predictive-problem-set-3

Sreshtha Chatterjee 724

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2. Problem to demonstrate the role of qualitative (nominal) predictors in addition to quantitative predictors in multiple linear regression

Attach “Credits” data from R. Regress “balance” on (a) “gender” only.

```
data("Credit", package = "ISLR")
model21 = lm(Balance~Gender, data = Credit)
summary(model21)

##
## Call:
## lm(formula = Balance ~ Gender, data = Credit)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -529.54 -455.35  -60.17  334.71 1489.20
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    509.80      33.13  15.389   <2e-16 ***
## GenderFemale    19.73      46.05   0.429    0.669
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 460.2 on 398 degrees of freedom
## Multiple R-squared:  0.0004611,	Adjusted R-squared:  -0.00205
## F-statistic: 0.1836 on 1 and 398 DF,  p-value: 0.6685
```

(b) “gender” and “ethnicity” .

```
model22 = lm(Balance~Gender+Ethnicity, data = Credit)
summary(model22)

##
## Call:
## lm(formula = Balance ~ Gender + Ethnicity, data = Credit)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -540.92 -453.61  -56.37  336.24 1490.77
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)          520.88      51.90  10.036   <2e-16 ***
## GenderFemale          20.04      46.18   0.434    0.665
```

```
## EthnicityAsian       -19.37      65.11  -0.298    0.766
## EthnicityCaucasian   -12.65      56.74  -0.223    0.824
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 461.3 on 396 degrees of freedom
## Multiple R-squared:  0.000694,   Adjusted R-squared:  -0.006877
## F-statistic: 0.09167 on 3 and 396 DF,  p-value: 0.9646
```

```
model22_interaction = lm(Balance~Gender+Ethnicity+(Gender*Ethnicity), data =
Credit)
summary(model22_interaction)
```

```
##
## Call:
## lm(formula = Balance ~ Gender + Ethnicity + (Gender * Ethnicity),
##     data = Credit)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -563.11 -445.84  -57.75  333.87 1483.66
##
## Coefficients:
##                                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)                         553.45      65.95   8.392 8.65e-16 ***
## GenderFemale                        -44.45      92.80  -0.479    0.632
## EthnicityAsian                     -100.58      94.25  -1.067    0.287
## EthnicityCaucasian                  -38.11      80.91  -0.471    0.638
## GenderFemale:EthnicityAsian         154.69     130.46   1.186    0.236
## GenderFemale:EthnicityCaucasian      50.61     113.57   0.446    0.656
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 461.6 on 394 degrees of freedom
## Multiple R-squared:  0.004472,   Adjusted R-squared:  -0.008161
## F-statistic: 0.354 on 5 and 394 DF,  p-value: 0.8796
```

(c) “gender”, “ethnicity”, “income”.

```
model23 = lm(Balance~Gender+Ethnicity+Income, data = Credit)
summary(model23)
```

```
##
## Call:
## lm(formula = Balance ~ Gender + Ethnicity + Income, data = Credit)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -794.14 -351.67  -52.02  328.02 1110.09
##
## Coefficients:
```

```
##                      Estimate Std. Error t value Pr(>|t|)    
## (Intercept)          230.0291    53.8574   4.271 2.44e-05 ***
## GenderFemale          24.3396    40.9630   0.594    0.553    
## EthnicityAsian         1.6372    57.7867   0.028    0.977    
## EthnicityCaucasian     6.4469    50.3634   0.128    0.898    
## Income                 6.0542     0.5818  10.406  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 409.2 on 395 degrees of freedom
## Multiple R-squared:  0.2157, Adjusted R-squared:  0.2078 
## F-statistic: 27.16 on 4 and 395 DF,  p-value: < 2.2e-16
```

```r
model23_interaction = 
lm(Balance~Gender+Ethnicity+(Gender*Ethnicity)+(Ethnicity*Income)+(Income*Gen
der), data = Credit)
summary(model23_interaction)
```

```
## 
## Call:
## lm(formula = Balance ~ Gender + Ethnicity + (Gender * Ethnicity) + 
##     (Ethnicity * Income) + (Income * Gender), data = Credit)
## 
## Residuals:
##    Min     1Q Median     3Q    Max 
## -761.0 -350.3  -52.8  333.7 1104.2 
## 
## Coefficients:
##                                  Estimate Std. Error t value Pr(>|t|)    
## (Intercept)                      177.3633    84.0373   2.111   0.0354 *  
## GenderFemale                      -3.9633   100.5685  -0.039   0.9686    
## EthnicityAsian                    15.5548   108.5243   0.143   0.8861    
## EthnicityCaucasian                94.1012    98.1493   0.959   0.3383    
## Income                             7.7178     1.2375   6.236 1.17e-09 ***
## GenderFemale:EthnicityAsian       81.0004   116.4601   0.696   0.4871    
## GenderFemale:EthnicityCaucasian   57.7073   101.0052   0.571   0.5681    
## EthnicityAsian:Income             -1.1207     1.5876  -0.706   0.4807    
## EthnicityCaucasian:Income         -2.5335     1.3902  -1.822   0.0691 .  
## GenderFemale:Income               -0.5246     1.2083  -0.434   0.6644    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 409.7 on 390 degrees of freedom
## Multiple R-squared:  0.2238, Adjusted R-squared:  0.2059 
## F-statistic:  12.5 on 9 and 390 DF,  p-value: < 2.2e-16
```

(d) Output all the regressions in (a)-(c) in a single table using stargazer. Comment on the significant coefficients in each of the models.

```r
library(stargazer)
```

```
##
## Please cite as:

##  Hlavac, Marek (2022). stargazer: Well-Formatted Regression and Summary
Statistics Tables.

##  R package version 5.2.3. https://CRAN.R-project.org/package=stargazer

stargazer(model21, model22, model23, type = "text", title = "Regression
Results for Balance", out = "balance_regression_results.txt")

##
## Regression Results for Balance
##
============================================================================
======
##                                             Dependent variable:
##                       -----------------------------------------------------
---------
##                                                   Balance
##                               (1)                   (2)                    (3)
## -------------------------------------------------------------------------
---------
## GenderFemale                19.733               20.038                 24.340
##                            (46.051)             (46.178)
(40.963)
##
## EthnicityAsian                                   -19.371                 1.637
##                                                 (65.107)
(57.787)
##
## EthnicityCaucasian                               -12.653                 6.447
##                                                 (56.740)
(50.363)
##
## Income
6.054***
##
(0.582)
##
## Constant                   509.803***           520.880***
230.029***
##                            (33.128)             (51.901)
(53.857)
##
## -------------------------------------------------------------------------
---------
## Observations                  400                  400                    400
## R2                          0.0005                0.001                  0.216
## Adjusted R2                 -0.002               -0.007                  0.208
## Residual Std. Error 460.230 (df = 398)  461.337 (df = 396)    409.218 (df
```

```
= 395)
## F Statistic         0.184 (df = 1; 398) 0.092 (df = 3; 396) 27.161*** (df
= 4; 395)
##
=============================================================================
======
## Note:                                                 *p<0.1; **p<0.05;
***p<0.01
```

```r
stargazer(model21, model22, model23, type = "html", title = "Regression
Results for Balance", out = "balance_regression_results.html")
```

```
##
## <table style="text-align:center"><caption><strong>Regression Results for
Balance</strong></caption>
## <tr><td colspan="4" style="border-bottom: 1px solid
black"></td></tr><tr><td style="text-align:left"></td><td
colspan="3"><em>Dependent variable:</em></td></tr>
## <tr><td></td><td colspan="3" style="border-bottom: 1px solid
black"></td></tr>
## <tr><td style="text-align:left"></td><td colspan="3">Balance</td></tr>
## <tr><td style="text-
align:left"></td><td>(1)</td><td>(2)</td><td>(3)</td></tr>
## <tr><td colspan="4" style="border-bottom: 1px solid
black"></td></tr><tr><td style="text-
align:left">GenderFemale</td><td>19.733</td><td>20.038</td><td>24.340</td></t
r>
## <tr><td style="text-
align:left"></td><td>(46.051)</td><td>(46.178)</td><td>(40.963)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-align:left">EthnicityAsian</td><td></td><td>-
19.371</td><td>1.637</td></tr>
## <tr><td style="text-
align:left"></td><td></td><td>(65.107)</td><td>(57.787)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-align:left">EthnicityCaucasian</td><td></td><td>-
12.653</td><td>6.447</td></tr>
## <tr><td style="text-
align:left"></td><td></td><td>(56.740)</td><td>(50.363)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-
align:left">Income</td><td></td><td></td><td>6.054<sup>***</sup></td></tr>
## <tr><td style="text-
align:left"></td><td></td><td></td><td>(0.582)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-
align:left">Constant</td><td>509.803<sup>***</sup></td><td>520.880<sup>***</s
up></td><td>230.029<sup>***</sup></td></tr>
## <tr><td style="text-
align:left"></td><td>(33.128)</td><td>(51.901)</td><td>(53.857)</td></tr>
```

```
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td colspan="4" style="border-bottom: 1px solid
black"></td></tr><tr><td style="text-
align:left">Observations</td><td>400</td><td>400</td><td>400</td></tr>
## <tr><td style="text-
align:left">R<sup>2</sup></td><td>0.0005</td><td>0.001</td><td>0.216</td></tr
>
## <tr><td style="text-align:left">Adjusted R<sup>2</sup></td><td>-
0.002</td><td>-0.007</td><td>0.208</td></tr>
## <tr><td style="text-align:left">Residual Std. Error</td><td>460.230 (df =
398)</td><td>461.337 (df = 396)</td><td>409.218 (df = 395)</td></tr>
## <tr><td style="text-align:left">F Statistic</td><td>0.184 (df = 1;
398)</td><td>0.092 (df = 3; 396)</td><td>27.161<sup>***</sup> (df = 4;
395)</td></tr>
## <tr><td colspan="4" style="border-bottom: 1px solid
black"></td></tr><tr><td style="text-align:left"><em>Note:</em></td><td
colspan="3" style="text-align:right"><sup>*</sup>p<0.1; <sup>**</sup>p<0.05;
<sup>***</sup>p<0.01</td></tr>
## </table>
```

```
stargazer(model21, model22_interaction, model23_interaction, type = "text",
title = "Regression Results for Balance Interaction", out =
"balance_regression_interaction_results.txt")
```

```
##
## Regression Results for Balance Interaction
##
=============================================================================
==================
##                                                        Dependent variable:
##                                   -----------------------------------------
--------------------
##                                                              Balance
##                                          (1)                 (2)
(3)
## ----------------------------------------------------------------------------
--------------------
## GenderFemale                            19.733              -44.449
-3.963
##                                        (46.051)            (92.796)
(100.569)
##
## EthnicityAsian                                              -100.577
15.555
##                                                             (94.251)
(108.524)
##
## EthnicityCaucasian                                           -38.109
94.101
##                                                             (80.907)
```

```
(98.149)
##
## Income
7.718***
##
(1.238)
##
## GenderFemale:EthnicityAsian                                 154.686
81.000
##                                                             (130.460)
(116.460)
##
## GenderFemale:EthnicityCaucasian                             50.609
57.707
##                                                             (113.566)
(101.005)
##
## EthnicityAsian:Income
-1.121
##
(1.588)
##
## EthnicityCaucasian:Income
-2.534*
##
(1.390)
##
## GenderFemale:Income
-0.525
##
(1.208)
##
## Constant                              509.803***          553.449***
177.363**
##                                        (33.128)            (65.947)
(84.037)
##
## -----------------------------------------------------------------------------
---------------------
## Observations                             400                 400
400
## R2                                      0.0005              0.004
0.224
## Adjusted R2                             -0.002              -0.008
0.206
## Residual Std. Error           460.230 (df = 398)  461.631 (df = 394)
409.699 (df = 390)
## F Statistic                   0.184 (df = 1; 398) 0.354 (df = 5; 394)
12.496*** (df = 9; 390)
##
```

```
==============================================================================
==================
## Note:
*p<0.1; **p<0.05; ***p<0.01
```

```r
stargazer(model21, model22_interaction, model23_interaction, type = "html",
title = "Regression Results for Balance Interaction", out =
"balance_regression_interaction_results.html")
```

```
##
## <table style="text-align:center"><caption><strong>Regression Results for
Balance Interaction</strong></caption>
## <tr><td colspan="4" style="border-bottom: 1px solid
black"></td></tr><tr><td style="text-align:left"></td><td
colspan="3"><em>Dependent variable:</em></td></tr>
## <tr><td></td><td colspan="3" style="border-bottom: 1px solid
black"></td></tr>
## <tr><td style="text-align:left"></td><td colspan="3">Balance</td></tr>
## <tr><td style="text-
align:left"></td><td>(1)</td><td>(2)</td><td>(3)</td></tr>
## <tr><td colspan="4" style="border-bottom: 1px solid
black"></td></tr><tr><td style="text-
align:left">GenderFemale</td><td>19.733</td><td>-44.449</td><td>-
3.963</td></tr>
## <tr><td style="text-
align:left"></td><td>(46.051)</td><td>(92.796)</td><td>(100.569)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-align:left">EthnicityAsian</td><td></td><td>-
100.577</td><td>15.555</td></tr>
## <tr><td style="text-
align:left"></td><td></td><td>(94.251)</td><td>(108.524)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-align:left">EthnicityCaucasian</td><td></td><td>-
38.109</td><td>94.101</td></tr>
## <tr><td style="text-
align:left"></td><td></td><td>(80.907)</td><td>(98.149)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-
align:left">Income</td><td></td><td></td><td>7.718<sup>***</sup></td></tr>
## <tr><td style="text-
align:left"></td><td></td><td></td><td>(1.238)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-
align:left">GenderFemale:EthnicityAsian</td><td></td><td>154.686</td><td>81.0
00</td></tr>
## <tr><td style="text-
align:left"></td><td></td><td>(130.460)</td><td>(116.460)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-
align:left">GenderFemale:EthnicityCaucasian</td><td></td><td>50.609</td><td>5
```

```
7.707</td></tr>
## <tr><td style="text-
align:left"></td><td></td><td>(113.566)</td><td>(101.005)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-
align:left">EthnicityAsian:Income</td><td></td><td></td><td>-1.121</td></tr>
## <tr><td style="text-
align:left"></td><td></td><td></td><td>(1.588)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-
align:left">EthnicityCaucasian:Income</td><td></td><td></td><td>-
2.534<sup>*</sup></td></tr>
## <tr><td style="text-
align:left"></td><td></td><td></td><td>(1.390)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-
align:left">GenderFemale:Income</td><td></td><td></td><td>-0.525</td></tr>
## <tr><td style="text-
align:left"></td><td></td><td></td><td>(1.208)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td style="text-
align:left">Constant</td><td>509.803<sup>***</sup></td><td>553.449<sup>***</s
up></td><td>177.363<sup>**</sup></td></tr>
## <tr><td style="text-
align:left"></td><td>(33.128)</td><td>(65.947)</td><td>(84.037)</td></tr>
## <tr><td style="text-align:left"></td><td></td><td></td><td></td></tr>
## <tr><td colspan="4" style="border-bottom: 1px solid
black"></td></tr><tr><td style="text-
align:left">Observations</td><td>400</td><td>400</td><td>400</td></tr>
## <tr><td style="text-
align:left">R<sup>2</sup></td><td>0.0005</td><td>0.004</td><td>0.224</td></tr
>
## <tr><td style="text-align:left">Adjusted R<sup>2</sup></td><td>-
0.002</td><td>-0.008</td><td>0.206</td></tr>
## <tr><td style="text-align:left">Residual Std. Error</td><td>460.230 (df =
398)</td><td>461.631 (df = 394)</td><td>409.699 (df = 390)</td></tr>
## <tr><td style="text-align:left">F Statistic</td><td>0.184 (df = 1;
398)</td><td>0.354 (df = 5; 394)</td><td>12.496<sup>***</sup> (df = 9;
390)</td></tr>
## <tr><td colspan="4" style="border-bottom: 1px solid
black"></td></tr><tr><td style="text-align:left"><em>Note:</em></td><td
colspan="3" style="text-align:right"><sup>*</sup>p<0.1; <sup>**</sup>p<0.05;
<sup>***</sup>p<0.01</td></tr>
## </table>
```

(e) Explain how gender affects “balance” in each of the models (a)- (c) .

ANSWER: In model for only gender, the R^2 is 0.0004611. Therefore, Gender alone is not a good predictor for Balance. In model for gender and ethnicity, the R^2 is 0.001. Therefore, R^2 is slightly higher than the previous model but it doe not explain the variance much better either. Hence, this model is also not very good, even though it is better than the previous one. In the model for Gender, Ethnicity and Income, the R^2 is 0.22. Hence, there is a significant portion of the variance in Balance compared to the previous two models. Therefore, it is a much better model than the previous two models.

(f) Compare the average credit card balance of a male African with a male Caucasian on the basis of model (b).

```
coef(model22)["EthnicityCaucasian"]

## EthnicityCaucasian
##          -12.65305
```

The coefficient of EthnicityCaucasian represents the expected difference in average credit card balance between a Caucasian male and an African American male.A negative value indicates African Americans carry higher balances.

(g) Compare the average credit card balance of a male African with a male Caucasian when each earns 100,000 dollars. For comparison, use the model in (c).

```
coef(model23)["EthnicityCaucasian"]

## EthnicityCaucasian
##           6.446938
```

When income is fixed at $100,000, a Caucasian male is expected to have an average credit card balance approximately 6.45 units higher than an African American male. Since income is held constant, this difference represents the independent effect of ethnicity on credit card balance.

(h) Compare and comment on the answers in (f) and (g)

The estimated difference in credit card balance changes from −12.53 in model (b) to +6.45 in model (c) after including income in the regression. This sign reversal indicates that income was a confounding variable. In the model without income, African American males appear to have higher balances because they tend to have lower income levels on average. After controlling for income, the true relationship is revealed: Caucasian males have slightly higher balances. Hence, failing to include income leads to a misleading conclusion.

(i) Based on the model in (c), predict the credit card balance of a female Asian whose income is 2000,000 dollars.

```
new_person <- data.frame(
  Gender = "Female",
  Ethnicity = "Asian",
  Income = 200
)

predict(model23, newdata = new_person)
```

```
##        1
## 1466.851
```

(j) Check the goodness of fit of the different models in (a) -(c) in terms of AIC, BIC and adjusted R2. Which model would you prefer?

```
AIC(model21, model22, model23)

##         df      AIC
## model21  3 6044.527
## model22  5 6048.434
## model23  6 5953.518

BIC(model21, model22, model23)

##         df      BIC
## model21  3 6056.501
## model22  5 6068.391
## model23  6 5977.466

summary(model21)$adj.r.squared

## [1] -0.002050271

summary(model22)$adj.r.squared

## [1] -0.006876514

summary(model23)$adj.r.squared

## [1] 0.207774
```

The model with the highest R^2 is preferred,i.e.,Model (C).

4. Problem to demonstrate the impact of ignoring interaction term in multiple linear regression

Consider a simulation setting where the data is generated as follows:

```
set.seed(123)

n <- 100
R <- 1000

config1 <- c(-2.5, 1.2, 2.3, 0.001)
config2 <- c(-2.5, 1.2, 2.3, 3.1)


run_simulation <- function(n, R, beta_params) {
  beta0 <- beta_params[1]
  beta1 <- beta_params[2]
  beta2 <- beta_params[3]
```

```
  beta3 <- beta_params[4]

  mse_correct <- numeric(R)
  mse_naive <- numeric(R)

  for (r in 1:R) {
    # Step 1: Generate x1 from Normal(0,1)
    x1 <- rnorm(n, mean = 0, sd = 1)

    # Step 2: Generate x2 from Bernoulli(0.3)
    x2 <- rbinom(n, size = 1, prob = 0.3)

    # Step 3: Generate response with interaction
    epsilon <- rnorm(n, mean = 0, sd = 1)
    y <- beta0 + beta1*x1 + beta2*x2 + beta3*(x1*x2) + epsilon

    # Step 4: Fit both models
    model_correct <- lm(y ~ x1 + x2 + x1:x2)
    model_naive <- lm(y ~ x1 + x2)

    # Computing MSE
    mse_correct[r] <- mean(residuals(model_correct)^2)
    mse_naive[r] <- mean(residuals(model_naive)^2)
  }

  return(list(
    avg_mse_correct = mean(mse_correct),
    avg_mse_naive = mean(mse_naive),
    mse_correct = mse_correct,
    mse_naive = mse_naive
  ))
}

run_simulation(100,1000,c(-2.5,1.2,2.3,0.001))

## $avg_mse_correct
## [1] 0.9631944
##
## $avg_mse_naive
## [1] 0.9739083
##
## $mse_correct
##    [1] 0.8511631 0.9602055 1.1195283 1.0043441 0.9340622 0.9593344
1.1642569
##    [8] 1.1217837 0.7978635 0.9382235 0.9265837 1.1240546 0.9878345
1.0495433
##   [15] 0.6668817 1.1806558 0.6025595 0.9339389 0.8400421 1.2028140
0.8633813
##   [22] 1.0396428 0.9655059 1.1248303 1.0716287 0.8190131 0.9240596
```

```
0.8843535
##  [29] 0.6744280 0.9594065 1.1208650 1.1101941 0.9813772 1.2152708
0.8430911
##  [36] 1.2270940 1.0253857 1.1018195 0.8591747 0.9338904 0.7121827
0.9649639
##  [43] 1.0669452 1.2806620 0.9574646 1.0840626 1.0005940 0.8219106
1.1961841
##  [50] 0.9380908 1.0340598 1.0619782 1.0055131 0.9719751 0.9622186
0.8783047
##  [57] 0.8798710 0.9631876 0.9282966 0.9892506 0.8134892 0.9759858
0.8908095
##  [64] 0.9958596 0.9997398 0.8272421 0.8642148 1.2440663 1.0556850
0.9838358
##  [71] 0.8523210 0.9755285 0.9104178 0.9629931 0.8014365 0.8772507
0.8260519
##  [78] 1.0479337 0.8041048 1.0284109 0.8043705 1.2396617 0.9307682
0.8792202
##  [85] 1.0052468 0.9068418 0.6728680 0.9841076 0.9301163 0.9200855
1.1005434
##  [92] 1.0220627 0.8038798 0.9072987 0.8796999 1.0559835 0.8851410
1.0213916
##  [99] 1.1977000 1.0053808 0.6830194 1.0359013 1.1257250 1.0222068
1.0414583
## [106] 0.7133538 0.8980224 0.9385607 1.0619702 0.9841011 0.8098888
0.8077855
## [113] 0.8377663 1.0108942 1.3325774 1.0718247 0.9465029 0.9248388
0.8482812
## [120] 0.9475343 0.9357535 0.9616382 1.2739665 0.7247223 0.9643491
0.8197339
## [127] 1.0164664 0.9169040 0.7566221 0.7616770 0.8755224 1.1988702
1.1218217
## [134] 0.9436985 1.0839092 0.9210328 0.8733366 0.9115994 1.0970721
1.3838465
## [141] 0.9299031 0.9842831 1.0557059 0.7613090 0.9428698 0.8144222
1.3202055
## [148] 0.7998735 1.0462277 1.0323445 1.1670110 0.8373549 0.9644205
0.9748160
## [155] 1.1487764 0.7440055 0.7344456 1.1146372 1.0718142 1.0083167
0.8633858
## [162] 0.8546137 0.8302891 0.9910272 1.1725070 0.8044684 1.0692872
1.1052489
## [169] 1.1927949 0.9967579 0.9502807 0.9110405 0.8373373 0.8212196
1.0750134
## [176] 1.1156961 0.9118338 0.8684574 1.1811836 1.0319844 0.8362548
0.9166377
## [183] 1.0031340 0.9571470 0.9113169 0.8251637 1.0658329 0.9817381
0.8999842
## [190] 0.9082262 0.8690223 1.0239034 0.8686320 0.7909859 0.9329321
0.7716948
## [197] 0.8502715 0.8908670 1.1475067 1.0671311 0.9745383 1.0073927
```

```
0.8874678
##  [204] 1.0623773 0.9593141 0.9410767 0.9402278 0.9953612 1.0018693
0.8984173
##  [211] 0.7673933 0.9177921 0.9331660 0.9592497 0.9233550 0.9109142
0.8657846
##  [218] 0.8371189 1.1997496 0.9982760 1.2596228 1.1094293 0.9587219
1.1618845
##  [225] 0.8311579 0.9505046 1.2102269 1.0348414 0.7664057 0.8399587
1.0520589
##  [232] 0.9983069 1.1139479 1.0459517 1.2298468 1.0324168 1.0209751
0.9087116
##  [239] 1.1001712 0.9487625 1.0699112 0.9862624 1.0493129 0.9003958
1.1446360
##  [246] 0.7327366 0.9183268 0.8345367 0.8838106 1.1416692 0.8760962
0.9274893
##  [253] 0.9197151 0.8739263 1.1645873 1.0964112 1.0141674 1.0373413
0.8671904
##  [260] 1.2037991 1.1102733 0.9891225 1.0396684 0.7478346 0.9276369
0.8243502
##  [267] 1.0483848 1.0784355 0.8196020 0.6435139 0.8449918 0.8434033
1.1376889
##  [274] 1.0931940 0.8355953 0.9446504 1.0107015 1.1004672 1.0825498
0.8803912
##  [281] 1.1148383 0.7658045 1.1650395 0.8071839 0.9216751 0.7879822
1.0347646
##  [288] 0.9931594 0.9786562 0.8456303 1.0753041 0.9549443 0.8768470
0.9790271
##  [295] 0.9512437 1.0090553 0.8963722 1.0677075 0.7730833 0.7030574
1.0353773
##  [302] 1.0073148 0.9551268 1.1039598 1.0512381 0.9397922 1.1298806
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##  [309] 0.8038165 0.9965239 0.8462117 1.0780739 0.9851992 0.9044195
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##  [316] 1.1185988 0.8645992 0.9040048 1.1661860 1.2244874 0.8245771
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##  [323] 0.8096475 1.0065176 1.1217337 0.9363156 0.7630746 1.1227853
0.9102020
##  [330] 1.0504810 1.1286700 0.8495741 0.9399326 0.9109247 0.8030193
0.7056881
##  [337] 0.9969815 0.9491856 0.7963832 0.9483114 1.0674882 0.9958003
0.9741195
##  [344] 0.9679985 1.0690359 0.9787183 0.7728853 0.5894895 0.9333693
1.0842927
##  [351] 1.0188193 0.8345744 0.6784259 0.8661270 1.1100438 0.9255922
0.9824966
##  [358] 1.0476429 0.7754814 1.1121418 1.0287572 0.9496679 0.8060536
0.8346197
##  [365] 0.9082714 0.9286577 1.0160974 1.1871341 1.0408925 0.8779071
1.0161434
##  [372] 0.8633521 0.8494342 1.0540352 0.8111737 0.8778540 0.9878683
```

```
0.9036609
## [379] 0.8771925 1.0909046 0.8986656 1.0345547 0.7580630 0.8752461
1.0748239
## [386] 0.8915372 0.8693073 0.7556820 0.9194303 0.9528398 0.9179080
0.9918571
## [393] 0.9469818 0.9222605 0.8353342 0.9543475 0.9143277 0.7988030
0.8229422
## [400] 1.0041790 0.8966792 0.9952749 1.0136736 0.9513469 1.1078721
0.7991733
## [407] 0.8907532 0.9910068 1.0564795 1.1911637 1.0178301 1.1040895
0.9739830
## [414] 0.9140624 0.8438129 0.9863337 0.8939198 0.9705782 1.2885807
1.3214273
## [421] 1.0550004 1.2033212 1.0307564 0.9121097 0.7604658 1.1415878
0.8322735
## [428] 0.7056745 1.0177417 0.9075746 1.2744340 0.8925297 0.9375638
0.7487907
## [435] 1.4760077 1.1083564 0.9683227 1.0487036 1.0623702 0.9978160
0.7456720
## [442] 0.7122337 0.7854537 0.7425151 1.0224450 0.7703527 0.9054747
0.8053234
## [449] 0.8955399 0.9563440 1.0888109 0.8895100 0.8622147 0.7438467
0.9037129
## [456] 0.9363023 0.7817102 0.6740603 0.7540881 1.0683877 0.9053403
0.8093094
## [463] 0.8542809 1.2809875 0.8475208 1.2059314 0.8413130 1.0053594
0.9892967
## [470] 0.9196343 0.7393179 1.0279381 0.7659777 0.8403848 0.9835555
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## [477] 0.9409488 0.9527642 1.0440132 0.8121112 1.0534595 0.6355414
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## [484] 0.9232794 1.0170377 1.2734881 0.7108981 0.9968929 1.1603135
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## [491] 1.1786981 0.9084437 1.2302191 0.8340054 0.8301872 0.9728867
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## [498] 0.7271535 0.9342151 0.8396705 0.8696465 0.8640846 1.0986029
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## [505] 1.0265974 1.0378058 0.9193949 0.8475196 1.2460689 0.9914062
1.0180326
## [512] 1.0140459 1.2193496 0.7834818 1.1263250 1.1837400 0.8767436
1.0659680
## [519] 1.0762163 1.1159727 0.9613519 0.9314537 0.9320698 1.0249718
0.9467831
## [526] 0.8492278 0.6990175 1.1091040 0.9335835 0.9885738 1.0453514
1.0578510
## [533] 1.2665251 0.8963095 1.0436355 0.7814509 0.9113549 0.9925145
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## [540] 0.9927381 0.8668994 1.1533584 1.0346731 1.1305932 1.0399505
0.9589033
## [547] 0.9966929 0.7424846 0.8055481 0.9842978 0.9972484 1.0619277
```

```
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## [554] 1.0033950 1.0588456 0.8145725 0.9088672 1.0786947 0.7443267
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## [561] 0.8432188 1.0290211 0.9210258 0.9157726 0.9766475 0.9701617
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## [568] 1.0336379 1.1166274 0.9629765 0.9361987 0.8590336 0.7840233
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## [575] 0.9425114 0.9908612 1.0760080 1.0788627 0.9548405 0.8160039
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## [589] 1.0746421 1.0905022 1.2502169 0.9082858 0.9659064 0.8259357
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## [596] 1.1267413 1.2185363 0.9990798 0.8076131 0.8175309 0.9699709
0.9827712
## [603] 0.9429004 0.7665865 0.7193629 0.8372963 0.9331224 0.9582386
0.6891521
## [610] 0.8608428 1.0824219 0.7895726 0.9942255 1.0856233 0.7993137
1.2728592
## [617] 0.9150314 1.3677203 0.9123550 0.8838979 1.2562597 0.6489036
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## [624] 1.1449130 1.0105841 0.9688973 1.0199609 1.0011243 1.3681754
0.7553080
## [631] 0.9857789 0.9609218 1.0623842 1.1011532 0.8657708 0.9542959
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## [680] 0.7815090 0.9297938 1.1987654 0.8930787 0.9222909 1.1696928
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## [687] 1.0047877 1.0498463 0.9419591 0.9085011 0.8936270 1.0532852
1.0379473
## [694] 1.0683057 0.9818992 0.8231210 0.9478612 1.0095956 1.0500767
0.9071455
## [701] 0.6429289 0.9320308 0.8419603 0.6949018 0.8743261 1.1624582
0.8377924
## [708] 0.8362051 0.9557119 1.1192818 1.1360605 0.7865803 1.0905291
0.9209087
## [715] 1.1467537 0.9667579 1.0983159 0.6884373 0.9669129 1.0570849
1.0045729
## [722] 1.2394085 0.7896253 0.8746715 0.9609754 1.0391212 1.0406079
```

```
1.1109063
## [729] 1.0377093 1.0551355 0.9245882 0.8875264 1.0267504 1.0265009
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## [736] 0.7441452 0.8162824 0.8085498 1.0719469 0.8206334 1.1952998
1.0618664
## [743] 1.0091641 0.9384406 1.0045541 0.7265244 0.9113332 1.1573979
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## [750] 0.9482847 1.1024732 1.1474175 0.8934700 0.9904544 1.1627627
0.8329686
## [757] 0.9741173 0.7301672 0.8744928 0.9585531 0.6310872 0.7891332
1.1067398
## [764] 1.0884137 1.0278045 0.8310355 1.1200030 1.1181239 1.0548566
0.7647872
## [771] 0.9658824 1.1120274 0.8786954 0.8188009 0.9709730 1.0561101
1.1203244
## [778] 1.0902559 0.8833024 1.1027794 0.9194011 0.9349235 0.9607619
1.0663334
## [785] 1.0081979 0.8195575 1.0655594 0.8416438 0.9437761 0.7891350
1.1704849
## [792] 1.0438620 0.9104020 0.8852160 1.2051778 0.9996888 0.9112317
1.0195133
## [799] 0.8597746 0.7730857 0.9961153 0.9370578 0.7500343 1.0872256
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## [806] 0.7056132 0.6716129 0.9419260 0.9277870 0.9695567 0.8486285
0.9223254
## [813] 1.0626918 1.0491288 1.1182080 0.8553636 0.8613231 0.9729924
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## [820] 0.9139486 0.8208689 1.1167198 0.8431280 1.2394092 1.2518124
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## [827] 0.9314023 1.0118985 1.0440131 1.1276552 1.2208124 0.9963020
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## [834] 0.7562642 0.8224575 1.0066630 1.0758256 0.8864422 1.2437492
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## [841] 0.8518498 0.9823101 0.9332222 0.8567361 0.9599755 0.9508922
0.8303164
## [848] 0.8369184 0.9321467 0.9115266 1.0358045 0.9714077 1.0846276
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## [855] 0.9389699 0.8808277 0.8604874 0.7675154 1.0175457 0.9215184
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## [862] 1.0357874 0.8716359 0.8246643 0.8716946 0.9794822 0.7374223
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## [869] 0.9550209 1.1335256 0.9925980 0.9064445 0.7448256 1.0856360
0.8620624
## [876] 1.0162574 1.0624302 0.9930108 0.6581080 0.7760515 1.0472793
0.8904384
## [883] 1.1019373 1.1373524 0.9099953 0.9182583 0.8890073 1.0744685
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## [890] 1.1325327 0.8810804 0.8261335 1.1465780 0.8649804 0.9735397
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## [897] 0.8287499 1.0089819 0.8338775 0.9478243 0.8511168 0.8891682
```

```
0.8008581
## [904] 0.8850360 1.1224686 0.9660277 1.0611539 0.9960375 0.9599898
0.8760305
## [911] 0.8662185 0.9491782 0.7913168 0.9621599 0.8677053 0.8277239
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## [918] 1.1293054 1.2831527 0.8082663 0.9805109 0.8799888 1.1984158
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## [925] 1.0903855 0.8505971 1.0193690 0.8426324 0.9528177 1.0611425
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## [932] 1.0683190 0.9487540 0.8355545 1.1285477 0.7979034 0.8571976
0.8656235
## [939] 1.1864508 0.9262073 1.0683090 0.7546839 0.7411063 1.1801814
0.9284578
## [946] 0.8846154 1.1013912 0.9734859 1.1242687 1.0421537 0.8023752
1.0436588
## [953] 0.8411009 0.9679811 0.9359572 1.2471643 1.0893277 1.0036793
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## [960] 0.9452838 1.2630859 1.0978421 0.9451154 0.8753061 0.6546946
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## [967] 0.8958048 0.8372799 0.6415689 1.1718064 0.8521984 0.8898518
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## [974] 1.0704282 0.7456072 1.1645761 0.9955257 0.9574365 0.9875092
1.0431884
## [981] 0.8470019 1.1464895 0.8778501 1.2708590 0.9530591 0.8307820
1.1465300
## [988] 0.7393223 1.1486756 0.9625048 1.0680470 1.2034204 0.8614590
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## [995] 0.8592475 0.9453090 1.0435888 1.0859716 1.0123363 1.0152866
##
## $mse_naive
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##    [8] 1.1280360 0.8096540 0.9382381 0.9268750 1.1874849 0.9937895
1.0773733
##   [15] 0.6671363 1.1877198 0.6597612 0.9908777 0.8471518 1.2052555
0.8641067
##   [22] 1.0785172 0.9664353 1.1251174 1.0726033 0.8191596 0.9310938
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##   [29] 0.6818754 0.9694225 1.1286457 1.1103167 0.9989993 1.2178650
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##   [36] 1.2284861 1.0277932 1.1149051 0.8907086 0.9620685 0.7165250
0.9662251
##   [43] 1.1021051 1.2829585 0.9641088 1.1032323 1.0010083 0.8267996
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##   [50] 0.9605973 1.0514178 1.0677463 1.0118074 0.9724955 1.0014961
0.9233793
##   [57] 0.9350516 0.9685093 1.0017758 1.0013012 0.8247659 0.9906849
0.8921092
##   [64] 0.9995053 1.0490817 0.8523082 0.8724316 1.2446259 1.1249906
0.9838753
```

```
##  [71] 0.8637732 0.9951065 0.9195045 0.9665087 0.8522498 0.8871895
0.8467198
##  [78] 1.0496464 0.8047934 1.0298787 0.8064342 1.2414703 0.9326416
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##  [85] 1.0125486 0.9080152 0.6737567 0.9883660 0.9315632 0.9202785
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##  [92] 1.0353937 0.8378892 0.9117919 0.8797002 1.0671732 0.9199436
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##  [99] 1.2038444 1.0057139 0.6904181 1.0709916 1.1290121 1.0249039
1.0437275
## [106] 0.7138803 0.9014737 0.9387776 1.0982531 0.9877121 0.8355818
0.8164830
## [113] 0.8390826 1.0182280 1.3417415 1.0752968 0.9557524 0.9434591
0.8616874
## [120] 0.9480702 0.9357643 0.9732485 1.2767985 0.7268378 0.9733451
0.8242632
## [127] 1.0214935 0.9197881 0.8023332 0.7652647 0.8819074 1.2088654
1.1287982
## [134] 0.9546868 1.0939713 0.9353922 0.9050584 0.9129690 1.1102062
1.3854038
## [141] 0.9376458 0.9889159 1.0560865 0.7624427 0.9502406 0.8157038
1.3292585
## [148] 0.8674612 1.0736978 1.0731477 1.1673907 0.8463322 0.9650959
1.0288242
## [155] 1.1527602 0.7476742 0.7517434 1.1224817 1.0865613 1.0084483
0.8981381
## [162] 0.8594193 0.8313961 1.0135403 1.1725363 0.8096480 1.0783203
1.1072220
## [169] 1.1932397 0.9980506 0.9695797 0.9122875 0.8476144 0.8428131
1.0759141
## [176] 1.1502744 0.9134141 0.8711609 1.1951598 1.0635584 0.8365036
0.9190144
## [183] 1.0062473 0.9880947 0.9152014 0.8268199 1.0679847 0.9830345
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## [190] 0.9366842 0.8725236 1.0285625 0.8686687 0.7931542 0.9394865
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## [197] 0.8543982 0.8910148 1.1486169 1.0772306 1.0018850 1.0101333
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## [204] 1.1046670 0.9822254 1.0024006 0.9421078 1.0292988 1.0026291
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## [211] 0.7713995 0.9211891 0.9565292 0.9604891 0.9236219 0.9226976
0.8659512
## [218] 0.8497493 1.2020009 0.9982824 1.2642255 1.1453828 0.9869642
1.1637325
## [225] 0.8481376 0.9586649 1.2133335 1.0357840 0.7664182 0.8404157
1.0534957
## [232] 1.0047657 1.1174753 1.0482023 1.2373666 1.0562207 1.0309016
0.9427818
## [239] 1.1042571 0.9490294 1.1183727 0.9979113 1.0514522 0.9015457
1.1509457
```

```
## [246] 0.7328157 0.9491026 0.8512627 0.8868813 1.1513216 0.8882463
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## [253] 0.9344304 0.8741609 1.1718351 1.0971549 1.0159924 1.0846576
0.8712832
## [260] 1.2065362 1.1120877 1.0044097 1.0538189 0.7495721 0.9292133
0.8249930
## [267] 1.0565522 1.0787855 0.8392769 0.6466759 0.9876407 0.8778538
1.1404885
## [274] 1.0933111 0.8393218 0.9465641 1.0151994 1.1004708 1.0826656
0.8863708
## [281] 1.1165051 0.7725410 1.1651829 0.8073696 0.9258529 0.8403815
1.0391816
## [288] 1.0250652 0.9844979 0.8472396 1.0816255 0.9549560 0.8868155
0.9821960
## [295] 0.9558658 1.0211161 0.9515184 1.0926122 0.7945212 0.7101900
1.0356712
## [302] 1.0198808 0.9581702 1.1118207 1.0980401 0.9397998 1.1445465
0.9988027
## [309] 0.8042020 1.0079397 0.8476477 1.0812867 1.0145095 0.9465956
1.0872385
## [316] 1.1740554 0.8668545 0.9057908 1.1667873 1.2380942 0.8287051
1.0565839
## [323] 0.8101724 1.0097650 1.1224967 0.9376969 0.7695354 1.1229524
0.9149508
## [330] 1.0536132 1.1603829 0.8618179 0.9435638 0.9110690 0.8193986
0.7061052
## [337] 1.0565307 0.9590554 0.7964032 0.9483187 1.0688503 0.9980228
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## [344] 0.9767239 1.0937631 0.9960672 0.7875896 0.6000690 0.9456365
1.1296876
## [351] 1.0191596 0.8465830 0.6909918 0.8687856 1.1106264 0.9396479
0.9857832
## [358] 1.0602249 0.7884911 1.1121590 1.0300941 0.9602206 0.8154703
0.8877661
## [365] 0.9319951 0.9375948 1.0266575 1.1930182 1.0448542 0.8940872
1.0259997
## [372] 0.8637941 0.8500520 1.0566542 0.8111739 0.8778763 0.9984490
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## [379] 0.8802898 1.0994006 0.9031053 1.0481855 0.7583821 0.8765721
1.1074268
## [386] 0.8921168 0.8730602 0.7590747 0.9222141 0.9643949 0.9397156
1.0005672
## [393] 0.9471282 0.9222763 0.8377234 0.9605276 0.9299507 0.7992107
0.8284993
## [400] 1.0085202 0.9001981 0.9965683 1.0193413 0.9560168 1.1133789
0.7995524
## [407] 0.8938415 1.0007633 1.0574992 1.2298006 1.0269873 1.1082010
0.9744083
## [414] 0.9216544 0.8611401 0.9888755 0.9030620 0.9847995 1.3080576
1.3223440
```

```
## [421] 1.0551070 1.2273748 1.0450596 0.9140513 0.7616664 1.1474989
0.8324469
## [428] 0.7059017 1.0445579 0.9110214 1.2783968 0.9330418 0.9491687
0.7487908
## [435] 1.4778342 1.1083851 0.9906470 1.0489679 1.0635993 1.0266146
0.7464954
## [442] 0.7222661 0.7878958 0.7443063 1.0530861 0.7822420 0.9071666
0.8076251
## [449] 0.8991922 0.9586903 1.0987297 0.8958953 0.8638371 0.7506096
0.9038033
## [456] 0.9371206 0.7872223 0.6846305 0.7543073 1.0734318 0.9092899
0.8131920
## [463] 0.8714202 1.2825211 0.8484584 1.2106424 0.8438090 1.0060766
0.9909833
## [470] 0.9234679 0.7655940 1.0358927 0.7697206 0.8411528 0.9964844
0.9652054
## [477] 0.9503741 0.9592740 1.0444209 0.8196998 1.0538318 0.6634054
0.9181114
## [484] 0.9307865 1.0275787 1.2970890 0.7300050 1.0250508 1.1645093
0.9128700
## [491] 1.2058930 0.9267579 1.2414480 0.8340090 0.8421853 0.9813677
0.7839482
## [498] 0.7587035 0.9389198 0.8446725 0.8707951 0.8745049 1.1148504
0.7653815
## [505] 1.0287877 1.0378163 0.9204172 0.8583129 1.2600397 1.0055814
1.0180500
## [512] 1.0163939 1.2213213 0.7842712 1.1467068 1.2035244 0.8774996
1.0659856
## [519] 1.0762627 1.1222615 0.9617461 0.9798649 0.9357351 1.0284576
0.9951168
## [526] 0.8548803 0.7113368 1.1297284 0.9415094 1.0326626 1.0511868
1.0580863
## [533] 1.2770761 0.9427876 1.0808345 0.8230944 0.9163478 1.0027718
1.0749440
## [540] 1.0741877 0.8809134 1.1546752 1.0488906 1.1421321 1.0416116
0.9642585
## [547] 1.0185195 0.7459591 0.8055966 0.9854831 0.9973993 1.0755038
0.9710880
## [554] 1.0033996 1.0618398 0.8232675 0.9102384 1.0895145 0.7473405
1.1424619
## [561] 0.8435223 1.0465506 0.9363828 0.9266835 0.9773559 0.9890944
0.9431003
## [568] 1.0337929 1.1437080 0.9673487 0.9387034 0.8711527 0.8015420
1.1995732
## [575] 0.9584291 1.0629214 1.0845470 1.0930796 0.9639623 0.8163718
0.9598799
## [582] 0.9426728 0.9174258 0.8117003 1.2128603 1.2018811 1.0112012
0.8011961
## [589] 1.1038775 1.0923561 1.2971816 0.9483979 0.9910454 0.8259725
1.0455629
```

```
##  [596] 1.1403619 1.2213656 1.0009253 0.8118545 0.8672389 0.9745643
0.9850076
##  [603] 0.9715046 0.7666297 0.7720854 0.8389536 0.9415472 0.9599344
0.7009493
##  [610] 0.8631263 1.0887829 0.8053857 1.0143166 1.0943797 0.7993162
1.2800370
##  [617] 0.9156379 1.3781858 0.9134266 0.8969553 1.2566927 0.6503143
0.9915746
##  [624] 1.1496256 1.0116068 0.9760524 1.0199626 1.0183910 1.3697865
0.7571422
##  [631] 0.9873534 0.9833588 1.0671287 1.1067931 0.8682387 0.9633466
0.9987198
##  [638] 1.1147253 1.1196865 1.0502554 0.9605438 1.0529398 1.0716859
1.0267279
##  [645] 0.9316333 0.9207433 0.9233069 0.9844603 0.9731045 1.0011995
1.1481276
##  [652] 0.8366453 1.0369689 0.8744040 0.7419529 1.0958338 1.0555270
0.9934483
##  [659] 0.9256468 0.9157851 0.8592632 1.1712736 0.8627417 0.8802022
0.9580467
##  [666] 0.7969809 1.0686131 1.0468672 0.6927830 0.8599540 0.9307236
0.8590547
##  [673] 1.1438406 0.9032198 0.8778404 0.9119161 1.1177340 0.9445205
0.9175530
##  [680] 0.7831658 0.9299504 1.2198416 0.8997115 0.9223045 1.1768601
0.8125211
##  [687] 1.0248663 1.0498760 0.9420304 0.9594153 0.9387283 1.0789108
1.0417421
##  [694] 1.1042207 0.9830336 0.8258165 0.9715133 1.0459984 1.0507018
0.9071877
##  [701] 0.6637136 0.9654875 0.8722057 0.6985662 0.9305787 1.1639359
0.8379658
##  [708] 0.8362150 0.9611879 1.1306876 1.1396246 0.7992114 1.1550100
0.9246871
##  [715] 1.1591966 0.9667680 1.1049115 0.7016378 0.9671306 1.0656229
1.0076149
##  [722] 1.2689259 0.7973434 0.8759518 0.9609892 1.0405633 1.0414005
1.1764489
##  [729] 1.0440818 1.0809327 0.9265387 0.8902256 1.0351364 1.0282148
0.8766990
##  [736] 0.7450887 0.8162932 0.8118342 1.0729294 0.8206800 1.2001866
1.0834426
##  [743] 1.0123707 0.9498801 1.0184491 0.7273890 0.9118181 1.1574679
1.1130756
##  [750] 0.9507255 1.1050234 1.1477582 0.9530502 0.9981133 1.1752710
0.8512130
##  [757] 0.9959186 0.7442756 0.8811635 0.9586478 0.6425220 0.7943146
1.1117287
##  [764] 1.0884588 1.0371396 0.8318934 1.1228528 1.1188722 1.0609649
0.7689137
```

```
## [771] 0.9662601 1.1122419 0.8788057 0.8194633 1.0168922 1.0565055
1.1784895
## [778] 1.1029036 0.8971518 1.1097438 0.9304455 0.9394492 0.9609774
1.0765664
## [785] 1.0120195 0.8365487 1.0738960 0.8432060 0.9469556 0.7977836
1.1962084
## [792] 1.0587447 0.9347311 0.8944270 1.2116215 1.0094990 0.9138958
1.0196629
## [799] 0.8898941 0.7847512 0.9966382 0.9398703 0.7667340 1.0872260
0.8618611
## [806] 0.7056496 0.7220644 0.9468175 0.9505328 0.9703546 0.8700967
0.9239650
## [813] 1.0689545 1.0492786 1.1195216 0.8643914 0.8657829 0.9742803
0.8538210
## [820] 0.9226265 0.8512042 1.1275235 0.8432064 1.2395092 1.2518125
1.2935528
## [827] 0.9428973 1.0268709 1.0453898 1.1438628 1.2208504 1.0040346
0.9900904
## [834] 0.7587356 0.8251138 1.0075136 1.0766458 0.8867657 1.2863825
1.0604600
## [841] 0.8518753 0.9837611 0.9344463 0.8677111 0.9661369 0.9866563
0.8306484
## [848] 0.8499370 0.9322052 0.9133004 1.0534351 0.9820143 1.0999641
0.9869359
## [855] 0.9405826 0.8813308 0.8610068 0.7905097 1.0185432 0.9255889
1.1362857
## [862] 1.0387987 0.8823332 0.8504344 0.8841840 0.9828191 0.7401638
1.1662430
## [869] 0.9627562 1.1404850 1.0416294 0.9778481 0.7465372 1.0968608
0.8677945
## [876] 1.0264724 1.0992874 0.9946758 0.6630985 0.7779015 1.0475250
0.8938373
## [883] 1.1039543 1.1635989 0.9102414 0.9209075 0.8893095 1.0989529
0.7073901
## [890] 1.1590700 0.8811212 0.8265946 1.1485188 0.8951863 0.9735569
1.0601792
## [897] 0.8375011 1.0165636 0.8343898 0.9551163 0.8513007 0.9113405
0.8020737
## [904] 0.9048276 1.1517662 0.9660278 1.0653617 1.0026553 0.9618279
0.8844522
## [911] 0.8783917 0.9497463 0.8009409 0.9672396 0.8860847 0.8280819
1.2453204
## [918] 1.1666692 1.3020120 0.8180182 0.9826560 0.8799992 1.2178025
0.6462672
## [925] 1.0967900 0.8508074 1.0252572 0.8483772 1.0231700 1.0629312
1.1981370
## [932] 1.0855794 0.9873546 0.8379503 1.1403405 0.8132239 0.8586995
0.8756737
## [939] 1.1881970 0.9287282 1.0742641 0.7546869 0.7497437 1.1881718
0.9402285
```

```
##  [946] 1.0210706 1.1023214 1.0061317 1.1413350 1.0424490 0.8307539
1.0437293
##  [953] 0.8492170 1.0135811 0.9388490 1.2586681 1.0959099 1.0037027
1.1159163
##  [960] 0.9580994 1.2679540 1.1334560 0.9456961 0.8753061 0.6554439
1.1421773
##  [967] 0.9041792 0.8394908 0.6419015 1.1870238 0.8556206 0.8924291
1.2116238
##  [974] 1.0764129 0.7481794 1.1647019 0.9982562 0.9593189 0.9882880
1.0499527
##  [981] 0.8619751 1.1645632 0.8833765 1.2746876 0.9541308 0.8337491
1.1466968
##  [988] 0.7394465 1.1590651 0.9778340 1.0681420 1.2036478 0.9341402
0.8994253
##  [995] 0.8602949 0.9469253 1.0583382 1.0874459 1.0171502 1.0154366
```

When the true interaction coefficient is negligible (β_3 = 0.001), omitting the interaction term has virtually **no impact** on model performance.

```
run_simulation(100,1000,c(-2.5,1.2,2.3,3.1))

## $avg_mse_correct
## [1] 0.9577982
##
## $avg_mse_naive
## [1] 2.863335
##
## $mse_correct
##    [1] 0.9361545 0.8990821 0.8566232 0.9398601 0.7623377 1.0244665
1.0927969
##    [8] 0.8856149 0.7915997 0.9207781 0.8624195 0.7888115 1.0326013
0.9159760
##   [15] 1.0669444 1.1187375 1.2615162 0.9316154 0.8317768 1.0679249
0.8696638
##   [22] 0.8524500 0.9713539 1.0152585 0.9934668 0.8122017 0.9048086
0.9264103
##   [29] 0.9954714 0.8660562 1.0785476 0.9752150 0.9175464 1.0012129
1.0334925
##   [36] 1.0672425 1.0688819 0.7946862 1.2908628 0.8706567 1.0668883
1.0030914
##   [43] 1.2888980 0.7544823 0.7214420 0.8893845 1.0886203 0.9427252
0.9291164
##   [50] 1.1003089 1.0845886 1.1180871 1.2392893 1.0029177 1.0108469
0.8070356
##   [57] 0.8064031 0.7093645 0.8566933 1.0079025 0.7409510 0.9532615
0.9182601
##   [64] 0.8500456 0.9764679 0.9320910 1.0800195 0.6774569 1.0899296
0.9725546
##   [71] 0.9321122 1.1354495 0.9044048 0.7843419 1.0901142 0.8032159
1.0948214
```

```
##  [78] 1.2040666 0.7739376 0.8875357 0.9301022 1.1106742 0.7988840
0.8212890
##  [85] 1.1118360 0.8559778 0.9329654 0.9083649 1.0091308 1.0682698
0.9438619
##  [92] 1.0816142 1.0258453 0.9546682 0.8410744 1.0324974 1.0265723
1.0486084
##  [99] 1.0467047 0.7269147 0.9910388 0.8849950 0.9231992 1.0574773
0.8270629
## [106] 0.7196046 0.7609113 0.8842548 1.0306970 0.8790769 1.0020971
0.9460150
## [113] 0.8853201 1.1917357 0.9268762 0.9011623 0.8879754 0.9437960
0.7581105
## [120] 0.9738525 1.1062614 1.2150910 0.9586216 0.8568369 0.8471034
1.0183479
## [127] 0.7577515 1.2114773 0.9677322 0.7423618 1.0019370 0.9230742
0.9643771
## [134] 0.7915705 0.9821467 0.9777995 1.5013810 1.0998828 0.7930936
0.8544296
## [141] 0.7861671 0.9389253 0.9201414 0.8587984 0.9698436 0.8854372
0.8768184
## [148] 1.3289878 1.0844564 0.9821031 1.0257839 0.9744673 0.9482338
1.1463660
## [155] 1.0844612 0.8827037 1.2549981 0.9623703 1.0391141 0.8846125
0.9769902
## [162] 0.7760356 1.0962084 1.2946534 1.1204317 1.0250022 0.8453335
0.9598585
## [169] 1.0131074 0.8387880 0.9069129 0.9678524 1.0275583 0.7888375
1.1717725
## [176] 0.9622671 0.8626203 1.1392128 0.7040935 0.8609095 0.9891224
0.9194963
## [183] 0.8904678 0.6334799 0.8222465 0.8332287 1.1229189 1.0053920
0.8575848
## [190] 0.8737016 1.0208760 0.9772593 0.9648910 0.8289035 0.8645470
1.0175362
## [197] 0.8917951 1.0294155 0.7151337 1.0031777 0.7198873 0.9474143
1.0384723
## [204] 1.0408619 0.8980446 0.9338481 0.7300061 1.1518613 1.0189672
0.7055504
## [211] 0.9641013 0.9038130 1.2304365 0.9981491 1.0283878 0.6450458
0.7618279
## [218] 1.2260050 0.9869282 0.8417837 0.9037629 1.0124335 1.3857188
0.8714561
## [225] 0.7609785 1.0057915 0.9135895 0.7111046 0.8964854 0.9698883
1.0777977
## [232] 0.7004975 0.9953234 0.6316139 1.2028539 1.1113403 1.2118171
1.0513480
## [239] 1.1356997 1.1116658 1.0129824 1.0437220 1.1342484 0.8691220
0.7327770
## [246] 1.1586254 0.9615696 0.7173504 0.9411977 0.7704434 0.9289765
0.9775554
```

```
## [253] 0.7487299 0.7824630 0.8274480 1.3002326 0.8938576 0.8035643
1.0993674
## [260] 1.2287194 0.8497772 0.9937745 0.8931644 1.0960179 1.1586650
0.8790313
## [267] 1.0573877 0.7693386 1.1817637 0.8361020 0.9150779 0.9569142
0.8564640
## [274] 0.9061593 1.0379216 0.9067828 1.0574433 1.0065283 1.0435883
0.9288803
## [281] 0.7633712 1.1455051 0.9207118 1.2124945 0.8532639 1.1009438
0.8555757
## [288] 0.9053347 0.8384516 0.6874961 1.1281134 0.8898763 0.8710059
0.8885668
## [295] 0.9575980 1.1575850 1.3817699 0.8340211 1.1088103 0.9161409
0.7920637
## [302] 0.9102233 1.0979247 0.9848324 1.0193055 0.7564745 0.8219685
1.3220606
## [309] 1.0924950 0.8111728 0.6944896 1.1547901 1.0256480 1.0032320
0.9093133
## [316] 0.8655303 0.9178649 0.9497310 0.8771125 1.0965853 0.8211081
0.8047440
## [323] 1.1069910 1.1536223 0.9691837 1.3330681 0.9347032 1.0971730
0.8734772
## [330] 1.1305873 0.9444613 1.2294365 0.9882098 0.9028136 1.1622214
1.0341283
## [337] 0.9548900 0.9563632 0.8774541 0.8500951 0.9355155 0.8207613
0.8667338
## [344] 0.9204586 0.8253232 0.7688911 1.0182378 0.7977529 1.1179595
0.8381644
## [351] 1.0759129 0.8597196 0.9946024 0.8572492 1.1186971 1.0283072
0.8634575
## [358] 0.9363954 0.9951528 1.0015006 0.8601266 0.9451882 1.0068546
1.1195834
## [365] 0.9546632 0.9382029 0.9872500 1.1233052 0.8810696 0.9729866
0.9518423
## [372] 1.0702567 1.0911178 0.8294476 0.9627929 1.0745020 0.8016776
0.9878672
## [379] 1.0648439 1.0362648 0.9220304 0.9232978 0.9889680 0.8337160
1.0981203
## [386] 0.8182776 1.0595772 1.1286169 0.7667520 1.1457544 0.8718021
0.8702621
## [393] 0.7159424 0.9295490 1.0892610 0.8987110 1.1618812 0.9114219
1.0358046
## [400] 0.7865745 0.9409889 0.9977586 1.2079069 0.9390894 0.9861883
0.9464881
## [407] 0.8432442 0.9151957 0.7813678 0.7431874 1.1154252 0.9265377
0.9542373
## [414] 0.8213268 1.0288713 0.7761481 1.0237507 0.9829092 1.0500058
1.2115333
## [421] 1.2908454 0.7736512 0.8380551 0.8810190 1.0659142 1.0887672
1.0577107
```

```
##  [428] 0.8355960 0.9189144 1.0750378 0.9011302 1.4429013 0.8436319
1.3172685
##  [435] 0.9624655 1.0289159 0.9979174 0.6698347 1.0045354 0.8387588
1.0983243
##  [442] 0.8512539 0.7609243 0.8932473 1.0240899 0.9295669 1.0871644
0.9270119
##  [449] 0.9512467 0.9370781 1.0093900 1.0227971 0.9494176 0.8998242
1.0101323
##  [456] 0.9554314 0.7990500 0.8447479 0.8493617 1.1020451 0.8341017
0.9236504
##  [463] 0.8858368 0.9675076 1.0037023 0.8917944 1.0889579 0.7324787
1.2385317
##  [470] 0.8670215 0.9592385 0.9703518 0.8358067 0.9829378 0.8363364
0.8407570
##  [477] 0.8287115 0.8359400 0.9866670 0.8592042 0.9624931 0.9105252
1.0587821
##  [484] 1.1070618 0.8802474 0.8520995 0.9041884 0.8529144 1.0392183
1.1629794
##  [491] 0.7494924 0.6825400 0.9755624 1.1127720 0.8601300 0.8815698
1.1421880
##  [498] 0.6956854 0.8787940 0.8479427 0.8118096 1.0781806 0.9081502
0.8314968
##  [505] 0.8672841 0.7164352 0.8966362 0.8892416 0.8948141 1.0261927
0.9103856
##  [512] 0.8465939 0.8379965 1.2658096 1.0028633 1.2195917 0.8407961
0.8869090
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##  [889] 3.060997 1.987377 2.932817 2.709428 1.892953 2.386577 2.417458
2.157212
##  [897] 2.537061 2.690525 2.858524 2.105420 2.504969 4.066607 2.747682
3.520235
##  [905] 2.675672 1.994059 2.812631 3.307778 2.673311 3.039028 2.602183
2.115906
##  [913] 2.922468 2.156698 2.344956 2.794430 2.598261 3.192036 2.461165
2.673478
##  [921] 2.285796 2.280578 2.983789 2.267631 3.766976 3.211847 2.645807
2.267646
##  [929] 3.803285 2.630454 2.419656 4.751555 2.261104 2.123737 2.242267
2.769411
##  [937] 2.776040 3.146272 2.960589 3.347170 2.878937 3.767437 2.223880
```

```
2.290369
##  [945] 3.838821 2.932433 2.936081 2.865833 2.267757 3.200144 1.975756
2.967974
##  [953] 2.912556 3.843412 1.654340 2.529568 3.308692 2.823633 3.148048
2.263552
##  [961] 2.715755 3.245715 3.675446 2.526412 2.492896 2.505958 3.299136
2.771571
##  [969] 3.148862 2.860408 3.168466 3.277974 2.294277 3.817873 3.190581
3.919228
##  [977] 2.567861 2.177704 2.713815 2.986358 3.900779 3.024350 3.442627
2.068665
##  [985] 2.996388 2.570318 2.949871 1.801064 1.916392 3.055145 3.616046
2.846076
##  [993] 2.371021 2.231460 3.013773 3.006388 2.660884 2.208140 2.573946
2.602913
```

When the true interaction coefficient is large ($\beta_3 = 3.1$), omitting the interaction term has a **big impact** on model performance.