Point mass equation of motion).

$$L = C_L \cdot \frac{1}{2}\rho V^2 \cdot S \tag{5}$$
$$D = C_D \cdot \frac{1}{2}\rho V^2 \cdot S \tag{6}$$
$$C_L = C_{L0}(Ma) + C_L'(Ma) \cdot \alpha \tag{7}$$
$$C_D = C_{D0}(Ma) + K(Ma) \cdot C_L^2 \tag{8}$$

The objective is to design adaptive control law in the presence of above mentioned uncertainties.

2 Control Law

2.1 Requirements

The adaptive control law should take the angle of attack of the vehicle and track a target flight path angle despite uncertainties in aerodynamic properties, mass of vehicle and noise in measurements.

2.2 Methodology

A Model Reference Adaptive Controller (MRAC) is used to track a desired flight path angle. The control law is :

$$\alpha = -\left(\theta_1 e + \theta_2 \dot{\gamma} + k_i \int_0^t e dt\right)$$

Where,

$$\alpha \;\; is \;\; the \;\; angle \;\; of \;\; attack$$

$$e = \gamma - \gamma_{ref}$$

$$\theta_1, \theta_2 \;\; are \;\; parameter \;\; gains$$

$$k_i \;\; is \;\; integral \;\; gain$$

And the update law for the parameters is :

$$\dot{\theta_1} = \lambda e$$

$$\dot{\theta_2} = \lambda e \dot{\gamma}_{ref}$$

Where,

$$\lambda \;\; is \;\; the \;\; adaptation \;\; rate$$

$$\gamma_{ref} \;\; is \;\; the \;\; reference \;\; flight \;\; path \;\; angle$$

2.3 Simulation

The following steps are followed for the simulation process :

2.3.1 Define vehicle dynamics

A class *HypersonicVehicle* is created which models the 3DoF dynamics of the system using the equaitons given in the problem statement. It calculates lift, drag and returns the time derivative of the state vector.

2.3.2 Define Reference Model

A class *ReferenceModel* is created which gives the controller a reference state to track. The reference model tracks the commanded state like a second order system, having a user-defined natural frequency and damping ratio.

2.3.3 Define Adaptive Controller

A class *AdaptiveController* is created for the controller, which uses a modified MRAC controller. Alongside two parameters associated with error and rate of error of the flight path angle, an integrator term is also present to remove any steady state error due to unfavorable dynamics which may occur due to uncertainties in the mass and aerodynamic properties of the vehicle.

2.3.4 Define Command Profile

A time varying function *gamma_cmd_function(t)* can be chosen for the commanded flight path which is to be tracked.

2.3.5 Set Simulation Parameters

Various settings like total simulation time, simulation time step, initial vehicle state, etc.

2.3.6 Run Simulation

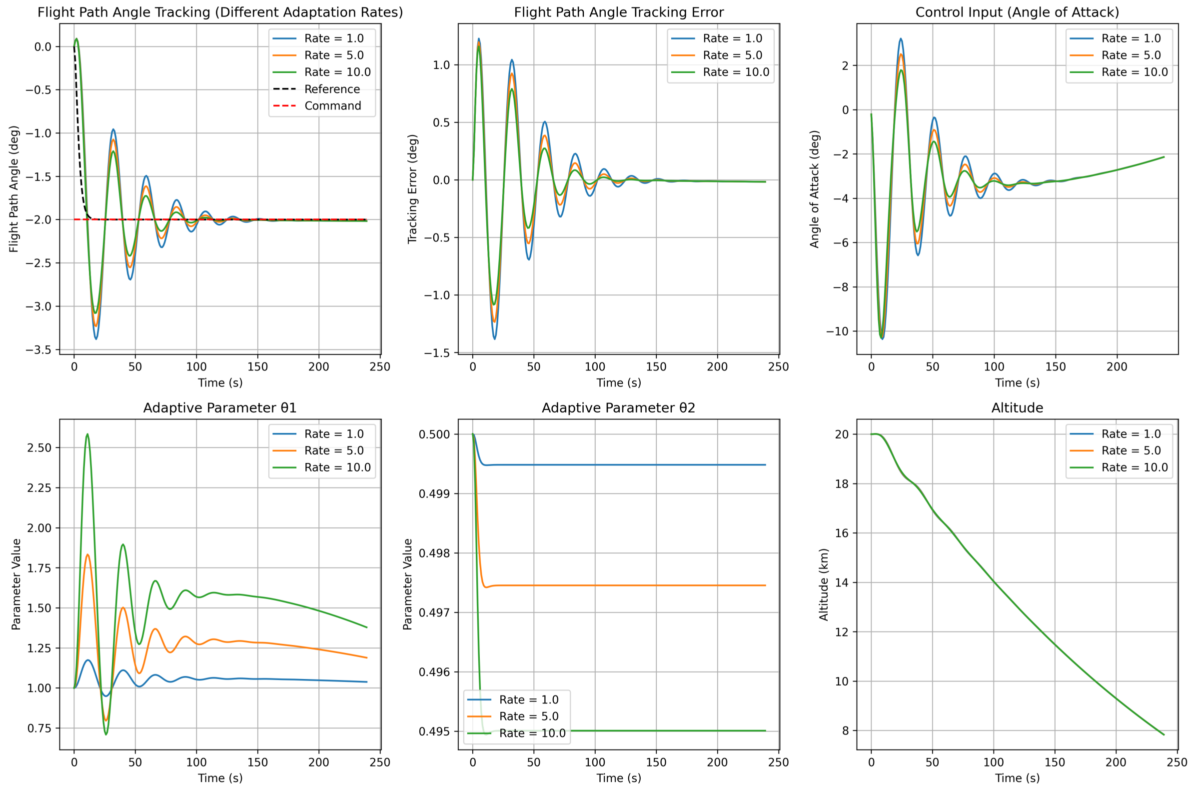
The simulation is run for 4 test cases:

1. Varying adaptation rate and no uncertainties
2. Varying mass uncertainty at constant adaptation rate
3. Varying aerodynamic uncertainty at constant adaptation rate
4. Varying mass and aerodynamic uncertainties at constant adaptation rate

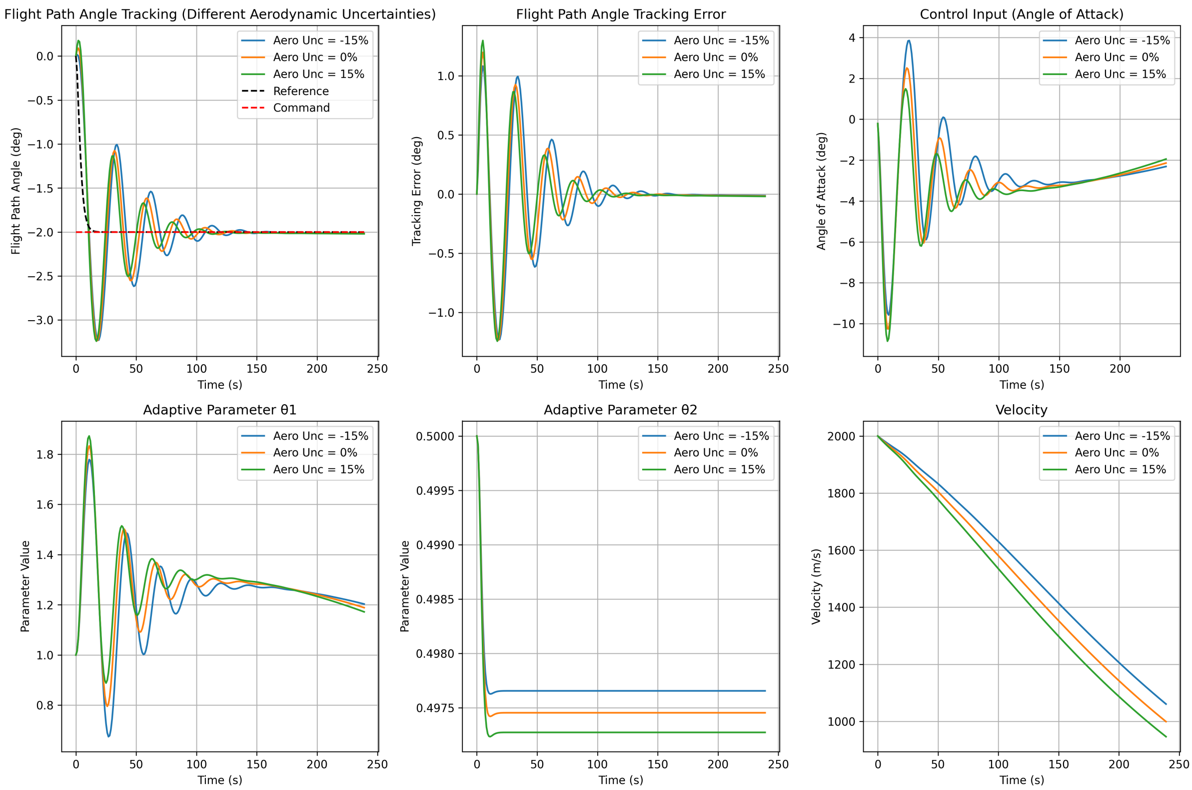
3 Numerical Examples

3.1 Constant Commanded Flight Path Angle

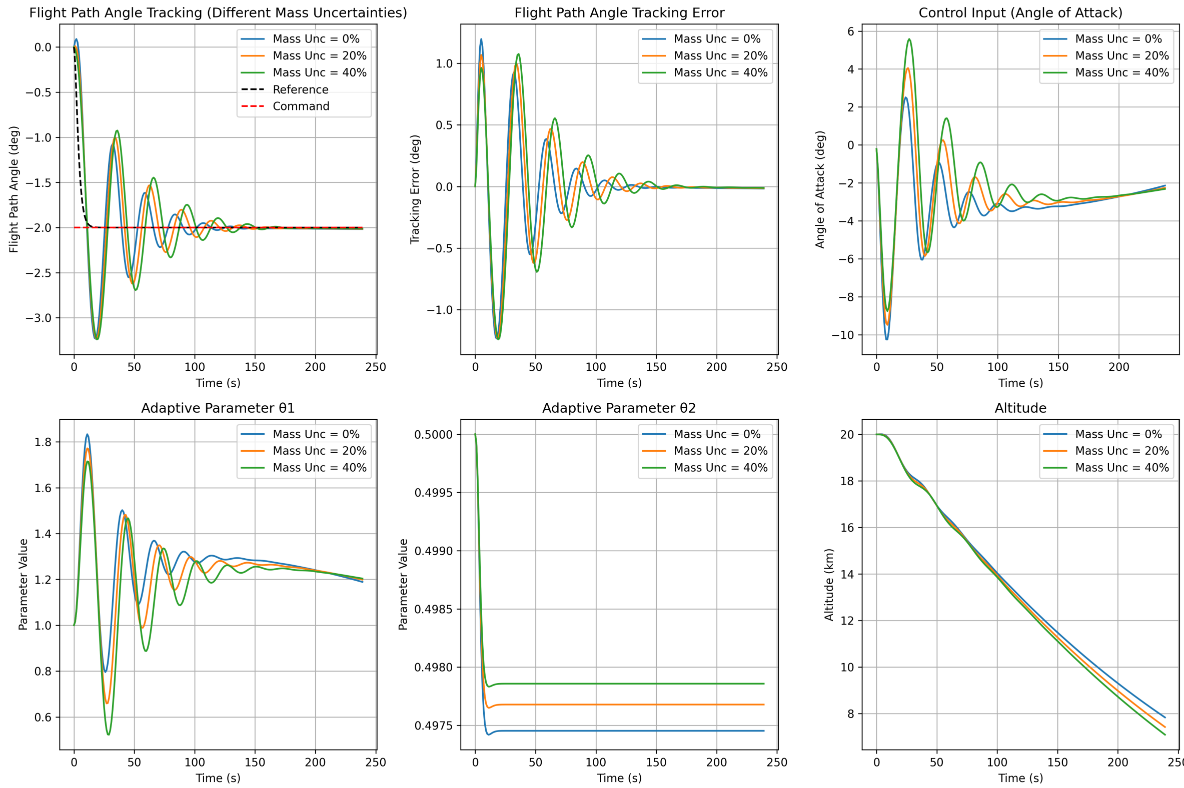
In this case study, a constant flight path angle of -2° is commanded.



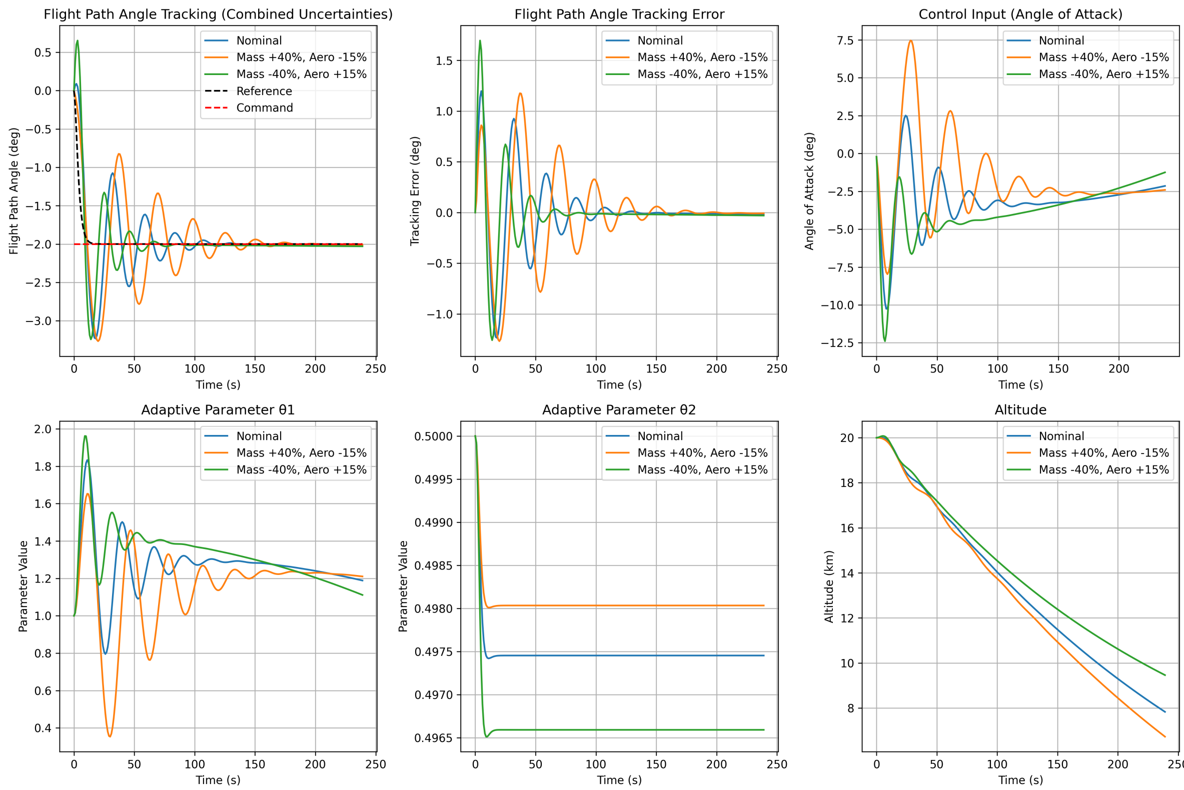
Simulation with varying adaptive gains and no uncertainties



Simulation with varying aerodynamic uncertainties and constant adaptation rate



Simulation with varying mass uncertainties and constant adaptation rate

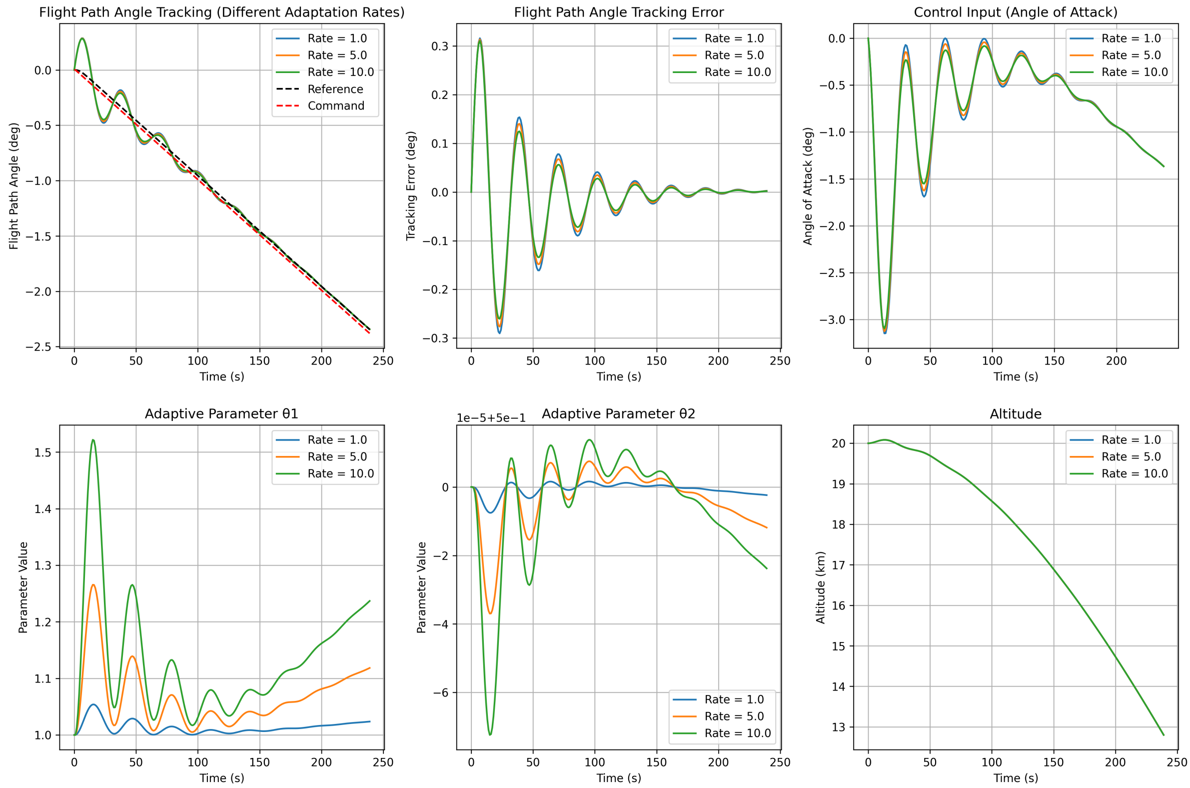


Simulation with varying aerodynamic and mass uncertainties and constant adaptation rate

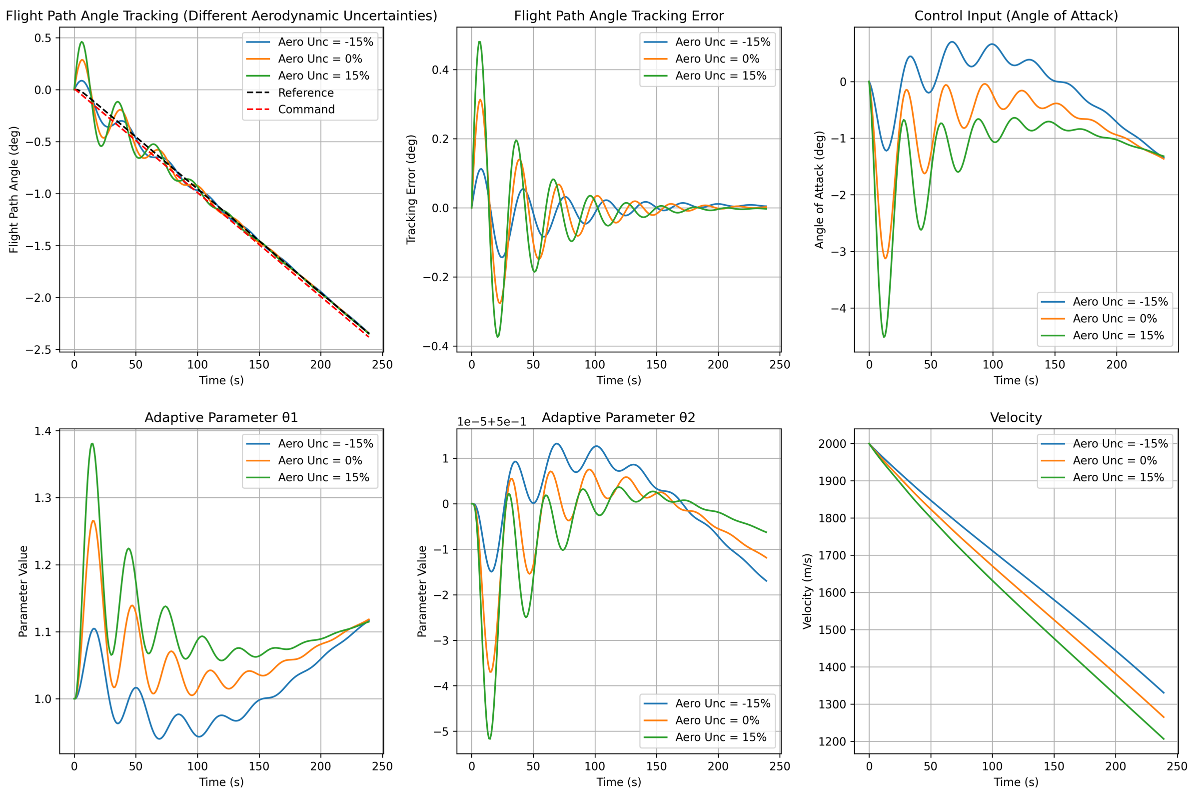
The reference model tracks the commanded model, and the system soon follows and settles at the desired steady-state value in all cases.

3.2 Ramped Flight Path Angle

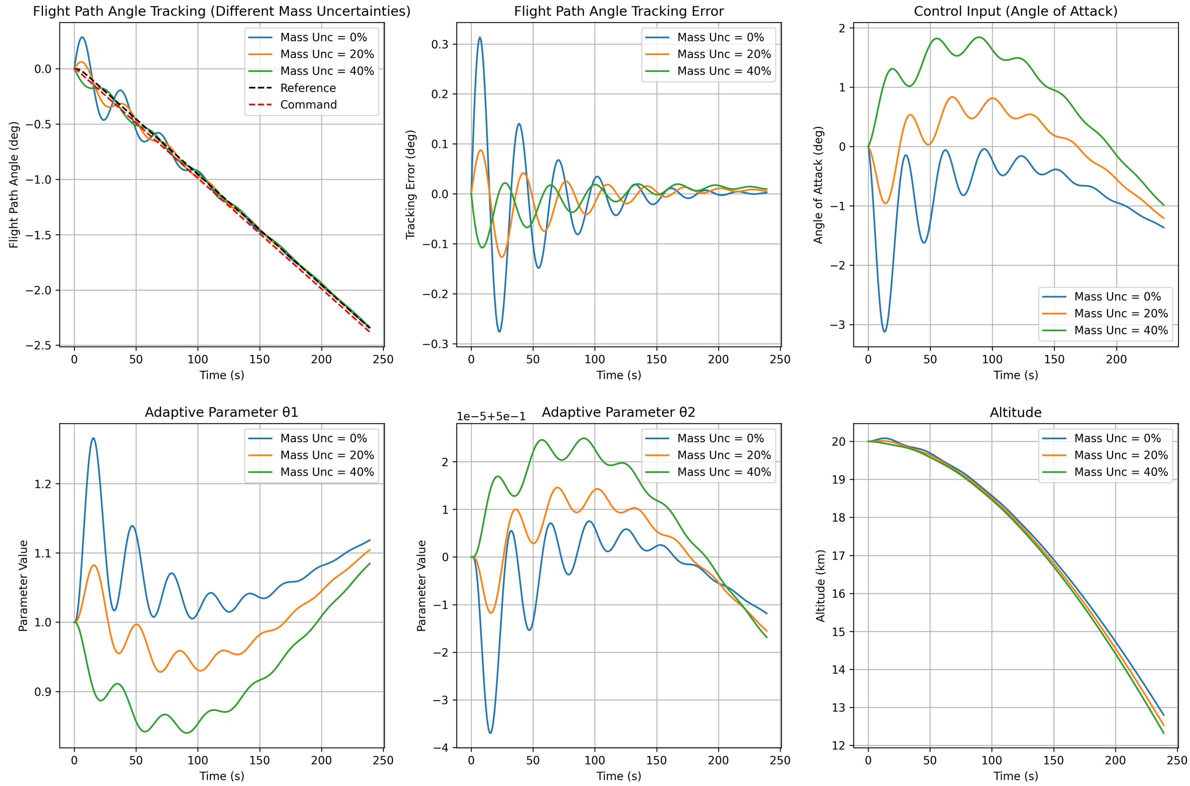
In this case study, a flight path angle of (-t/100)° is commanded.



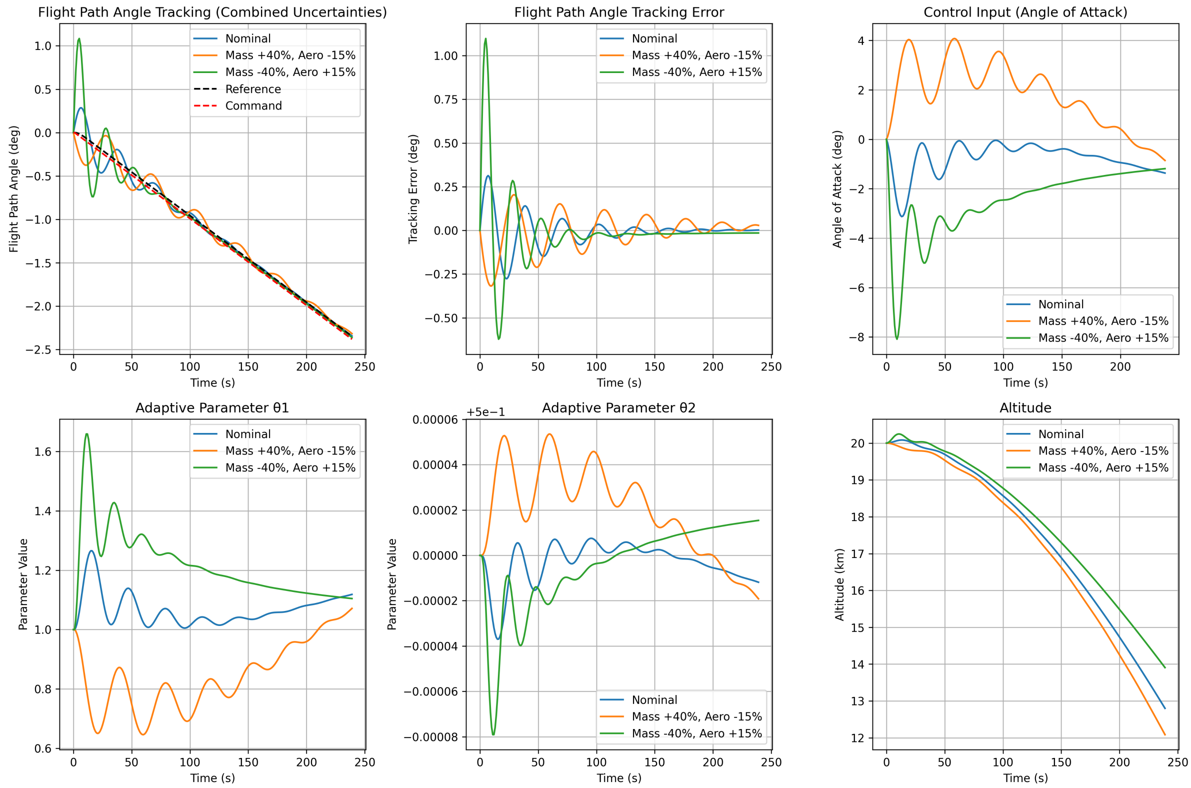
Simulation with varying adaptive gains and no uncertainties



Simulation with varying aerodynamic uncertainties and constant adaptation rate



Simulation with varying mass uncertainties and constant adaptation rate

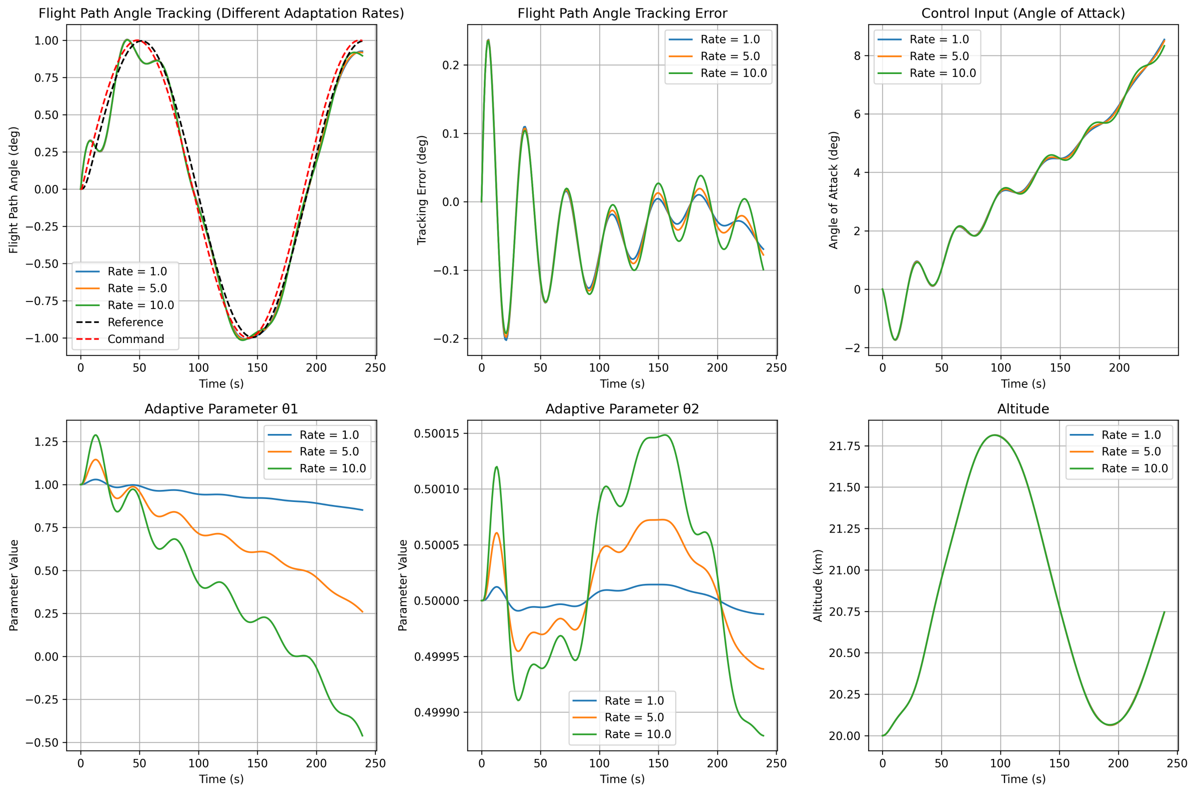


Simulation with varying aerodynamic and mass uncertainties and constant adaptation rate

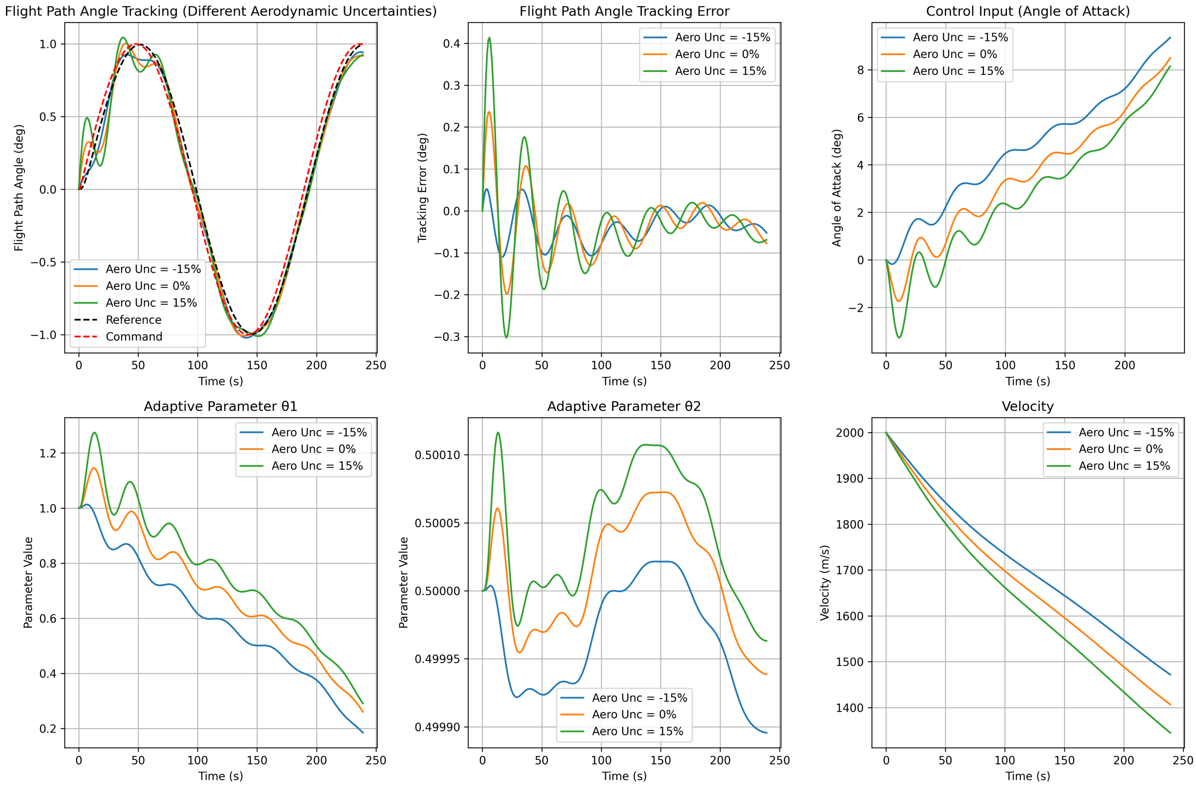
Since the reference model tracks the commanded model as a second order system, a constant steady state error is maintained between the reference and commanded fight path angles. The system successfully tracks the reference flight path angle, but obviously also ends up sharing the same steady state error as the reference model to the commanded model.

3.3 Sinusoidal Flight Path Angle

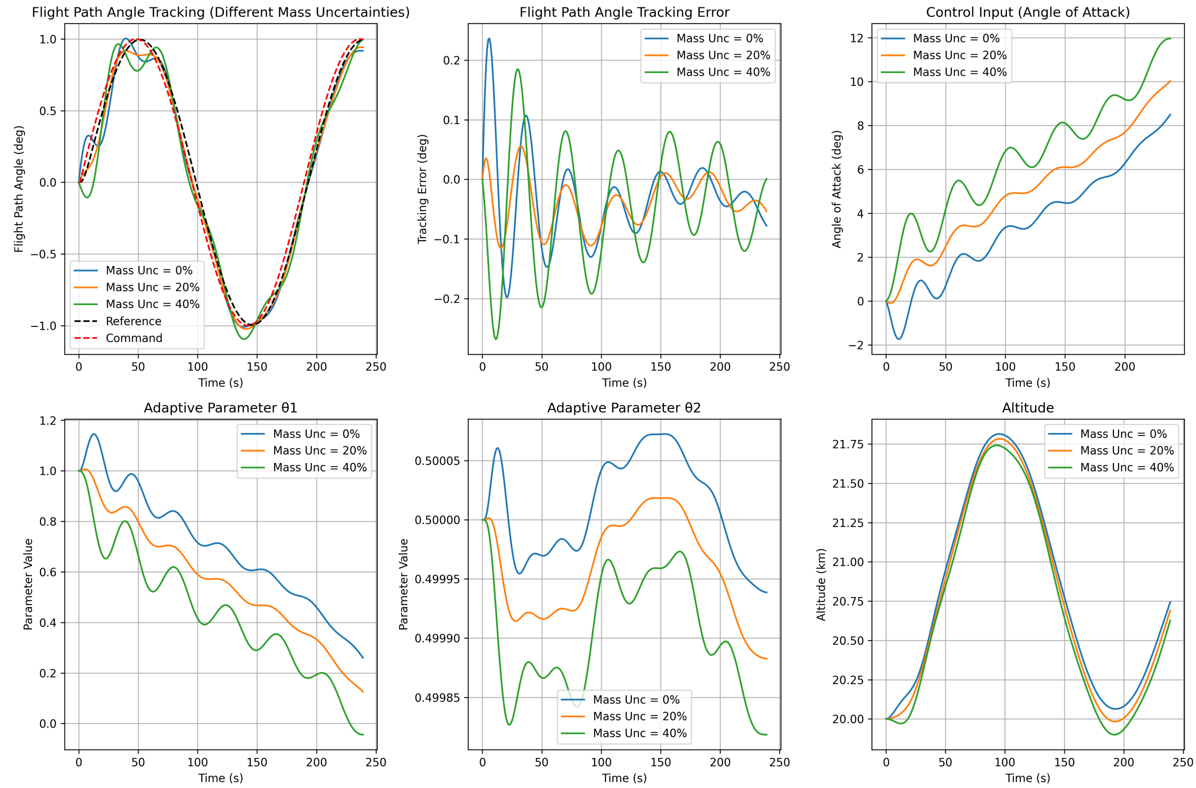
In this case study, a flight path angle of sin((t/30)°) is commanded.



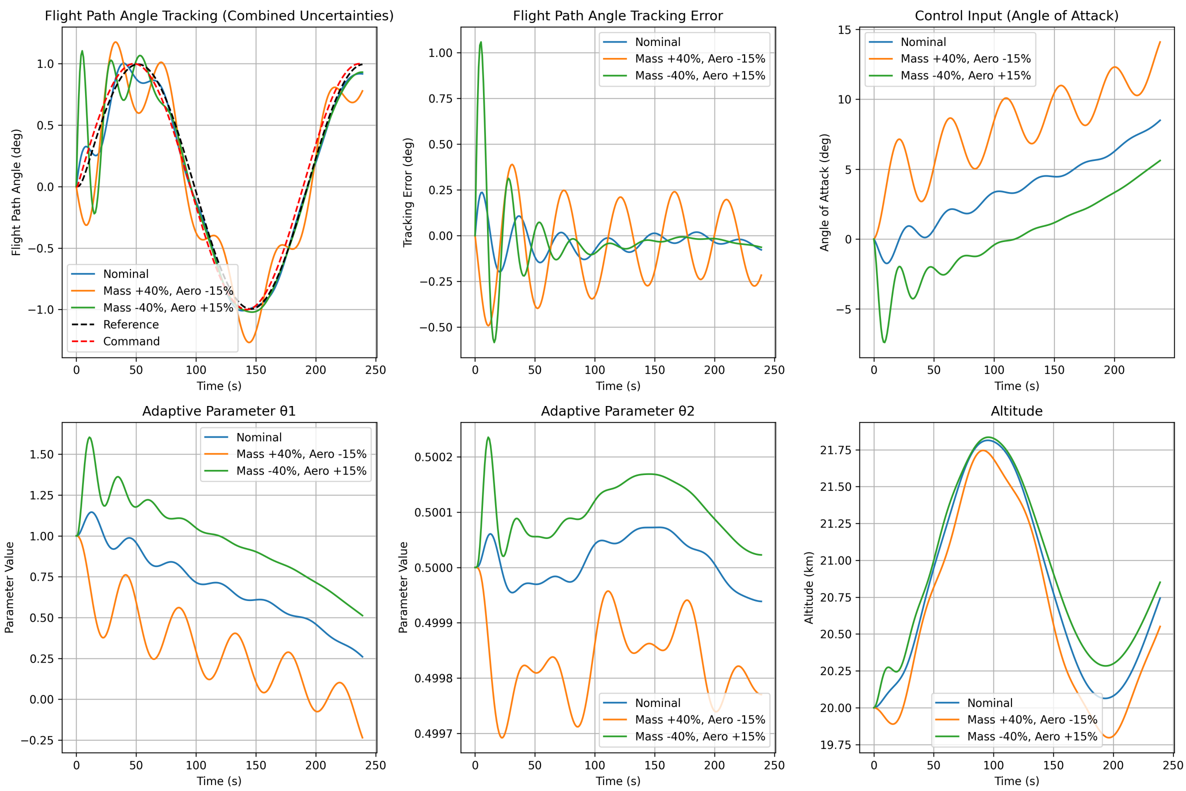
Simulation with varying adaptive gains and no uncertainties



Simulation with varying aerodynamic uncertainties and constant adaptation rate



Simulation with varying mass uncertainties and constant adaptation rate

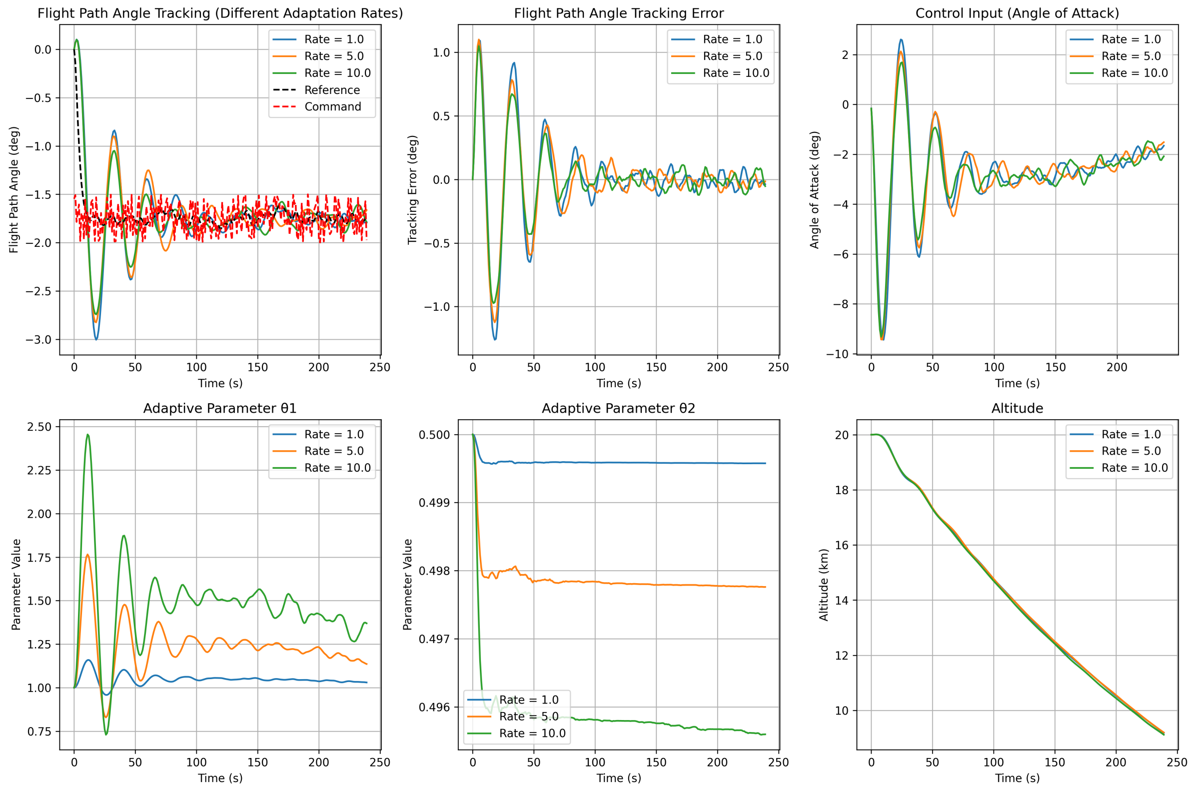


Simulation with varying aerodynamic and mass uncertainties and constant adaptation rate

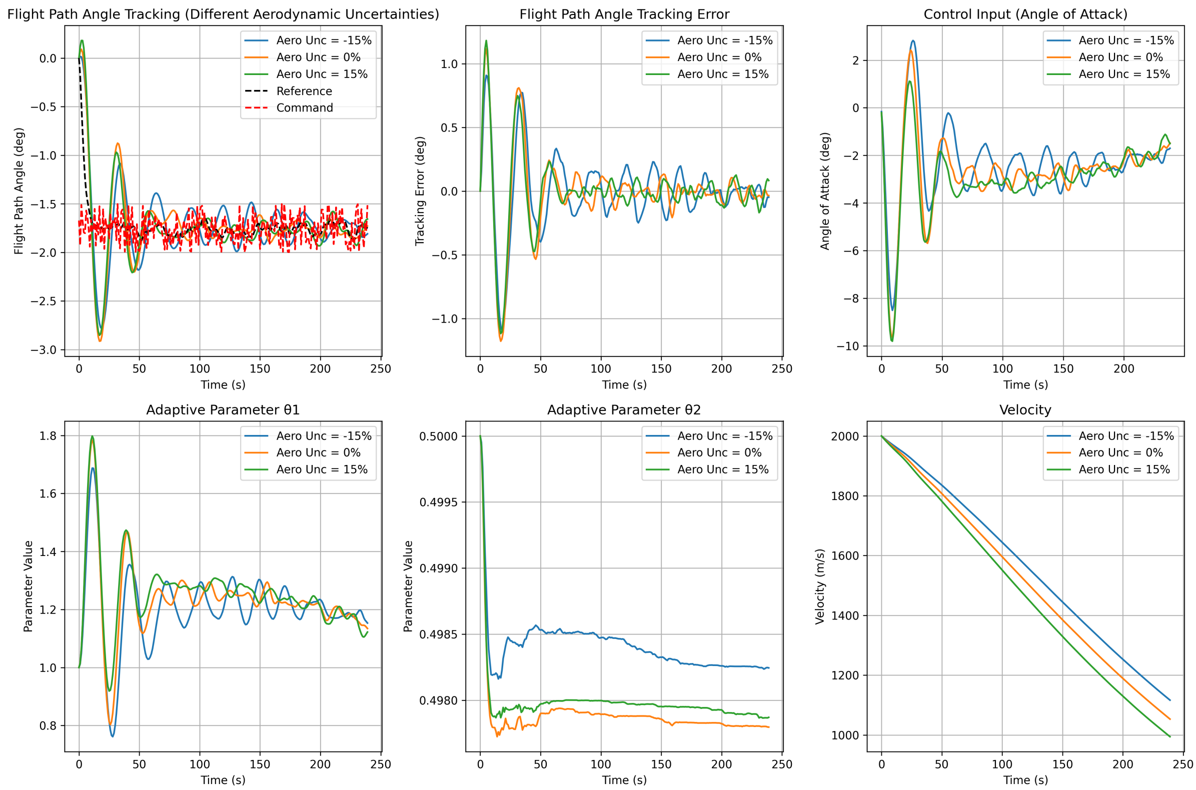
Once again, the reference model does not perfectly track the commanded angle, but the system tracks the reference model fairly well. In case of no uncertainties, the system converges to and tracks the reference model. In case of uncertainties, the system oscillates about the reference model values but still tracks them, which is expected.

3.4 Effect of Noise

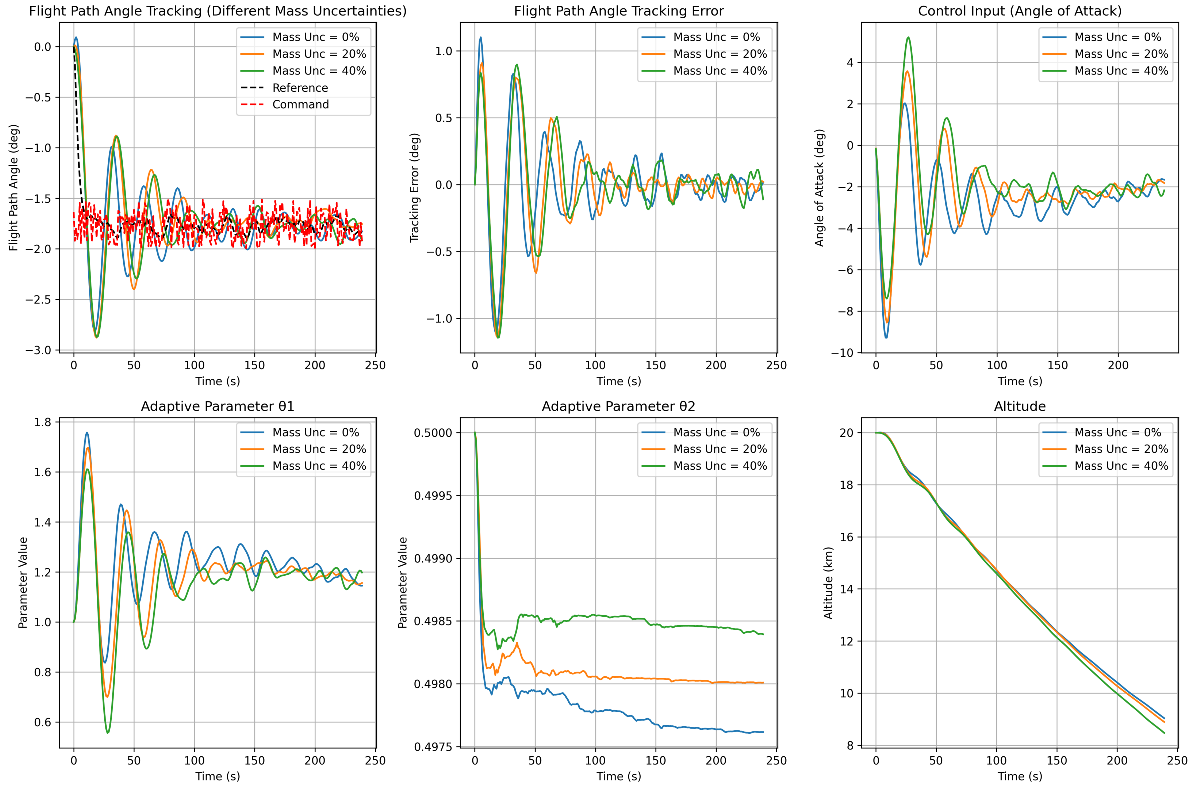
In this case study, the commanded flight path angle is -2°, but has a significant amount of high-frequency noise. The objective is to find the response of the controller to high-frequency signals and ensure stability.



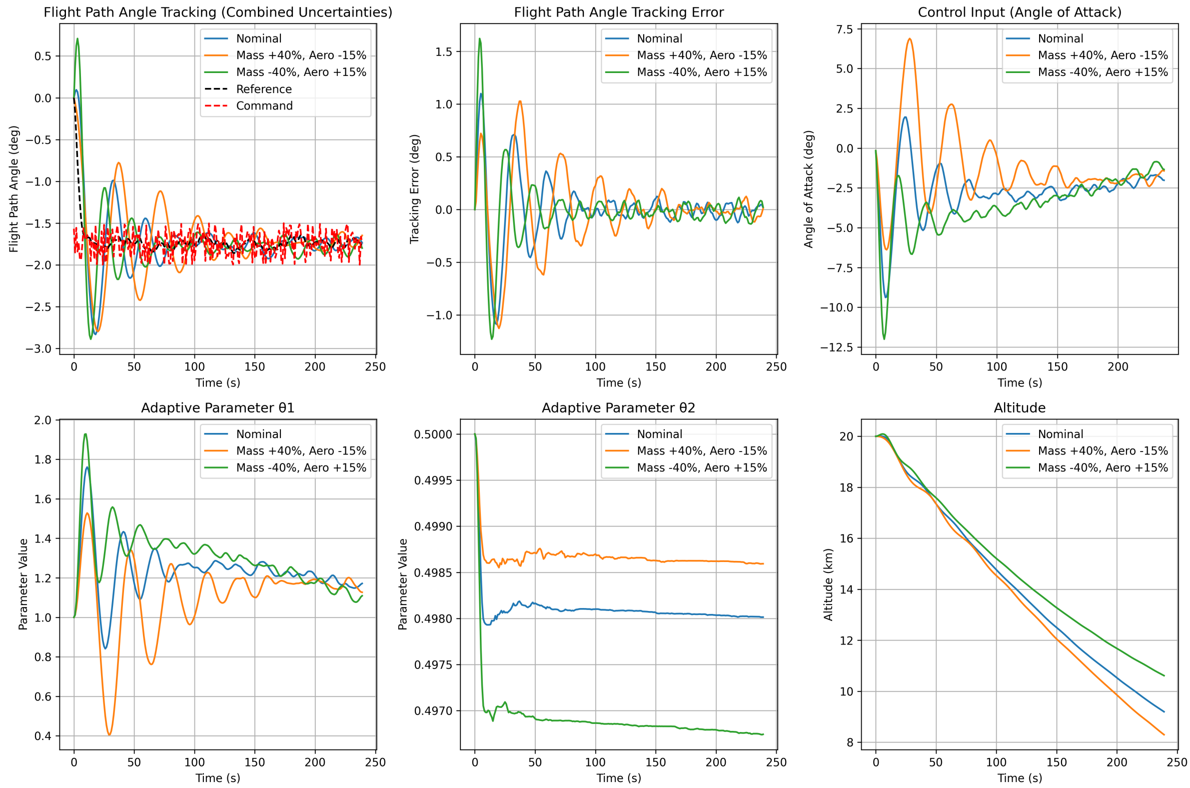
Simulation with varying adaptive gains and no uncertainties



Simulation with varying aerodynamic uncertainties and constant adaptation rate



Simulation with varying mass uncertainties and constant adaptation rate



Simulation with varying aerodynamic and mass uncertainties and constant adaptation rate

As we can see, the controller is not significantly effected by the addition of high-frequency noise. This is because the system is actually tracking the reference model, and not the commanded model. Since the reference model is tracking the commanded angle as a second order system, it filters out most of the high-frequency component.

4 Conclusions

4.1 Observations

- The Model Reference Adaptive Control (MRAC) law was able to track a variety of reference flight path angle profiles (constant, ramped, and sinusoidal) effectively, even in the presence of significant uncertainties in mass and aerodynamic coefficients.
- The addition of an integral term to the control law improved steady-state performance and reduced the residual error, particularly for ramp inputs where the reference model itself had a steady-state error.
- Simulations showed that increasing the adaptation rate λ resulted in faster convergence and improved tracking accuracy, though very high values can lead to chattering and instability in some cases.
- The MRAC controller was resilient to high-frequency noise in the commanded signal, as the reference model acted as a low-pass filter, preventing noise from corrupting the actual tracking behavior.

4.2 Limitations

- The MRAC formulation assumes ideal parameter convergence without projection or normalization, which may not hold in a real-world scenario with unmodeled dynamics or actuator saturation.
- The adaptation laws used are simple gradient-based updates and do not include robustness-enhancing techniques like σ-modification or dead zones, which could improve stability margins.
- Only scalar gains (θ_1, θ_2) were adapted. A more complete adaptive law might require estimation of full aerodynamic model parameters.
- The simulations assume perfect state measurements (except velocity), while in practice, measurements (like γ and $\dot{\gamma}$) may be noisy or delayed.
- The reference model parameters (natural frequency and damping) were tuned manually and are not adapted or optimized online.

4.3 Scope for Future Work

- Incorporating robust adaptive laws such as σ-modification, projection algorithms, or composite adaptation to prevent parameter drift and ensure boundedness in the presence of unmodeled dynamics or disturbances.
- Extension of the controller to a full 6-DoF model to account for attitude dynamics and control surface effects.
- Integration with state estimation techniques (e.g., Kalman filter, complementary filter) to deal with noisy or biased measurements, especially for γ and $\dot{\gamma}$.

- Investigating neural network or machine learning-based adaptive elements (e.g., reinforcement learning or adaptive critics) to improve performance over a wider range of uncertainties.