

# Community Structure in Networks

Social Networks Analysis and Graph Algorithms

Prof. Carlos Castillo — <https://chato.cl/teach>

# Sources

- A. L. Barabási (2016). Network Science – Chapter 09
- D. Easley and J. Kleinberg (2010). Networks, Crowds, and Markets – Chapter 03
- F. Menczer, S. Fortunato, C. A. Davis (2020). A First Course in Network Science – Chapter 06
- URLs cited in the footer of slides

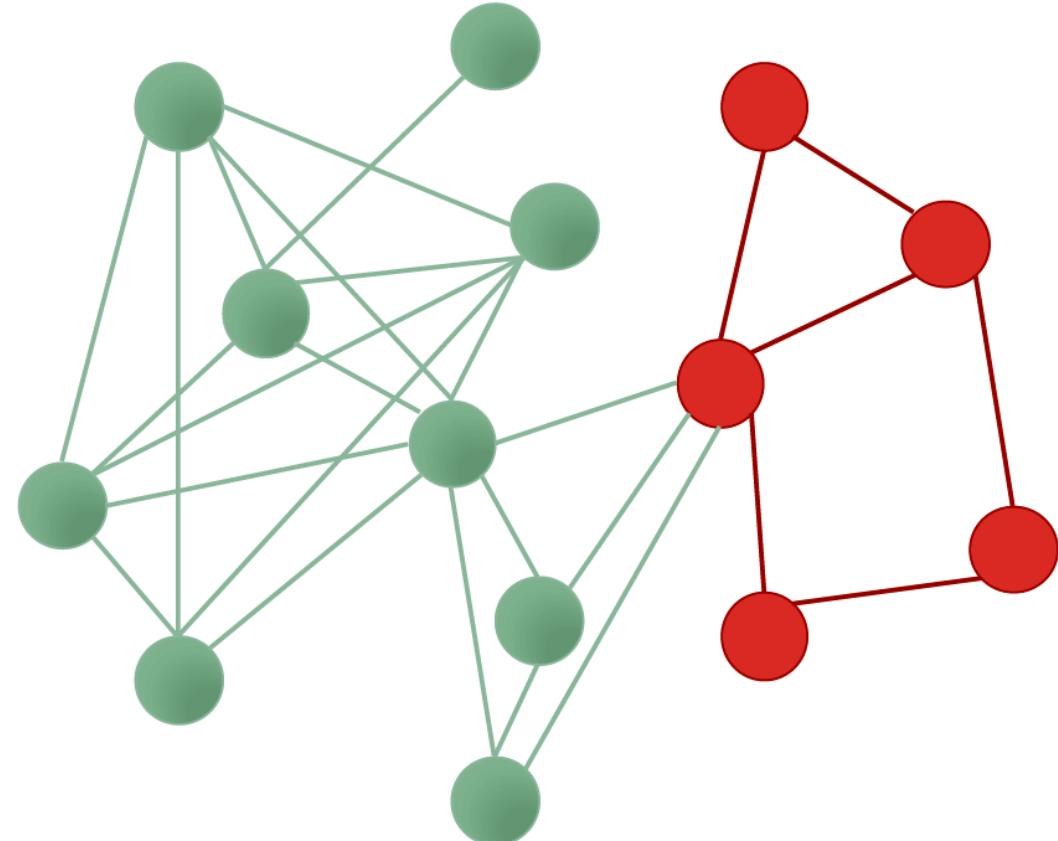
# Typical community structures

- **One** dense sub-graph
  - embedded somewhere within a larger graph
- **Two** groups (polarization)
  - plus perhaps some ambiguous nodes
- **Multiple** communities

# One dense sub-graph

# Reminder: sub-graph

A sub-graph is a subset of nodes, and all of the edges among those nodes



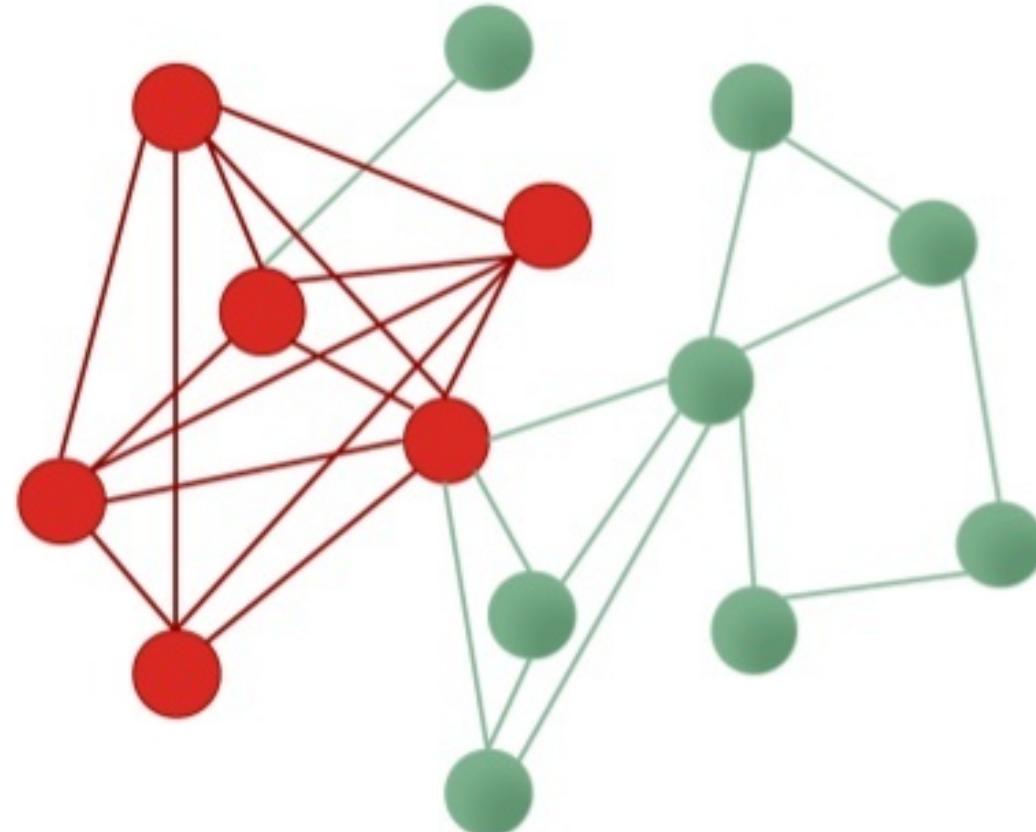
# Densest sub-graph

Sub-graph having the maximum **density** according to some measure

E.g.:

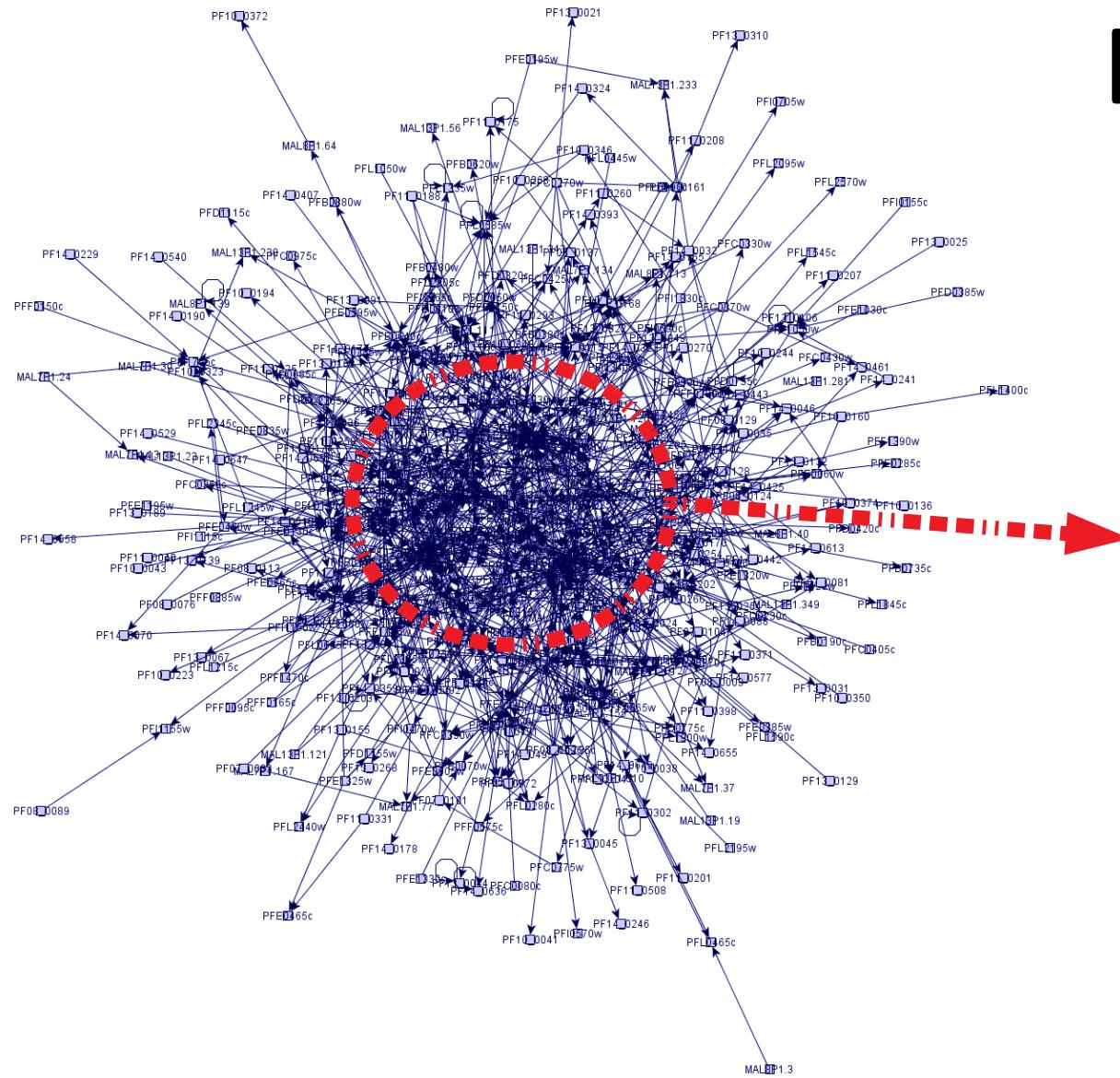
largest  $|E|/|V|$

largest  $(2|E|)/(|V|(|V|-1))$

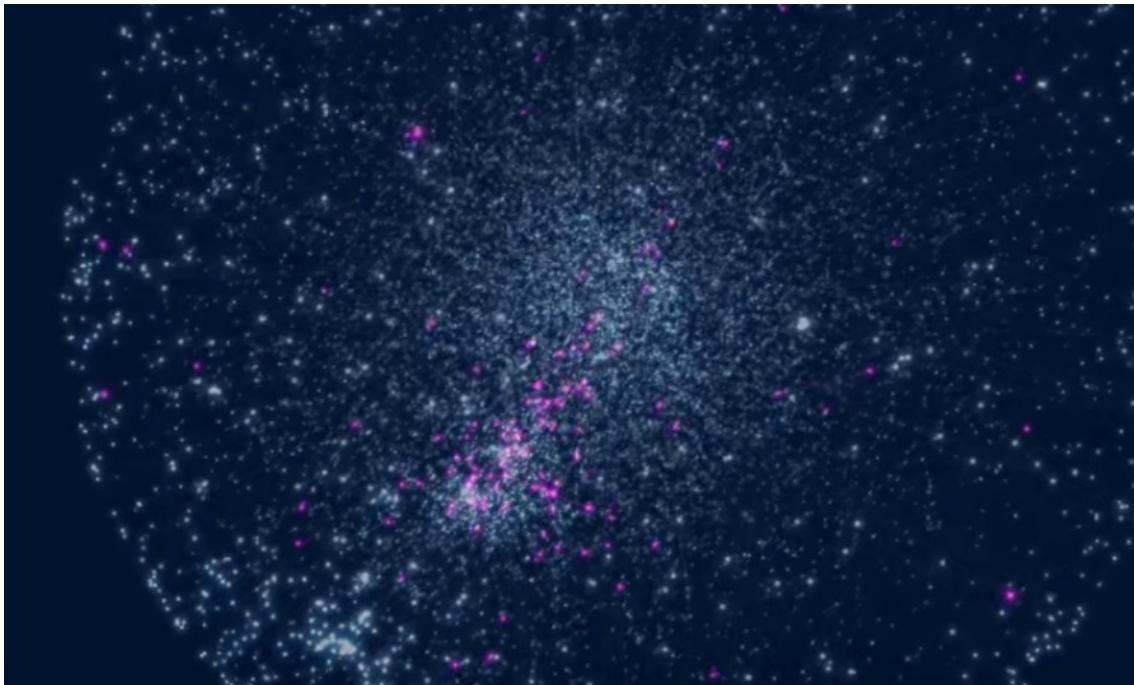


Many graphs look like “hairballs”

Sometimes, at the center these graphs may have an interesting dense sub-graph



# Asthma-related genes



[https://www.youtube.com/watch?v=VU\\_7FHAKMgA](https://www.youtube.com/watch?v=VU_7FHAKMgA)

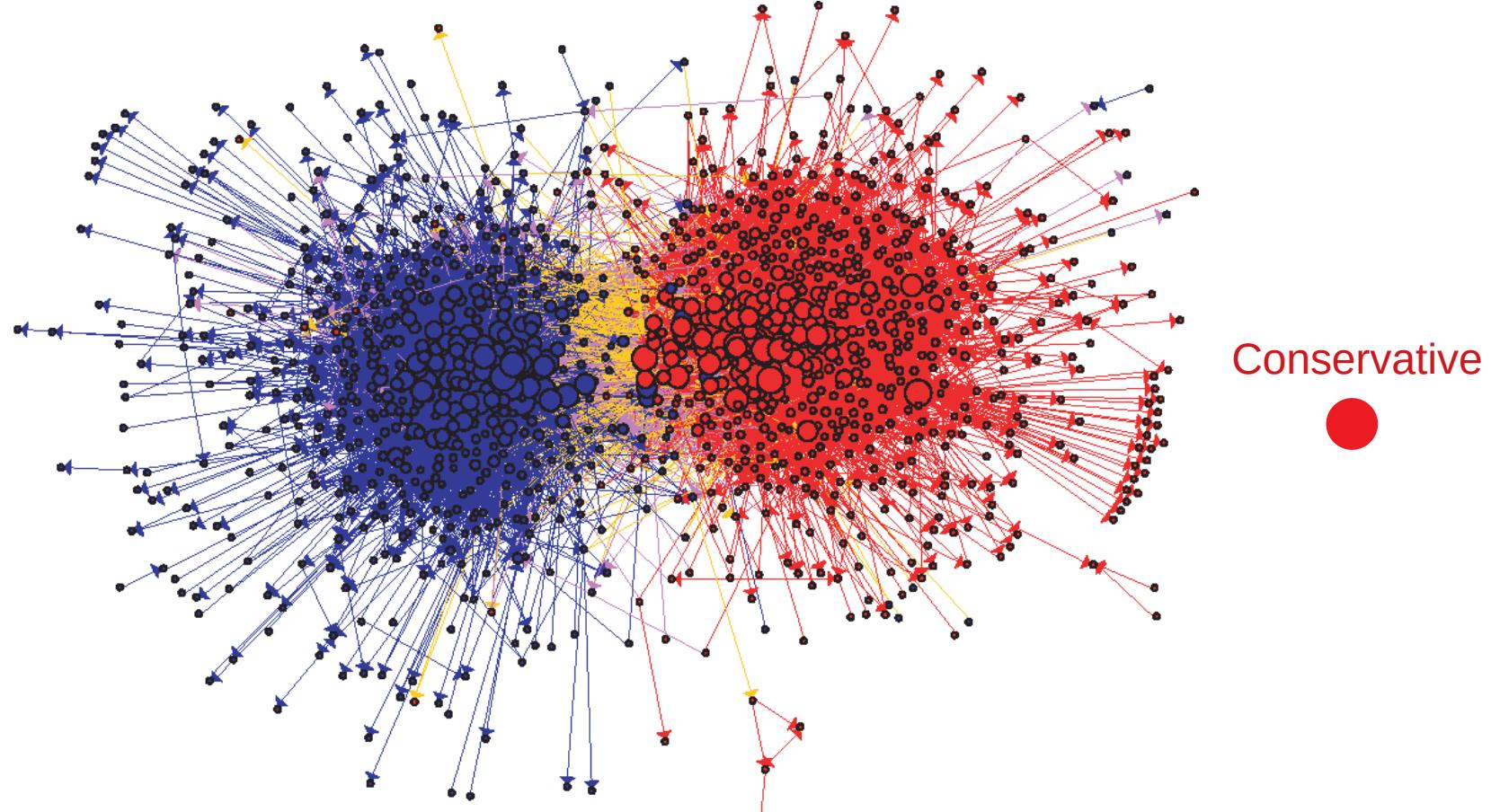
# Two groups (polarization)

# US Political Blogs (2004)

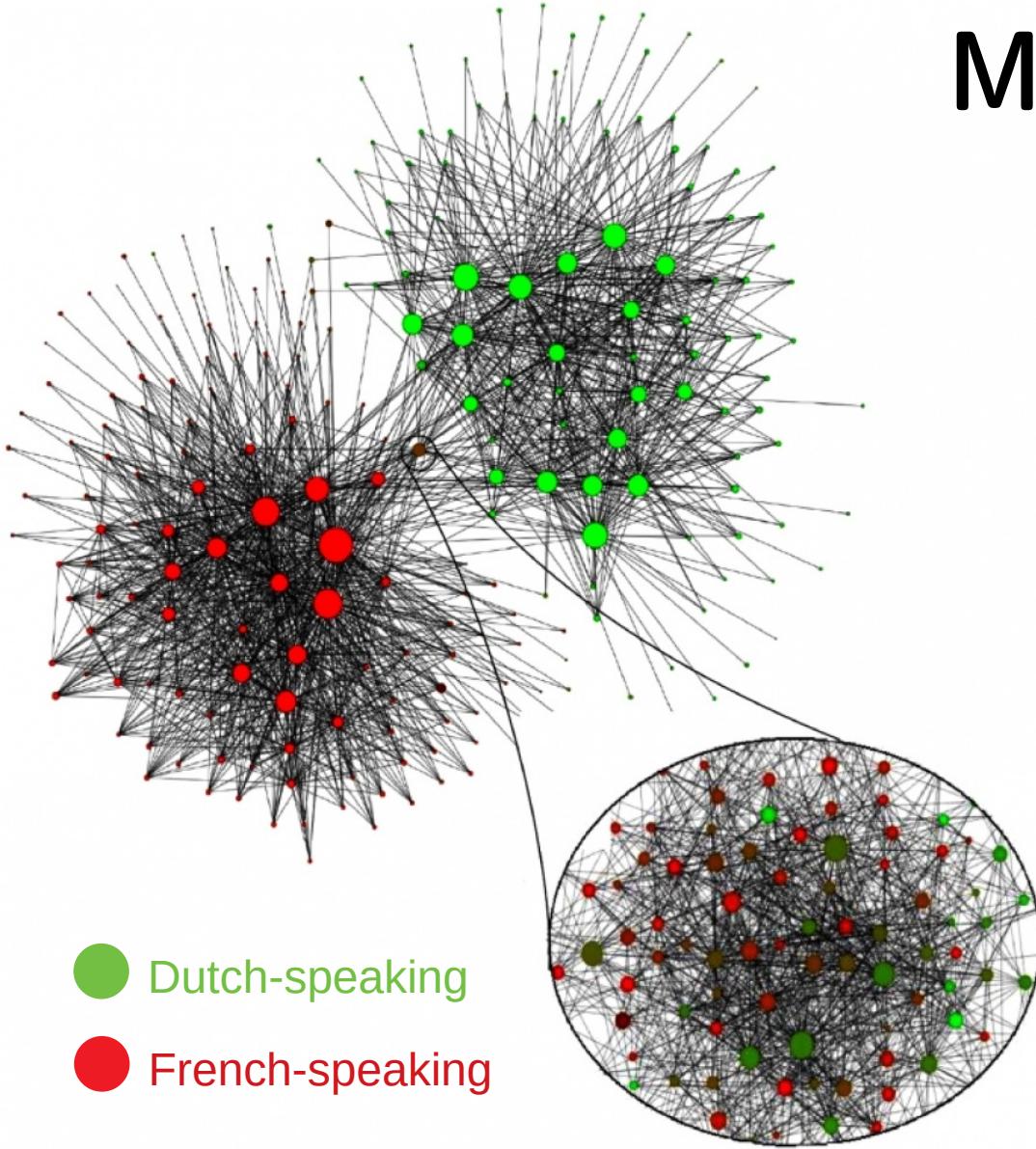
Liberal



Conservative



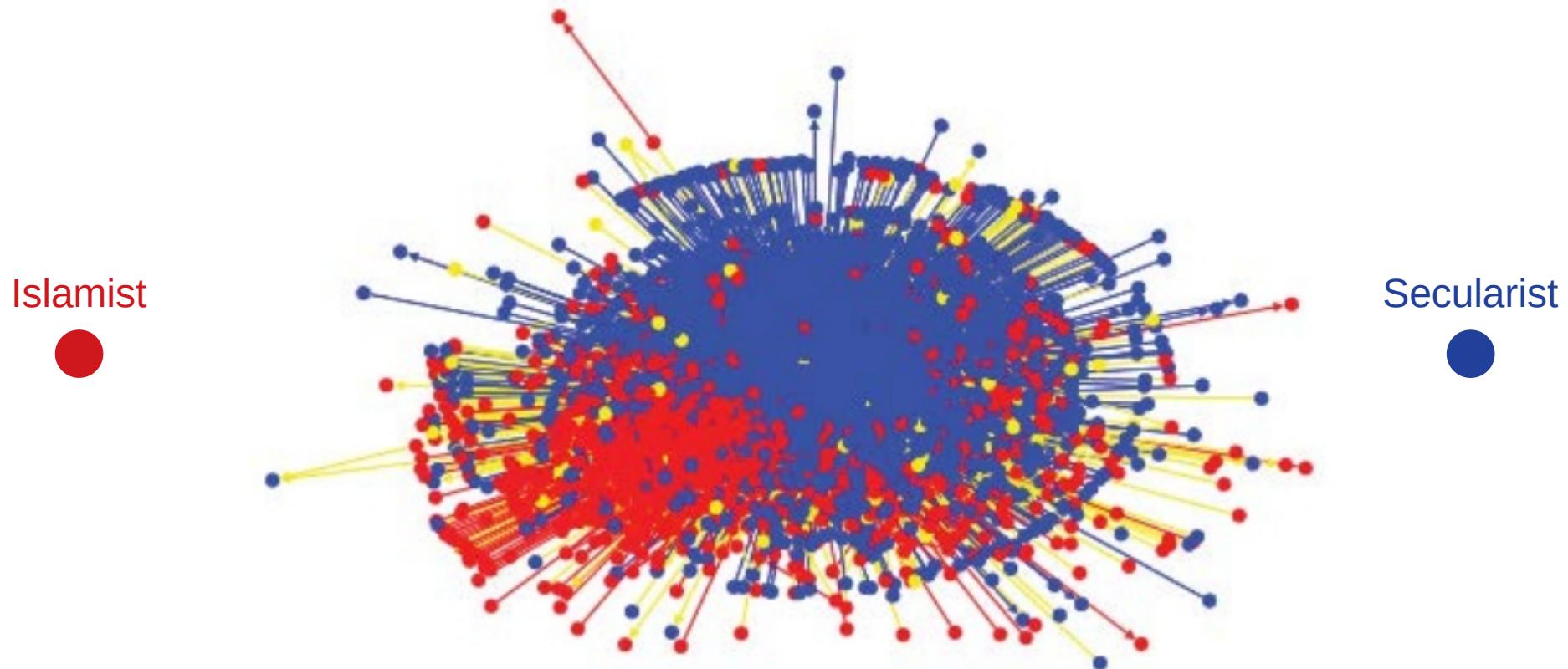
# Mobile phone users in Belgium (2008)



Each node is a community of 100 mobile users or more that tend to call each other

V. D. Blondel, J.-L. Guillaume, R. Lambiotte, and E. Lefebvre. Fast unfolding of communities in large networks. *J. Stat. Mech.*, 2008.

# Egyptian Twitter Users (2013)



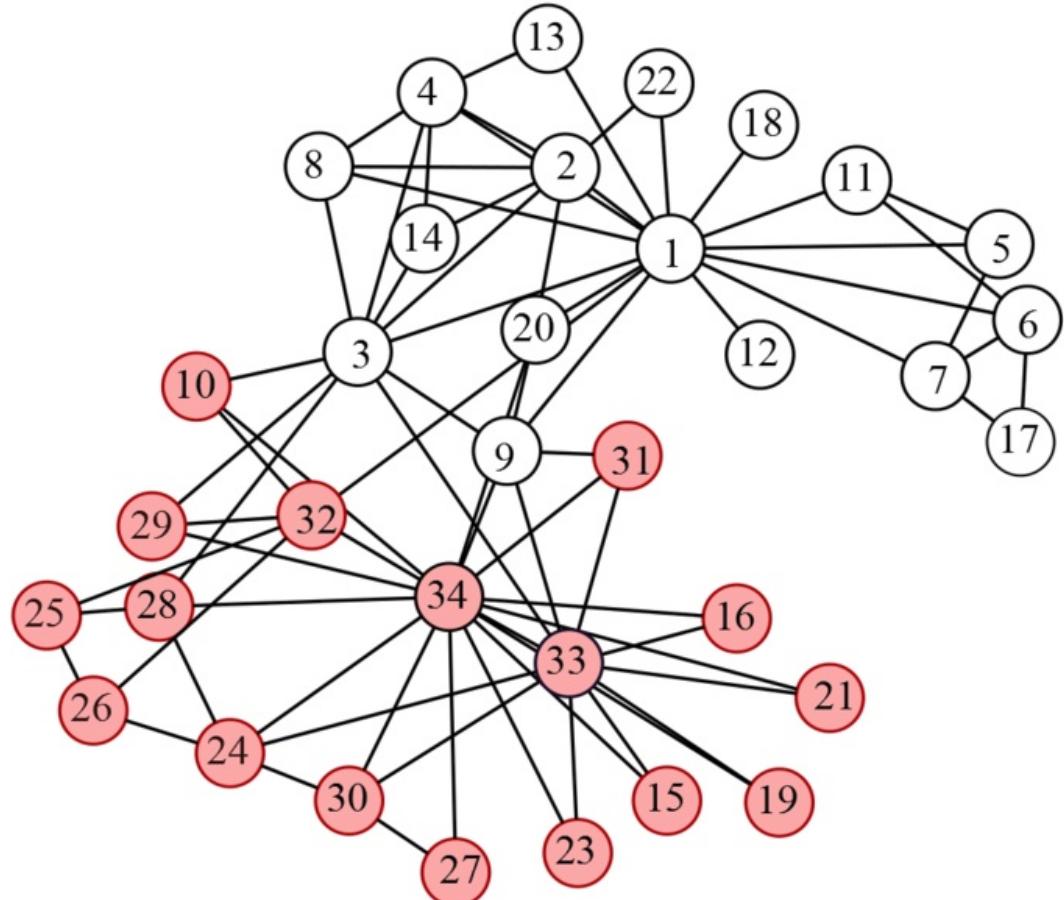
# Political Books



Source:  
Valdis Krebs  
& The Economist

# Wayne Zachary's PhD Thesis (1972)

- Studied 34 members of a karate club
- Found 78 links between members who regularly interacted outside the club
- The club splitted in two during the study
- 1=sensei, 34=president



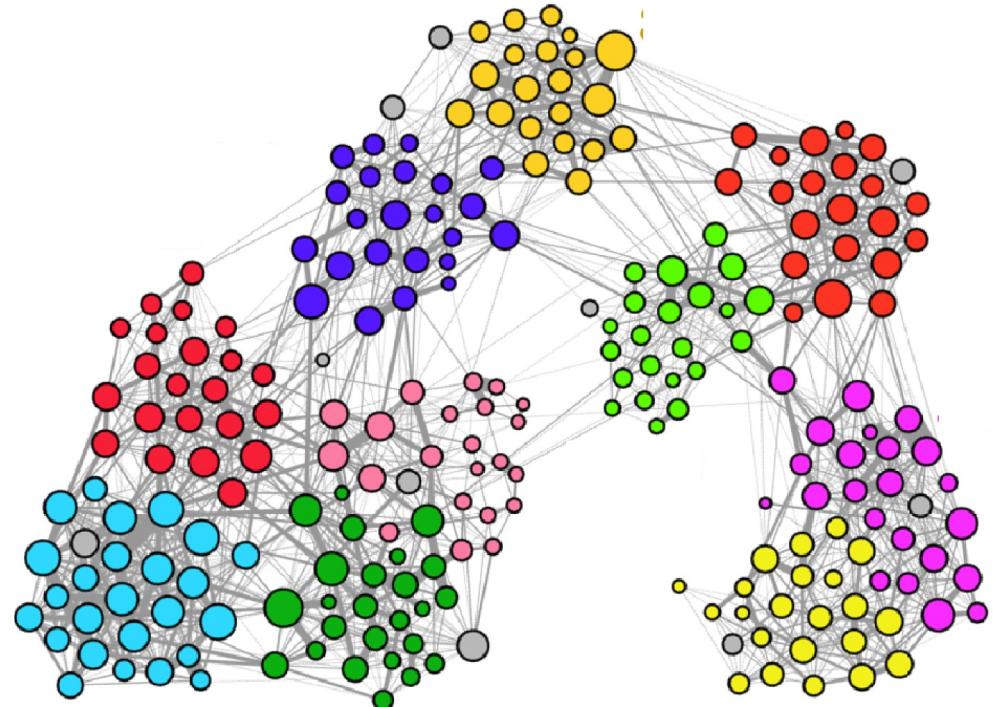
# Multiple communities

# Primary school contacts

Links connect students who spent more than two minutes face to face

Students wore RF-ID badges hanging on their chest, which have a range of about 1.0-1.5 meters

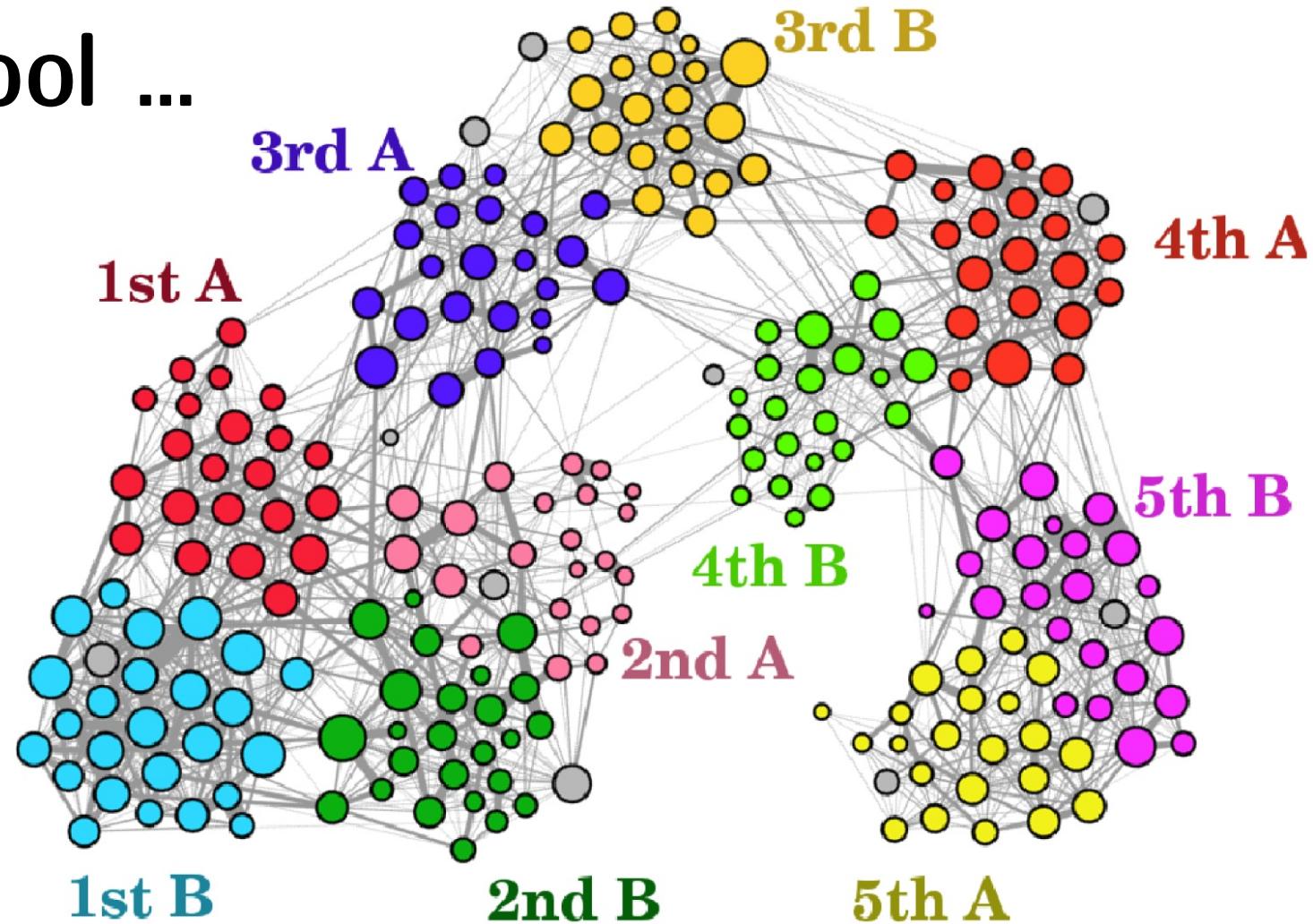
What do you think the colors represent in this visualization?



# Primary school ...

Colors represent classes. Teachers are shown in gray color.  
Node sizes are number of connections.

Note: these communities are hierarchical (more on this later)



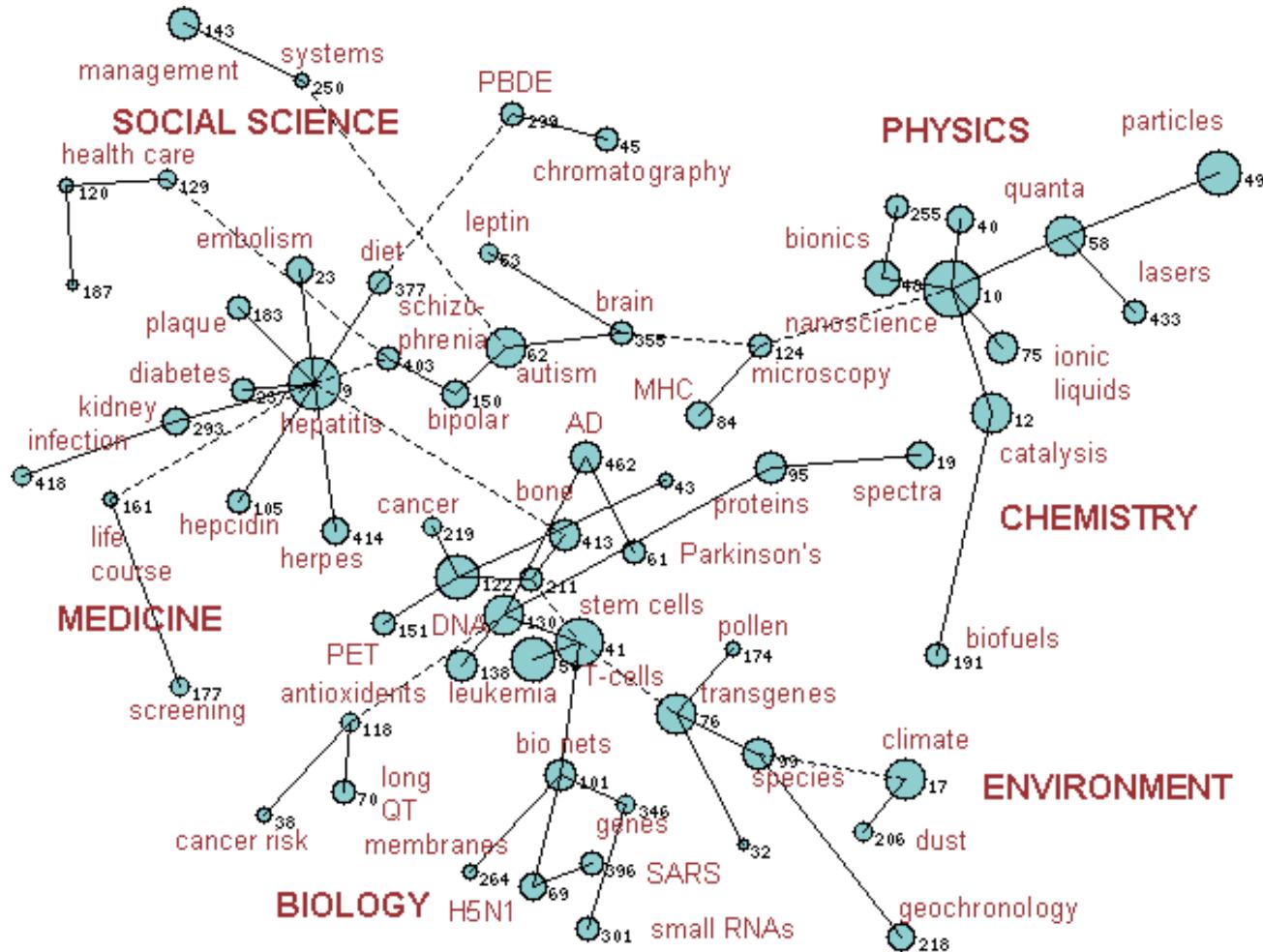
Stehlé, J., et al. (2011).

High-resolution measurements of face-to-face contact patterns in a primary school.

PloS one, 6(8), e23176.

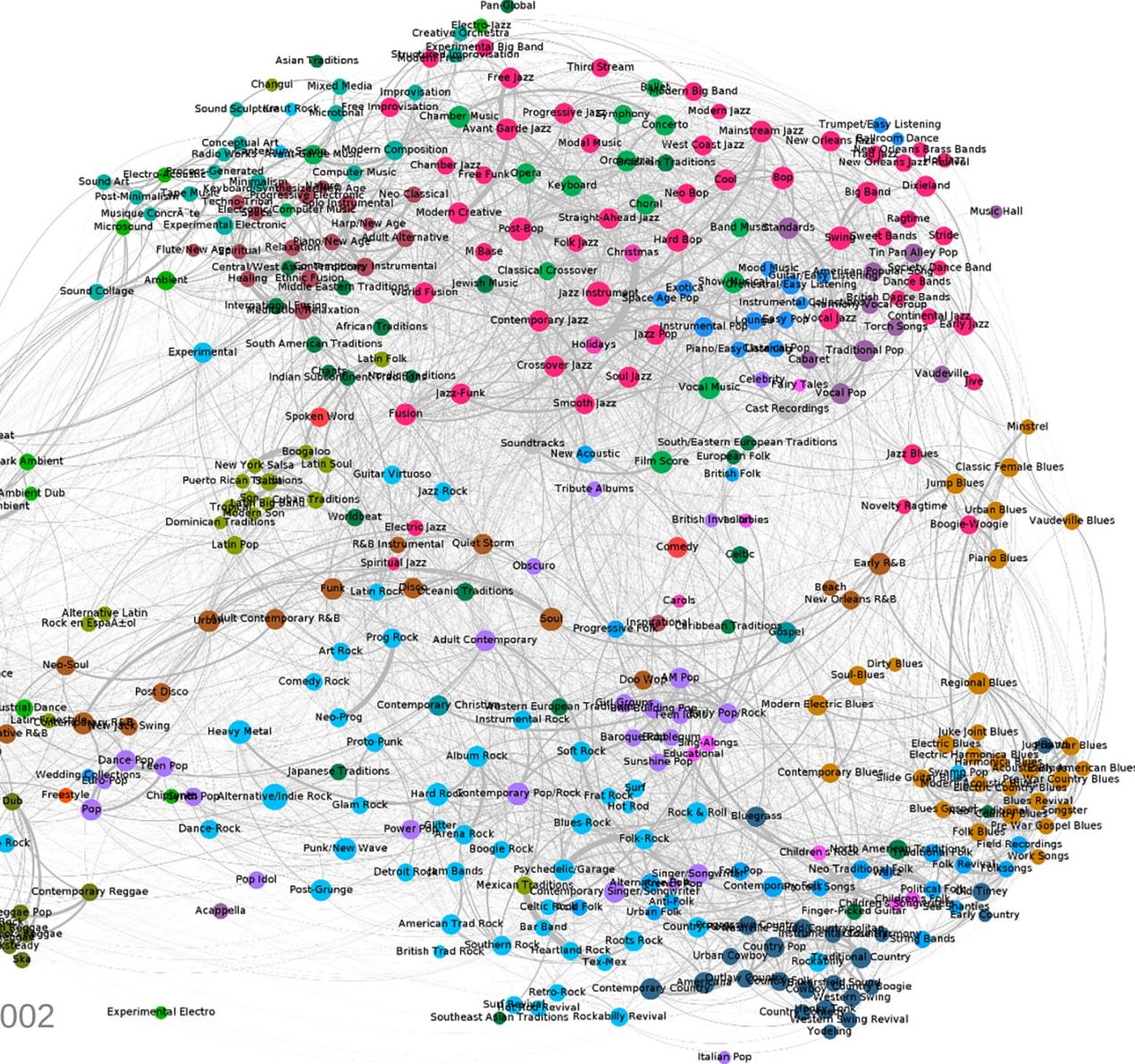
# Science

Two topics  $T_1$ ,  $T_2$ , are connected if there is at least one paper that cites:  
a paper  $u$  in  $T_1$  and  
a paper  $v$  in  $T_2$ .

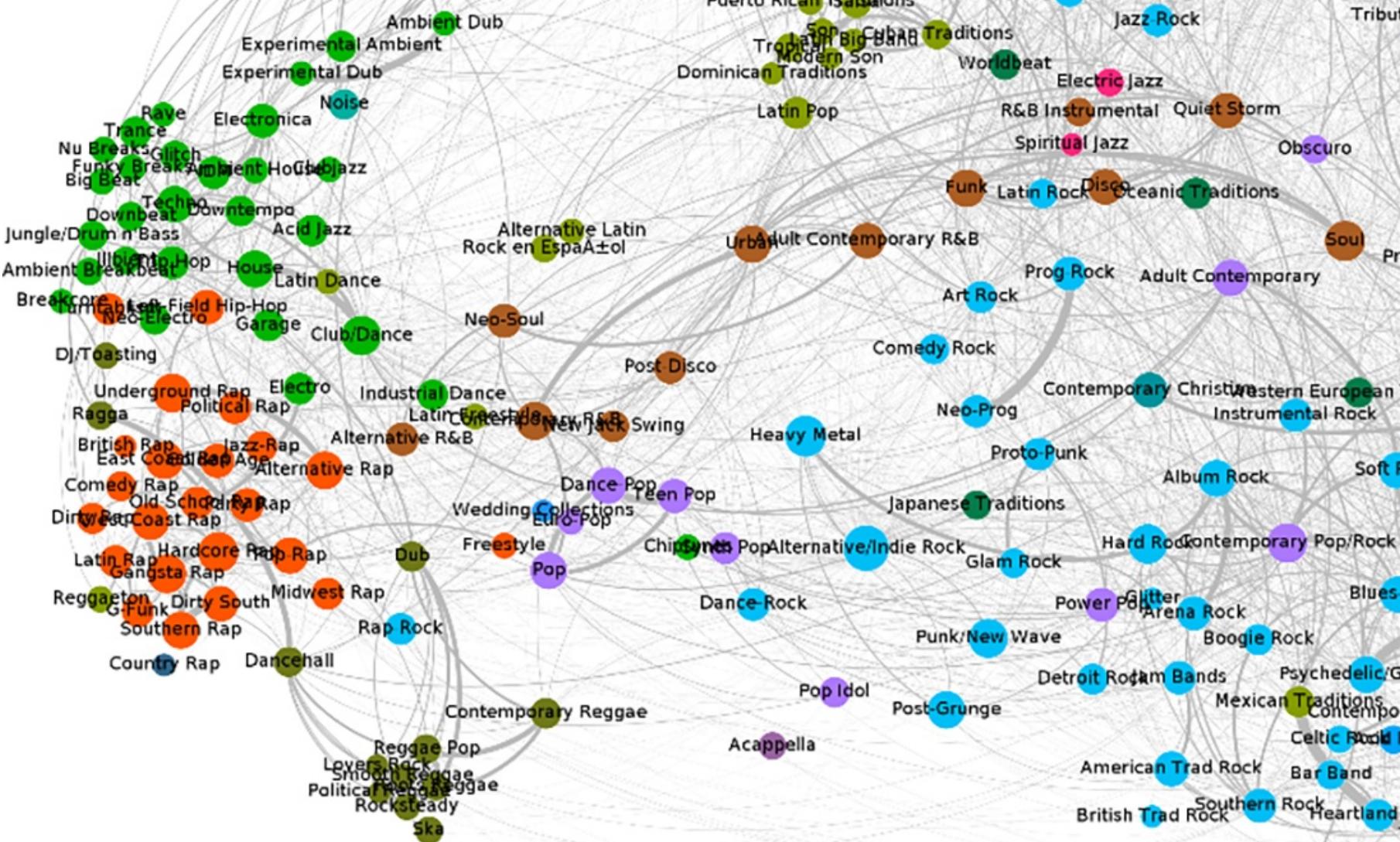


# Music

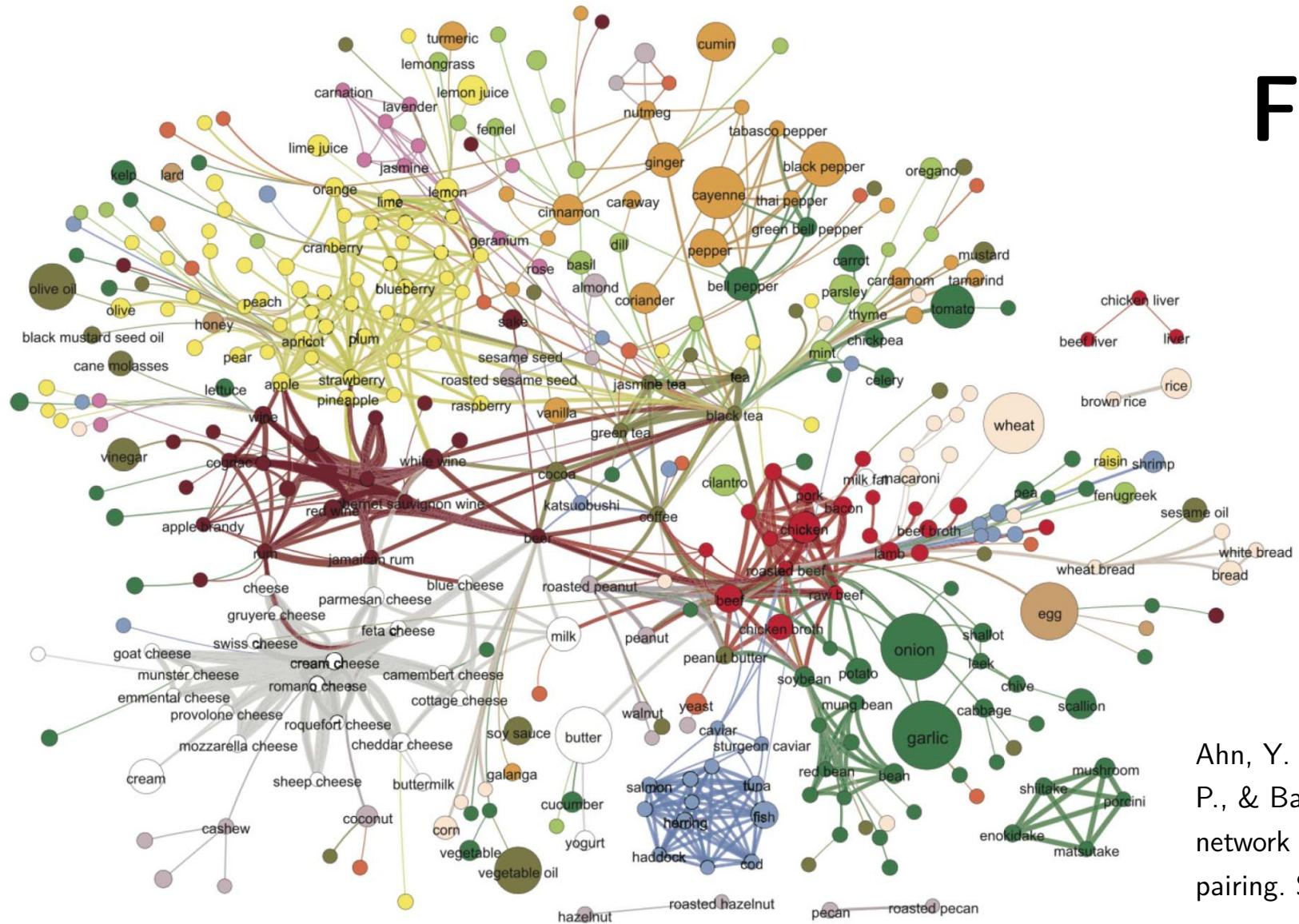
Two Genres, G1,  
G2, are connected  
if there is a  
musician producing  
tracks in both  
genres; width of  
link is number of  
musicians



# Music (cont.)

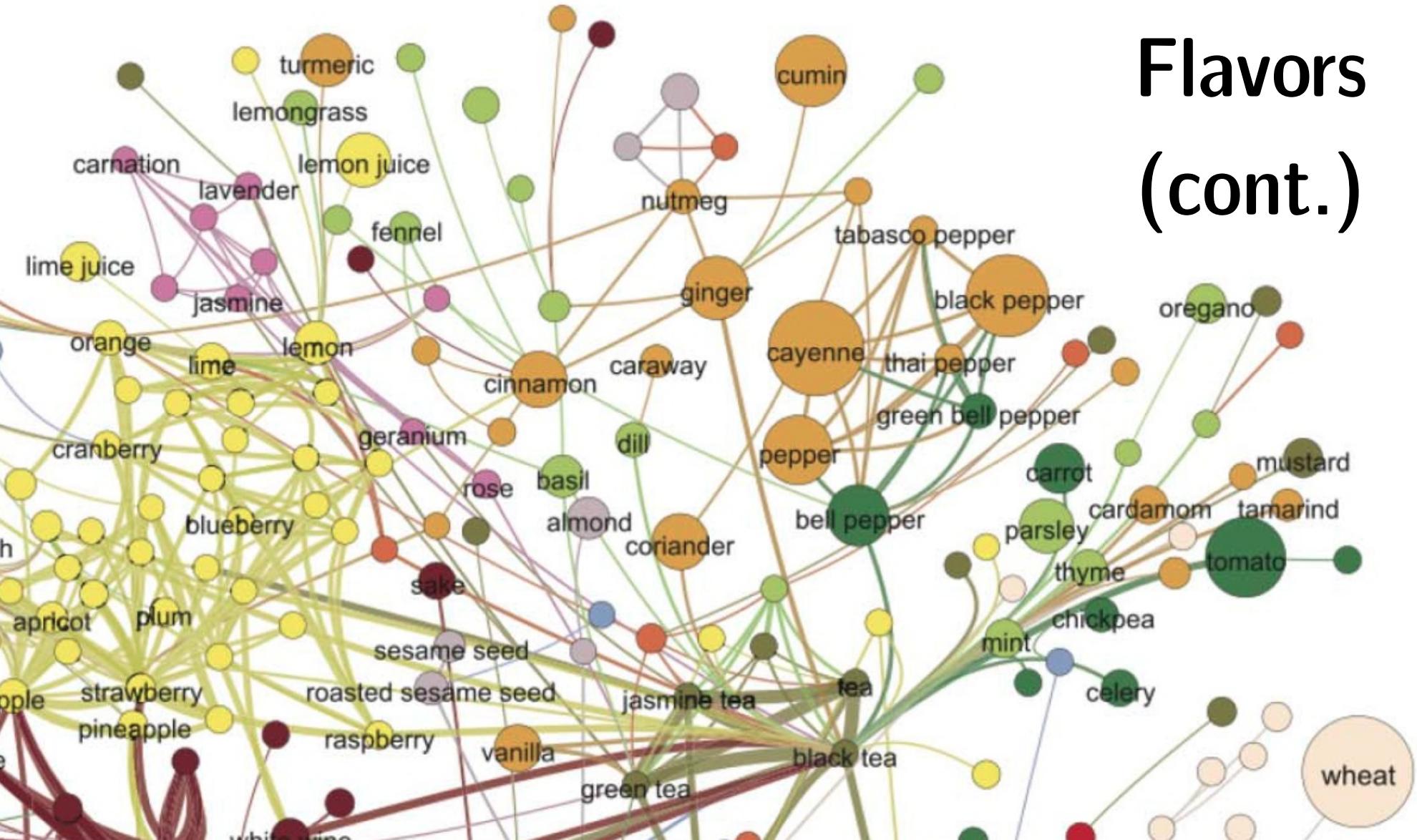


# Flavors

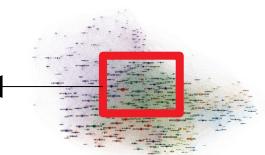
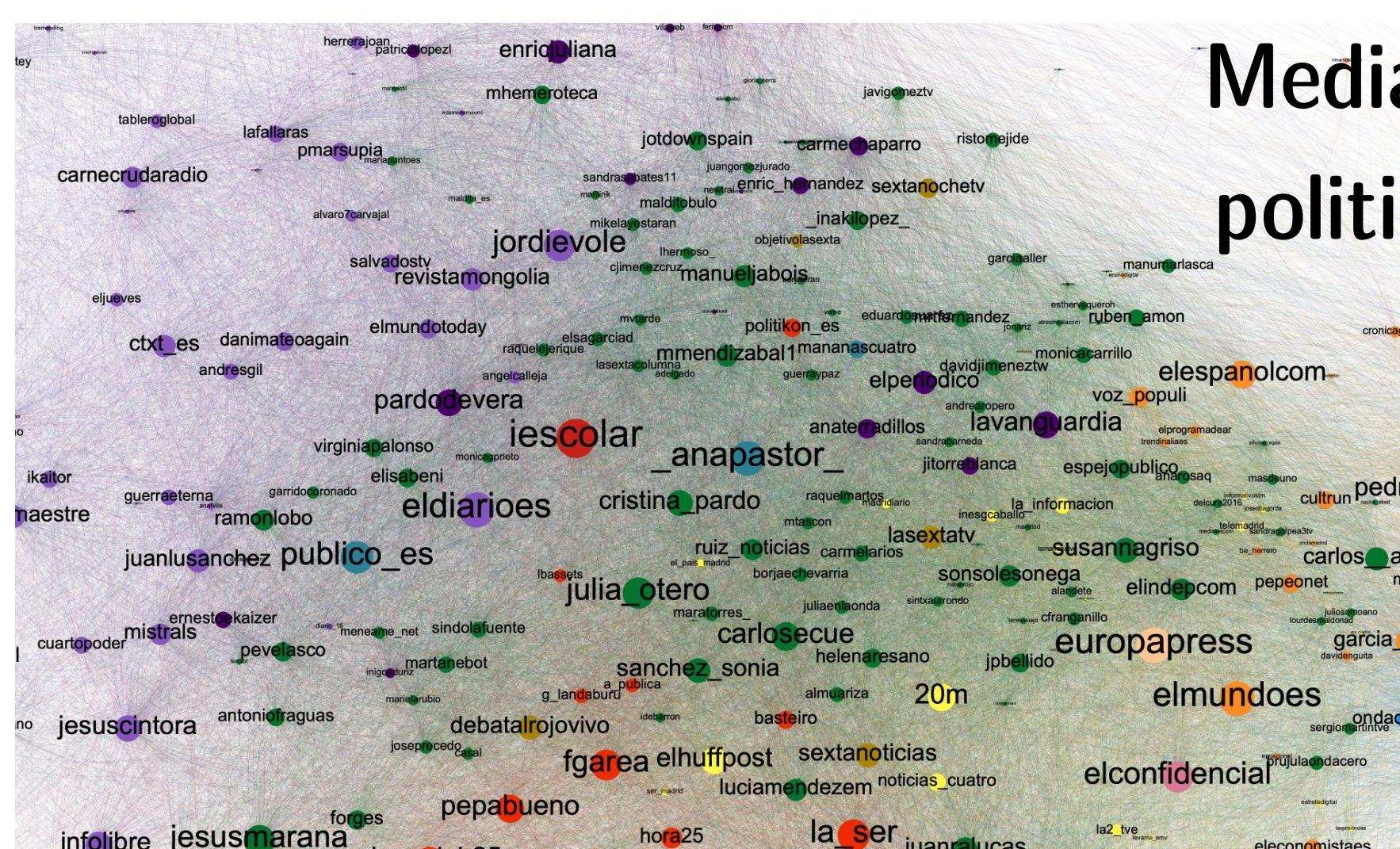


Ahn, Y. Y., Ahnert, S. E., Bagrow, J. P., & Barabási, A. L. (2011). Flavor network and the principles of food pairing. *Scientific reports*, 1, 196.

# Flavors (cont.)



# Media and politicians

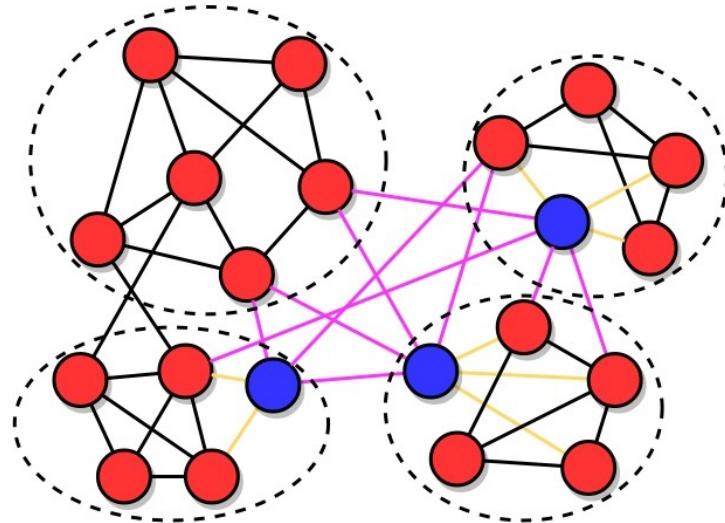


<https://twitter.com/jbo/status/1120444347772821504/photo/1>

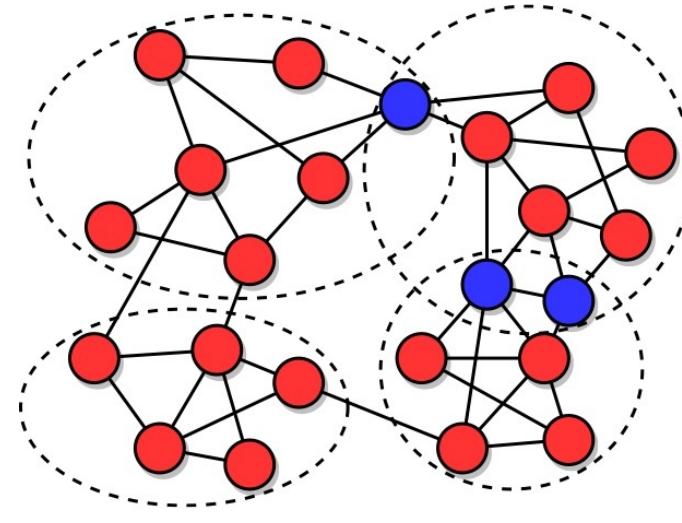
# Partitions vs Overlapping communities

## Hierarchical communities

# Partition vs Overlapping communities



Partition, or *hard* clusters

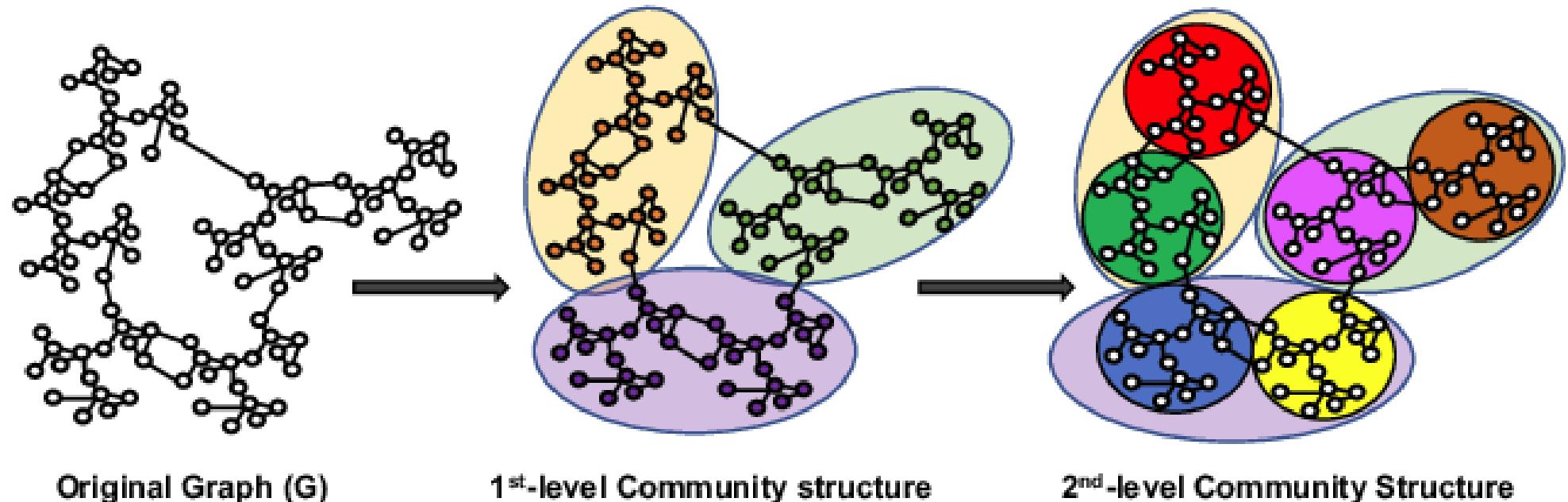


Overlapping communities, or *soft* clusters

What's special about blue nodes?

Blue nodes are in more than one community

# Hierarchical communities



Original Graph ( $G$ )

1<sup>st</sup>-level Community structure

2<sup>nd</sup>-level Community Structure

How to generate a graph having  
community structure?

# Exercise

Invent a variant of the ER model that generates graphs having **two communities**, but not necessarily disconnected



Standard ER Model:

- Generate N nodes
- For each of the  $N(N-1)/2$  pairs of nodes:
  - Perform a Bernoulli trial with probability  $p$
  - If the trial succeeds, connect those nodes

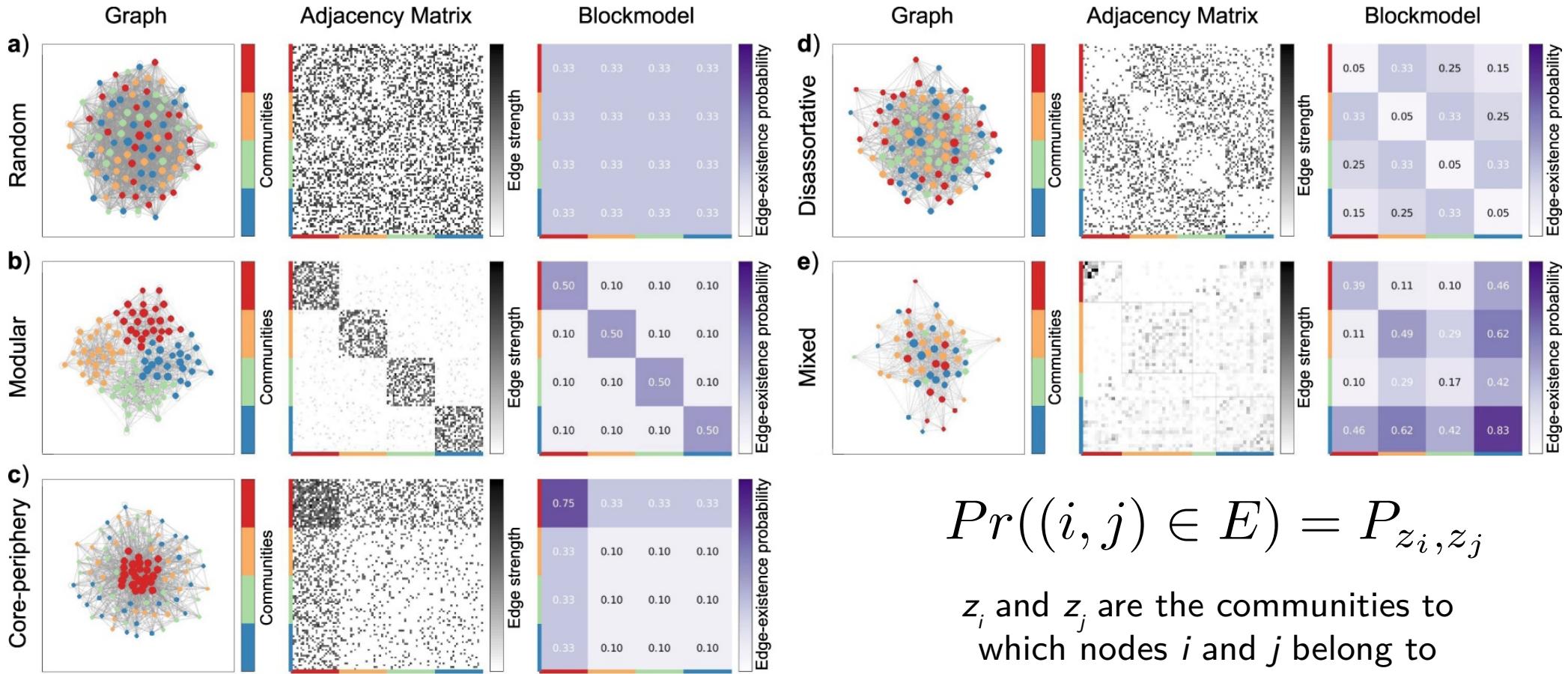
Pin board: <https://upfbarcelona.padlet.org/chato/tt20-community-structure-yhk7nozup8xbiejb>



# How to generate community structure?

- The **stochastic block model** generates graphs with community structure
  - It can also be used for inference, but we will not see that in this course
- Can be described as a variation of the ER model in which:
  - There are  $m$  groups
  - Link probability scalar  $p$  becomes an  $m \times m$  matrix  $P$  that contains in position  $(i,j)$  the probability of a link between a node in group  $i$  and a node in group  $j$

# Examples of stochastic block model



# Summary

# Things to remember

- Many networks have community structure
- Sometimes it's:
  - One dense sub-graph
  - Two communities (polarization)
  - Multiple communities
- Partitions vs overlapping communities
- Hierarchical communities