Summer School in Quantitative Fisheries Stock Assessment

Day 2: Biomass dynamic models

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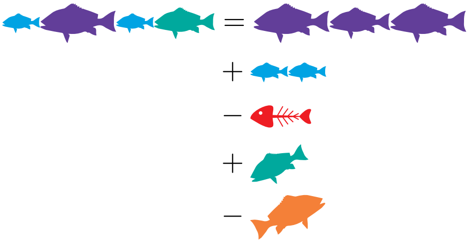
Outline

Demographic variables

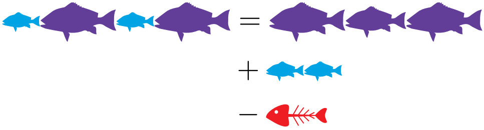


$$N_{t+1} = N_t + B_t - D_t + I_t - E_t$$

where

- N_t is the population number at time t 
- B_t is the number of births 
- D_t is the number of deaths 
- I_t is the number immigrating 
- E_t is the number emigrating 

Closed population assumption

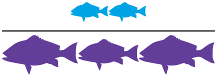


$$N_{t+1} = N_t + B_t - D_t + \cancel{I_t} - \cancel{E_t}$$

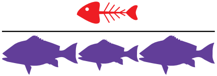
Birth and death rates

Define <u>constant</u> per-capita rates of:

- birth

$b =$ 

- death

$d =$ 

Population growth

Can now write

$$\begin{aligned} N_{t+1} &= N_t + bN_t - dN_t \\ &= N_t + (b - d)N_t \end{aligned}$$

Define <u>constant</u> per-capita population growth rate

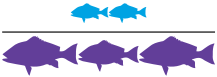
$$r = (b - d)$$

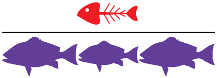
So

$$\begin{aligned} N_{t+1} &= N_t + rN_t \\ &= (1 + r)N_t \end{aligned}$$

Population growth

Here

$b =$  $= \frac{2}{3}$

$d =$  $= \frac{1}{3}$

$$r = (b - d) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

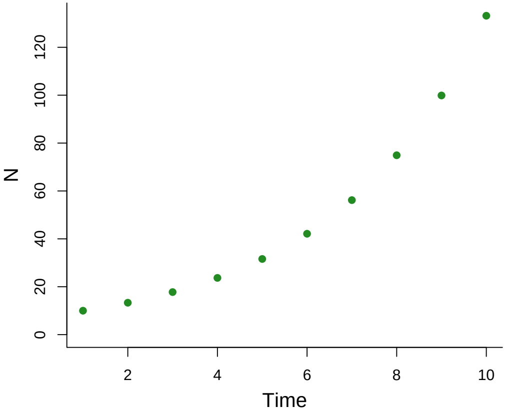
Population growth

$$
\begin{aligned}
N_1 &= N_0\left(1+\frac{1}{3}\right)\\
N_2 &= N_1\left(1+\frac{1}{3}\right) = N_0\left(1+\frac{1}{3}\right)\times\left(1+\frac{1}{3}\right)\\
N_3 &= N_2\left(1+\frac{1}{3}\right) = N_0\left(1+\frac{1}{3}\right)\times\left(1+\frac{1}{3}\right)\times\left(1+\frac{1}{3}\right)\\
&\vdots\\
N_t &= N_0\left(1+\frac{1}{3}\right)^t
\end{aligned}
$$

Geometric population growth

$$N_t = N_0\left(1+r\right)^t$$

Geometric population growth



"Population, when unchecked, increases in a geometrical ratio".

Malthus(1798)

Geometric population growth

Populations will be limited though through competition for limited:

- Food
- Space
- Light
- ...

Need to include effect of competition on the population growth rate, i.e., density dependence

Outline

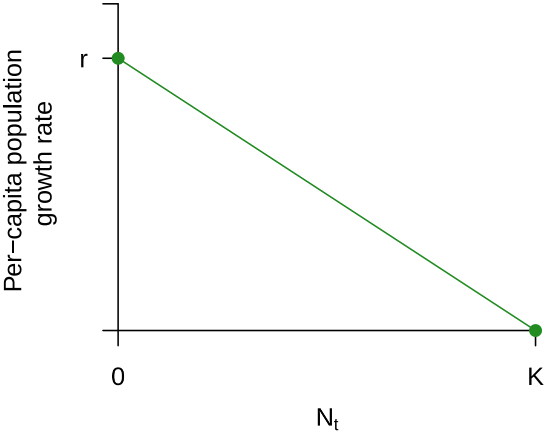
Intraspecific competition

For geometric population growth:

$$\begin{aligned} N_{t+1} &= N_t + rN_t \\ \frac{N_{t+1} - N_t}{N_t} &= r \end{aligned}$$

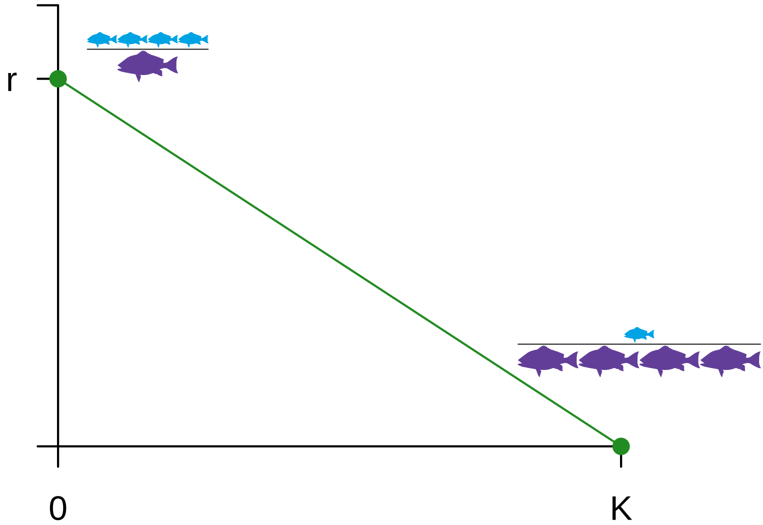
Now let the per-capita growth rate be a **decreasing** function of population size, i.e., not constant

Intraspecific competition

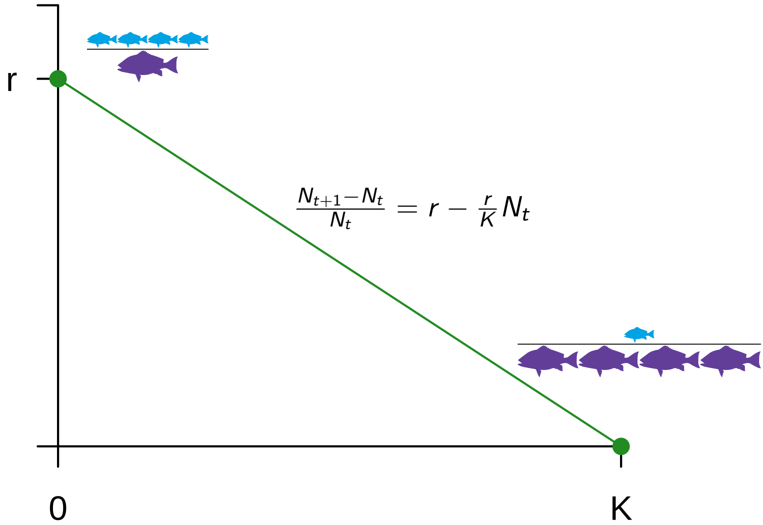


Increase in per-capita rate of population growth compensates for reduced population size

Intraspecific competition



Intraspecific competition



$$\frac{N_{t+1}-N_t}{N_t} = r - \frac{r}{K} N_t$$

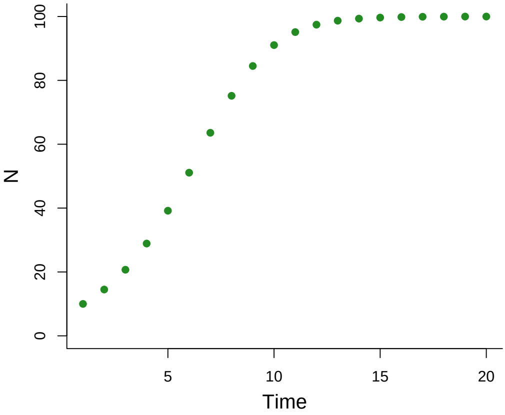
Intraspecific competition

$$
\begin{aligned}
\frac{N_{t+1} - N_t}{N_t} &= r - \frac{r}{K} N_t \\
N_{t+1} &= N_t + r N_t \left(1 - \frac{N_t}{K}\right)
\end{aligned}
$$

- Termed the discrete logistic population model
- Cornerstone of initial theory of compensation
- Often used in stock assessment of harvested populations
- Note N_t not disaggregated by age (would be $N_{a,t}$)

Limited population growth

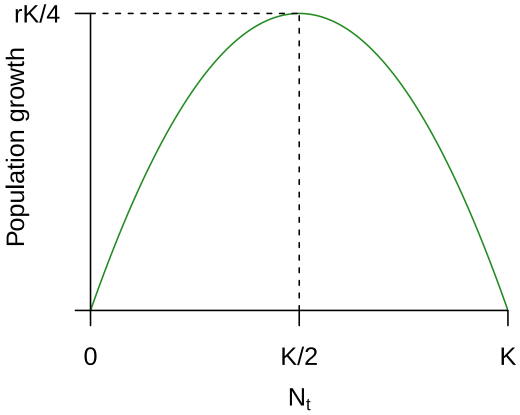
Logistic model



Verhulst(1838), Perl and Reed (1920)

Overall population growth rate

Population growth (per-capita × population size) has an optimum



Outline

Russell (1931)

Russell's Fishery equation:

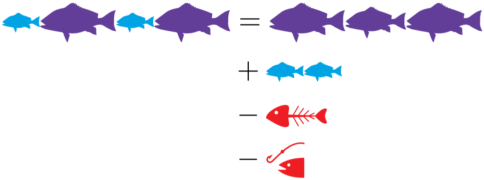
$$S_2 = S_1 + (R + G) - (M + F)$$

where

- S is the stock size
- R is recruitment of new individuals
- G is somatic growth
- M is natural mortality
- F is fishing mortality

Russell (1931)

Can cast in terms of our simple population model



$$B_{t+1} = B_t + R_t - D_t - C_t$$

where here

- B_t is now the biomass of the population (numbers *times* mass)
- R_t is the biomass of new recruits
- D_t is the biomass dying naturally
- C_t is mass caught (catch)

Include limited population growth

Graham-Schaefer biomass dynamic model

Difference (non-continuous) version

$$B_{t+1} = B_t + rB_t\left(1 - \frac{B_t}{K}\right) - C_t$$

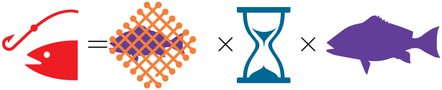
Often called just the Schaefer model. Widely-used model in fisheries assessment

Note

The Schaefer biomass dynamic model can be derived simply from a linear decrease in the per-capita growth rate over abundance with catch subtracted.

A first catch equation

Catch can be modelled as:



$$C = qEB$$

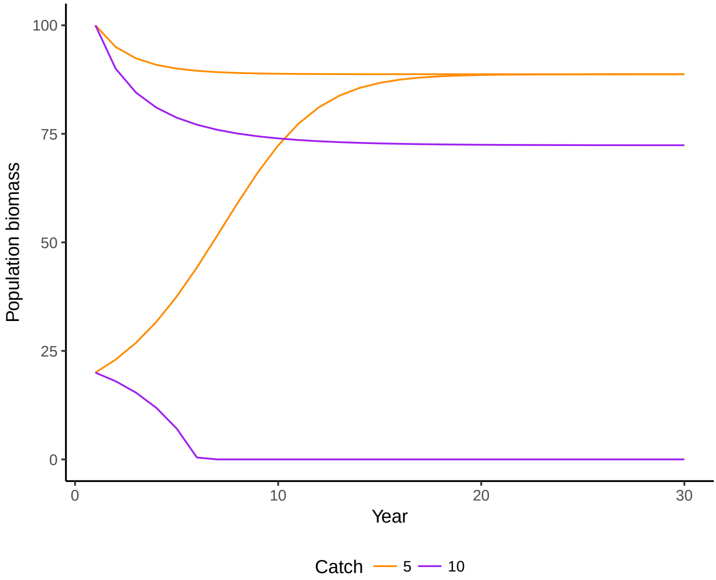
Where

- C is the catch
- q is catchability, defined as the proportion of the population removed per unit effort
- E is effort (e.g., fishing days, kW days, hooks)

What is a sustainable catch?

Q: For a given r and K parameters what values of catch are sustainable?

Catch and starting biomass



Catch and starting biomass

Note

Sustainability of a given catch depends on what level of biomass you start from.

Better to work with a rate of removal of the population

Fishing mortality

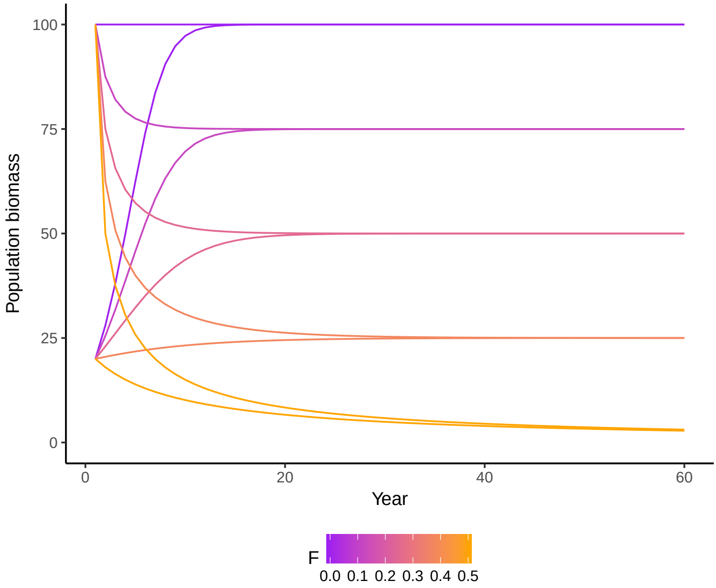
Note

Catchability and effort are often combined into the fishing mortality

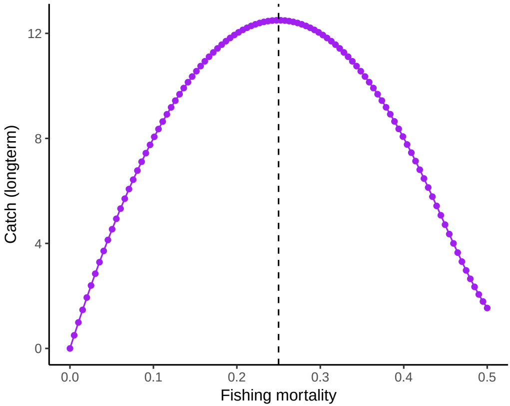
$$F = qE$$

which is usually expressed as an instantaneous rate but used here to denote the proportion of the population removed by fishing each year (often termed the harvest rate, sometimes denoted u).

What is a sustainable rate of fishing mortality?



Try different values for the harvest rate



- $F_{MSY} = \frac{r}{2}$ in the Schaefer model
- $K/2$ termed B_{MSY} - long-term biomass obtained if fishing at $F_{MSY} = r/2$

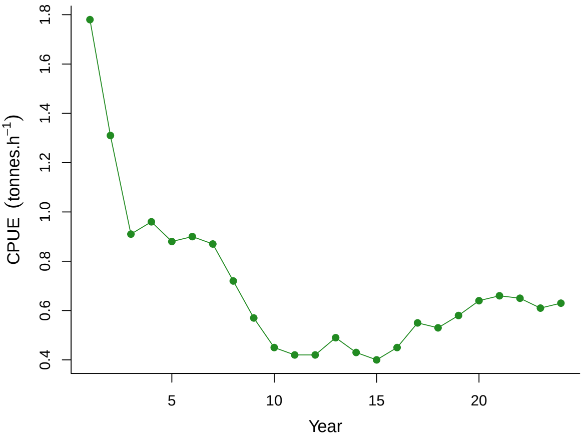
More on reference points later . . .

Outline

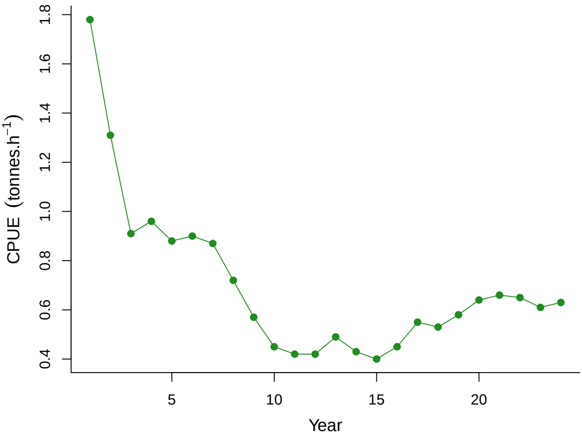
Need to estimate the parameters of the biomass dynamic model from:

- Biomass index I, e.g.
  - Survey index
  - Standardised CPUE index
- Catch C

Biomass index



Biomass index



Assume

$$I = qB$$

Index is proportional to biomass

Predicting the biomass index

Say we guessed the parameters, we would predict the index values via

$$\hat{B}_{t=1} = \hat{B}_0$$

$$\hat{B}_{t+1} = \hat{B}_t + \hat{r}\hat{B}_t\left(1 - \frac{\hat{B}_t}{\hat{K}}\right) - C_t$$

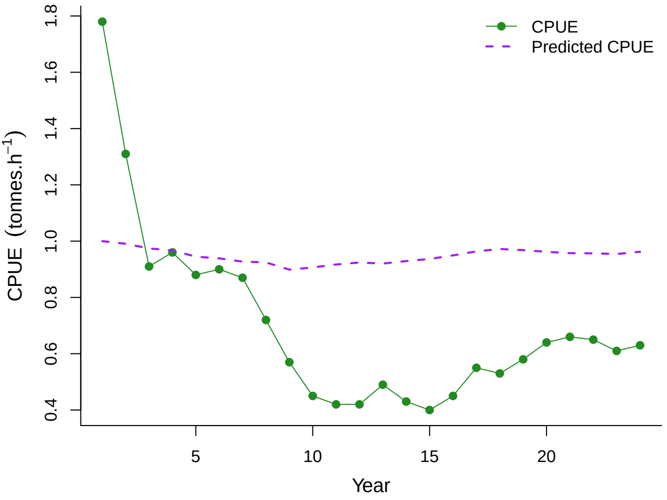
$$\hat{I}_t = \hat{q}\hat{B}_t$$

Note

This is an observation error approach

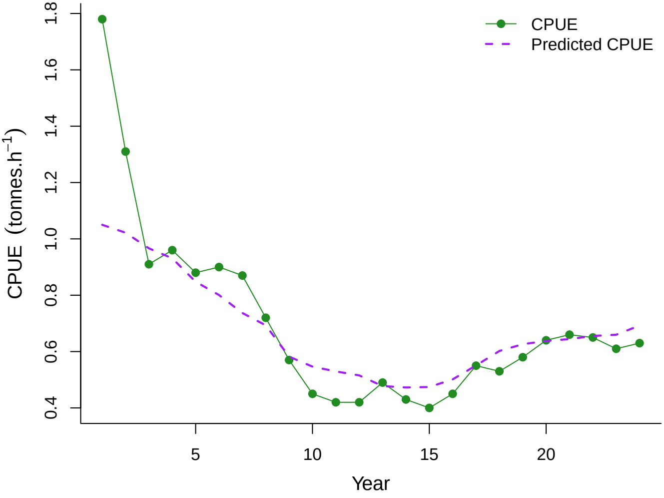
Predicting the biomass index

Say we guessed $\hat{r} = 0.5,\ \hat{K} = 10,000,\ \hat{B}_0 = 10,000,\ \hat{q} = 0.0001$



Another guess

Say we guessed $\hat{r} = 0.3$, $\hat{K} = 3{,}500$, $\hat{B}_0 = 3{,}500$, $\hat{q} = 0.0003$



Estimation

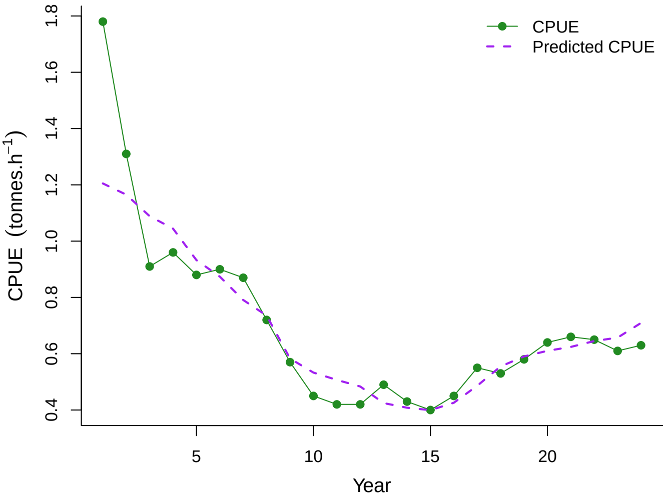
Like to do better than guess, so we estimate
Define the likelihood

$$L(r, K, B_0, q, \sigma|\mathbf{I}) = \prod_{t=1}^{T} N(\ln(I_t) - \ln(\hat{I}_t), \sigma^2)$$

which we maximise with respect to the parameters to obtain the maximum likelihood estimates

Maximum likelihood estimates: observation error

$\hat{r} = 0.37, \ \hat{K} = \hat{B}_0 = 2{,}823, \ \hat{q} = 0.00042, \ \hat{\sigma} = 0.125$



Process error approach

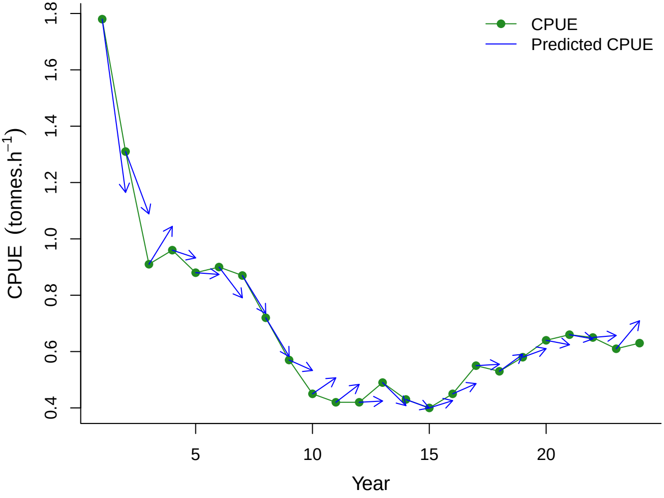
Rather than predict all the way through, a process error only model assumes the observations are perfect and the only error is in the biomass dynamic model

Note

A process error only approach assumes only source of error is in the biomass dynamics, not the observations.

Maximum likelihood estimates: process error

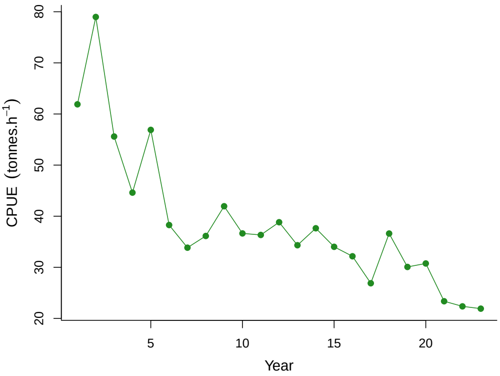
$$\hat{r} = 0.31,\ \hat{K} = \hat{B}_0 = 3{,}577,\ \hat{q} = 0.00025,\ \hat{\sigma} = 0.1$$



Outline

Contrast

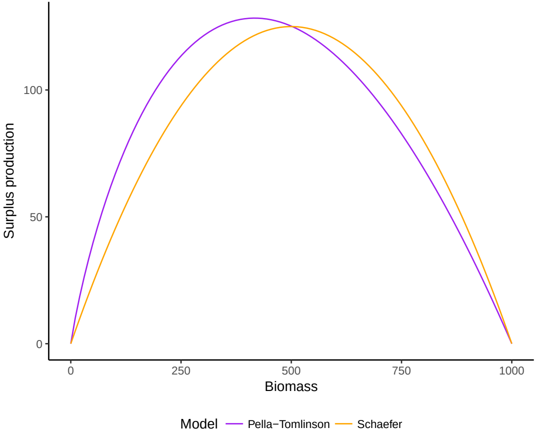
Need historical variation in stock size and fishing pressure to properly estimate the parameters of the biomass dynamic model. Beware of one-way trip data like this



- r estimation requires points at low stock size and low effort
- K estimation requires high stock sizes and low effort
- Ideally there would be variation in both

Alternative forms

A common alternative form is the Pella-Tomlinson model



$$B_{t+1} = B_t + \frac{r}{n} B_t \left(1 - \left(\frac{B_t}{K}\right)^n\right) - C_t$$

State space formulation

Switch to spacing out slides

State space formulation

Recently, much interest in estimation including both observation and process error (`spict`)

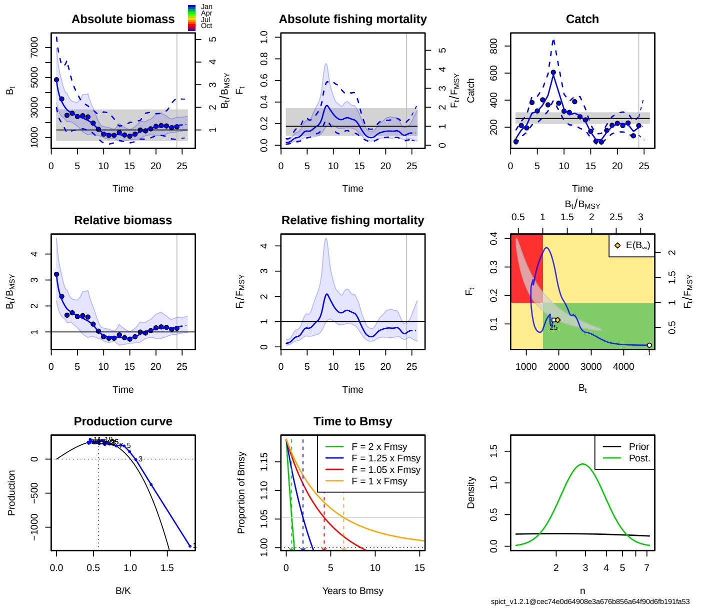
$$
\begin{aligned}
\text{Process equations:} \quad & B_{t+1} = \left(B_t + rB_t\left(1 - \frac{B_t}{K}\right) - C_t\right) e^{\eta_t} \\
& F_{t+1} = F_t e^{\epsilon_t} \\
\text{Observation equations:} \quad & I_t = qB_t e^{\varepsilon_t} \\
& C_t = F_t B_t e^{\zeta_t} (\text{roughly})
\end{aligned}
$$

where η_t, ε_t are process and measurement errors on biomass and the index; ϵ_t, ζ_t are process and measurement errors on fishing

Note

`spict` also includes a rich output but care needed in terms of the number of parameters estimated and given data quality. Make sure to check parameter correlation and diagnostics.

State space formulation



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- Logistic population growth can be derived from a simple linear decrease in the per-capita growth rate with increasing abundance

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- A number of ways of estimating parameters (obs error, process error only)
- Quality of fit highly dependent on data quality and contrast
- Reference points straightforward to obtain from a good fit
- Advanced models provide options for multiple indices, sources of error, asymmetric production
- Critical evaluation of the assessment fit and diagnostics very important with aggregate biomass dynamic models

References

Hilborn, R., and Walters, C. J. (2013). Quantitative fisheries stock assessment: choice, dynamics and uncertainty. Springer Science & Business Media. [Ch. 8]

Meyer, R., and Millar, R. B. (1999). BUGS in Bayesian stock assessments. Canadian Journal of Fisheries and Aquatic Sciences, 56(6), 1078-1087.

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