Summer School in Quantitative Fisheries Stock Assessment

Day 4: An introduction to Statistical Catch-at-Age models

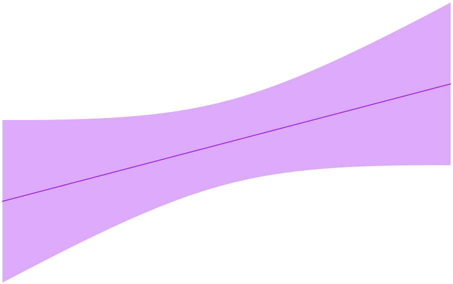
Cóilín Minto, Chato Osio, Alessandro Orio, Alessandro Ligas, Alessandro Mannini

July 19, 2018

Outline

Goal

Build a fully statistical model to estimate the parameters of the stock assessment, including uncertainty of estimates



Cohort

Cohort progression

$$
\begin{array}{cc}
 & \text{Age} \longrightarrow \\
\rotatebox{-90}{Time $\longrightarrow$} &
\begin{matrix}
N_{a1,t1} & N_{a2,t1} & N_{a3,t1} & N_{a4,t1} & N_{a5,t1} \\
N_{a1,t2} & N_{a2,t2} & N_{a3,t2} & N_{a4,t2} & N_{a5,t2} \\
N_{a1,t3} & N_{a2,t3} & N_{a3,t3} & N_{a4,t3} & N_{a5,t3} \\
N_{a1,t4} & N_{a2,t4} & N_{a3,t4} & N_{a4,t4} & N_{a5,t4} \\
N_{a1,t5} & N_{a2,t5} & N_{a3,t5} & N_{a4,t5} & N_{a5,t5}
\end{matrix}
\end{array}
$$

Yesterday with VPA & XSA we worked backwards through the cohort from the terminal value

Cohort

Cohort progression

$$\begin{array}{cccccc}
 & & & \text{Age} \longrightarrow & & \\
 & N_{a1,t1} & N_{a2,t1} & N_{a3,t1} & N_{a4,t1} & N_{a5,t1} \\
 & N_{a1,t2} & N_{a2,t2} & N_{a3,t2} & N_{a4,t2} & N_{a5,t2} \\
\text{Time} \downarrow & N_{a1,t3} & N_{a2,t3} & N_{a3,t3} & N_{a4,t3} & N_{a5,t3} \\
 & N_{a1,t4} & N_{a2,t4} & N_{a3,t4} & N_{a4,t4} & N_{a5,t4} \\
 & N_{a1,t5} & N_{a2,t5} & N_{a3,t5} & N_{a4,t5} & N_{a5,t5}
\end{array}$$

Today we will work forwards

Cohort

Cohort progression

$$
\begin{array}{c|ccccc}
 & \multicolumn{5}{c}{\text{Age} \longrightarrow} \\
\hline
 & N_{a1,t1} & N_{a2,t1} & N_{a3,t1} & N_{a4,t1} & N_{a5,t1} \\
 & N_{a1,t2} & N_{a2,t2} & N_{a3,t2} & N_{a4,t2} & N_{a5,t2} \\
\text{Time} \downarrow & N_{a1,t3} & N_{a2,t3} & N_{a3,t3} & N_{a4,t3} & N_{a5,t3} \\
 & N_{a1,t4} & N_{a2,t4} & N_{a3,t4} & N_{a4,t4} & N_{a5,t4} \\
 & N_{a1,t5} & N_{a2,t5} & N_{a3,t5} & N_{a4,t5} & N_{a5,t5}
\end{array}
$$

$$N_{a+1,t+1} = N_{a,t} e^{-(F_{a,t}+M_{a,t})}$$

Starting a cohort

How do we start the cohorts?

$$
\begin{array}{c|ccccc}
 & \multicolumn{5}{c}{\text{Age} \longrightarrow} \\
\hline
\text{Time} \downarrow & N_{a1,t1} & N_{a2,t1} & N_{a3,t1} & N_{a4,t1} & N_{a5,t1} \\
 & N_{a1,t2} & N_{a2,t2} & N_{a3,t2} & N_{a4,t2} & N_{a5,t2} \\
 & N_{a1,t3} & N_{a2,t3} & N_{a3,t3} & N_{a4,t3} & N_{a5,t3} \\
 & N_{a1,t4} & N_{a2,t4} & N_{a3,t4} & N_{a4,t4} & N_{a5,t4} \\
 & N_{a1,t5} & N_{a2,t5} & N_{a3,t5} & N_{a4,t5} & N_{a5,t5}
\end{array}
$$

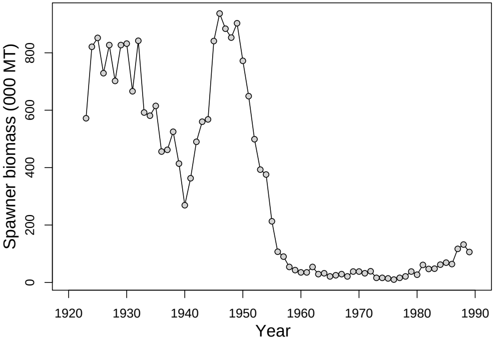
Starting a cohort

Need to relate $N_{a1,t}$ to N_{t-a1}, that is, we need to relate the number of new individuals to the population (spawners, spawning potential) when they were spawned.

That's called recruitment and in age-structured populations, it's where compensation enters.

Historical interlude

East Anglia herring collapse: recruitment overfishing



Enter: Cushing, Ricker, Beverton, and Holt

Theory of recruitment predecessor

THE BALANCE OF ANIMAL POPULATIONS

BY A. J. NICHOLSON, D.Sc.

(Division of Economic Entomology, Commonwealth Council for Scientific and Industrial Research, Canberra, Australia.)

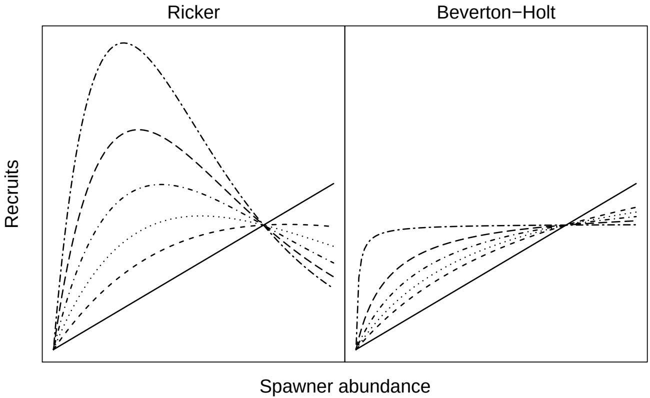
(With eleven Figures in the Text.)

Alexander Nicholson

- Main thesis: competition as the agent of population regulation

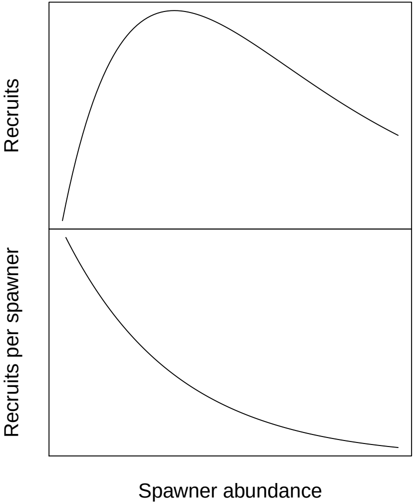
Theory of recruitment

Population regulation: $R = \alpha S f(S)$
Where $f(S)$ is a density-dependent function



Theory of recruitment

All have compensation at their core

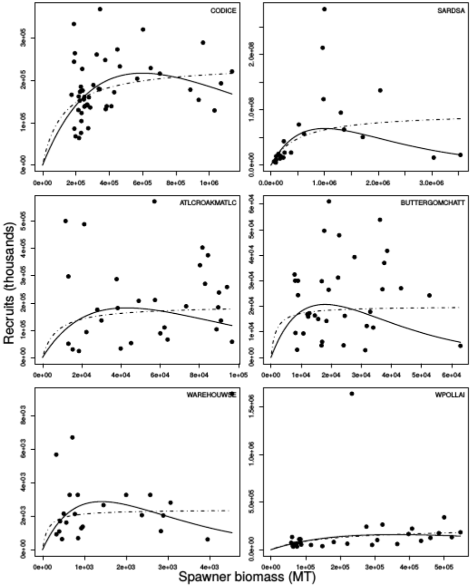


Theory of recruitment

By specifying a stock-recruitment relationship, we allow our theoretical model to replenish itself
Population described by

$$N_{a+1,t+1} = N_{a,t} e^{-(F_{a,t}+M_{a,t})}$$
$$N_{a1,t} = aS_{t-a1} f(S_{t-a1})$$

In reality, we have . . .



The 'good'

The 'bad'

The 'ugly'

Starting a cohort

$$
\begin{array}{c|ccccc}
 & & & \text{Age} \longrightarrow & & \\
\hline
\text{Time} \downarrow & N_{a1,t1} & N_{a2,t1} & N_{a3,t1} & N_{a4,t1} & N_{a5,t1} \\
 & N_{a1,t2} & N_{a2,t2} & N_{a3,t2} & N_{a4,t2} & N_{a5,t2} \\
 & N_{a1,t3} & N_{a2,t3} & N_{a3,t3} & N_{a4,t3} & N_{a5,t3} \\
 & N_{a1,t4} & N_{a2,t4} & N_{a3,t4} & N_{a4,t4} & N_{a5,t4} \\
 & N_{a1,t5} & N_{a2,t5} & N_{a3,t5} & N_{a4,t5} & N_{a5,t5}
\end{array}
$$

Here 5 parameters to be estimated[1]

[1] Or a smooth function, e.g., in a4a

Year one

We also need to start all ages in the first year

$$\begin{array}{cc} & \text{Age} \longrightarrow \\ \text{Time} \downarrow & \begin{array}{ccccc} N_{a1,t1} & N_{a2,t1} & N_{a3,t1} & N_{a4,t1} & N_{a5,t1} \\ N_{a1,t2} & N_{a2,t2} & N_{a3,t2} & N_{a4,t2} & N_{a5,t2} \\ N_{a1,t3} & N_{a2,t3} & N_{a3,t3} & N_{a4,t3} & N_{a5,t3} \\ N_{a1,t4} & N_{a2,t4} & N_{a3,t4} & N_{a4,t4} & N_{a5,t4} \\ N_{a1,t5} & N_{a2,t5} & N_{a3,t5} & N_{a4,t5} & N_{a5,t5} \end{array} \end{array}$$

Year one

We also need to start all ages in the first year

$$
\begin{array}{c|ccccc}
 & \multicolumn{5}{c}{\text{Age} \longrightarrow} \\
\hline
 & N_{a1,t1} & N_{a2,t1} & N_{a3,t1} & N_{a4,t1} & N_{a5,t1} \\
 & N_{a1,t2} & N_{a2,t2} & N_{a3,t2} & N_{a4,t2} & N_{a5,t2} \\
\text{Time} \longrightarrow & N_{a1,t3} & N_{a2,t3} & N_{a3,t3} & N_{a4,t3} & N_{a5,t3} \\
 & N_{a1,t4} & N_{a2,t4} & N_{a3,t4} & N_{a4,t4} & N_{a5,t4} \\
 & N_{a1,t5} & N_{a2,t5} & N_{a3,t5} & N_{a4,t5} & N_{a5,t5}
\end{array}
$$

Here 4 parameters to be estimated

How about Fishing mortality at age?

$$\begin{array}{c|ccccc} & & & \text{Age} & & \\ & F_{a1,t1} & F_{a2,t1} & F_{a3,t1} & F_{a4,t1} & F_{a5,t1} \\ & F_{a1,t2} & F_{a2,t2} & F_{a3,t2} & F_{a4,t2} & F_{a5,t2} \\ \text{Time} \longrightarrow & F_{a1,t3} & F_{a2,t3} & F_{a3,t3} & F_{a4,t3} & F_{a5,t3} \\ & F_{a1,t4} & F_{a2,t4} & F_{a3,t4} & F_{a4,t4} & F_{a5,t4} \\ & F_{a1,t5} & F_{a2,t5} & F_{a3,t5} & F_{a4,t5} & F_{a5,t5} \end{array}$$

How about Fishing mortality at age?

Age

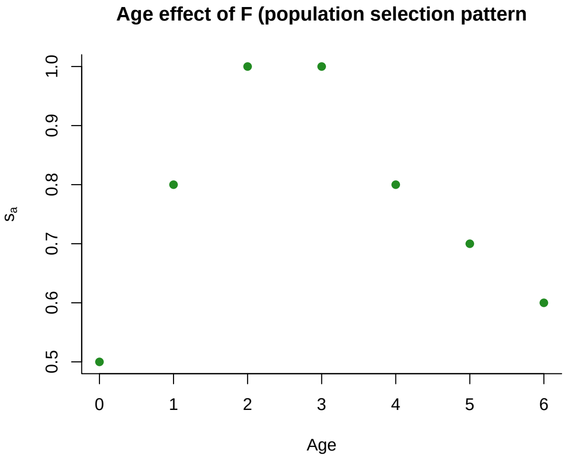
$$
\text{Time} \downarrow \quad
\begin{array}{ccccc}
F_{a1,t1} & F_{a2,t1} & F_{a3,t1} & F_{a4,t1} & F_{a5,t1} \\
F_{a1,t2} & F_{a2,t2} & F_{a3,t2} & F_{a4,t2} & F_{a5,t2} \\
F_{a1,t3} & F_{a2,t3} & F_{a3,t3} & F_{a4,t3} & F_{a5,t3} \\
F_{a1,t4} & F_{a2,t4} & F_{a3,t4} & F_{a4,t4} & F_{a5,t4} \\
F_{a1,t5} & F_{a2,t5} & F_{a3,t5} & F_{a4,t5} & F_{a5,t5}
\end{array}
$$

If all are free requires 25 parameters here (one for each age in each year)
that's a lot of parameters!

How about F?

Need to conserve degrees of freedom in the model but also need the **F** matrix. One approach is a separable model

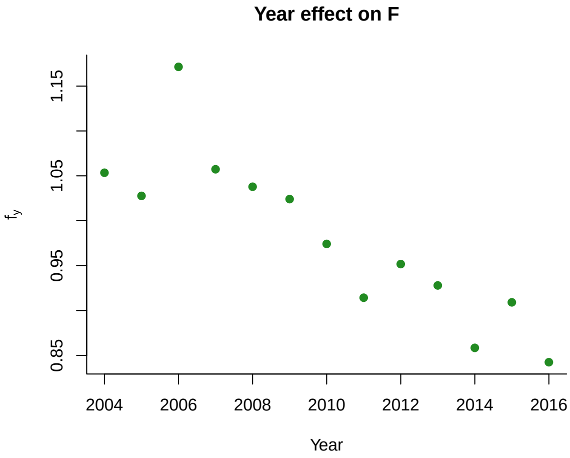
Model the age effects assumed constant across <u>time</u>



How about F?

Need to conserve degrees of freedom in the model but also need the **F** matrix. One approach is a separable model

Model year effects assumed constant across ages



How about F?

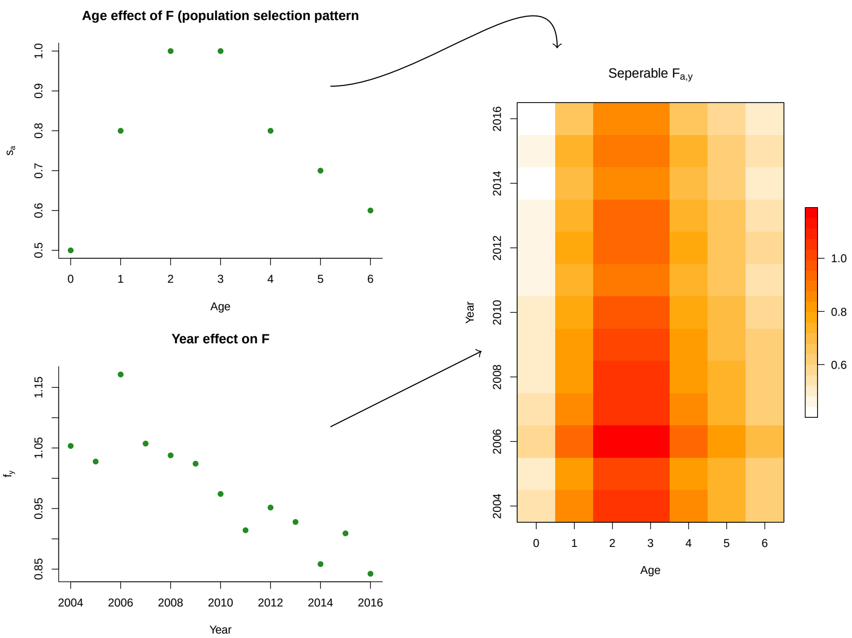
Need to conserve degrees of freedom in the model but also need the **F** matrix. One approach is a separable model

$$\mathbf{F} = \begin{pmatrix} s_{a1} & s_{a2} & s_{a3} & s_{a4} & s_{a5} \\ s_{a1} & s_{a2} & s_{a3} & s_{a4} & s_{a5} \\ s_{a1} & s_{a2} & s_{a3} & s_{a4} & s_{a5} \\ s_{a1} & s_{a2} & s_{a3} & s_{a4} & s_{a5} \\ s_{a1} & s_{a2} & s_{a3} & s_{a4} & s_{a5} \end{pmatrix} \star \begin{pmatrix} f_{t1} & f_{t1} & f_{t1} & f_{t1} & f_{t1} \\ f_{t2} & f_{t2} & f_{t2} & f_{t2} & f_{t2} \\ f_{t3} & f_{t3} & f_{t3} & f_{t3} & f_{t3} \\ f_{t4} & f_{t4} & f_{t4} & f_{t4} & f_{t4} \\ f_{t5} & f_{t5} & f_{t5} & f_{t5} & f_{t5} \end{pmatrix}$$

How about F?

Need to conserve degrees of freedom in the model but also need the **F** matrix. One approach is a separable model

$$\mathbf{F} = \begin{pmatrix} s_{a1}f_{t1} & s_{a2}f_{t1} & s_{a3}f_{t1} & s_{a4}f_{t1} & s_{a5}f_{t1} \\ s_{a1}f_{t2} & s_{a2}f_{t2} & s_{a3}f_{t2} & s_{a4}f_{t2} & s_{a5}f_{t2} \\ s_{a1}f_{t3} & s_{a2}f_{t3} & s_{a3}f_{t3} & s_{a4}f_{t3} & s_{a5}f_{t3} \\ s_{a1}f_{t4} & s_{a2}f_{t4} & s_{a3}f_{t4} & s_{a4}f_{t4} & s_{a5}f_{t4} \\ s_{a1}f_{t5} & s_{a2}f_{t5} & s_{a3}f_{t5} & s_{a4}f_{t5} & s_{a5}f_{t5} \end{pmatrix}$$



Age effect of F (population selection pattern
s_a
Age
Year effect on F
f_y
Year
Seperable $F_{a,y}$
Year
Age

Separable model

So the parameters we have in the model are:

- $N_{1,t=1\ldots T}$ number of recruits
- $N_{a=2\ldots A,t=1}$ numbers in first year
- $s_{a=1\ldots A}$ exploitation pattern (age effects)
- $f_{t=1\ldots T}$ fishing mortality level (year effects)

Predictions

From this set of parameters we can predict $\hat{\mathbf{F}}$ and $\hat{\mathbf{N}}$ from which we can predict catch

$$\hat{C}_{a,t} = \frac{\hat{F}_{a,t}}{\hat{F}_{a,t} + M_{a,t}} \left(1 - e^{-(\hat{F}_{a,t}+M_{a,t})}\right) \hat{N}_{a,t}$$

Or, with the addition of catchability, predict a survey index

$$\hat{I}_{a,t} = q_{a,t} \hat{N}_{a,t}$$

Estimation

We observe $C_{a,t}$, $I_{a,t}$ so in principle we can estimate the parameters by maximising the likelihood of the parameters given the data

$$L(\mathbf{N_{a=1}}, \mathbf{N_{t=1}}, \mathbf{s}, \mathbf{f}, \mathbf{q}, \boldsymbol{\sigma} | \mathbf{C}, \mathbf{I}) = \prod_{a=1}^{A} \prod_{t=1}^{T} (\mathrm{N}(\ln(I_{a,t}) - \ln(\hat{I}_{a,t}), \sigma_i^2) \\ \times \mathrm{N}(\ln(C_{a,t}) - \ln(\hat{C}_{a,t}), \sigma_c^2))$$

We maximise the (log) likelihood to estimate the best fitting parameters of the model given the data!

Can use less parameters in SCA by using smoothers (`a4a`)