Q1 We begin by permanently labeling node 1 and assigning temporary labels to other nodes: [0* 2 8 ∞ ∞]. Then we give node 2 a permanent label: [0* 2* 8 ∞ ∞]

Node	Temporary Label (* denotes next assigned permanent label)
3	min{8,2+5} = 7
4	min{∞, 2+4} = 6*
5	min{∞,2+12} = 14

Now labels are [0* 2* 7 6* 14]

Node	Temporary Label (* denotes next assigned permanent label)
3	min{7} = 7*
5	min{14,6+10} = 14

Now labels are [0* 2* 7* 6* 14]. Since there is no node joining the newest permanently labeled node (node 3) to node 5, we may give make node 5 a permanent label. We obtain [0* 2* 7* 6* 14*]. Since c_{25} = 14 - 2 and c_{12} = 2 - 0 we find the shortest path from node 1 to 5 to be 1-2-5(length 14).

Q2 We begin by giving node 1 a permanent label [0* 2 1 ∞]. Next node 3 obtains a permanent label: [0* 2 1*∞]. Node 4 now obtains a new temporary label of min{∞,1+1} = 2, and node 2 obtains a permanent label yielding [0* 2* 1* 2]. Finally node 4's temporary label becomes permanent and we obtain [0* 2* 1* 2*] which yields the shortest" path 1-3-4. Of course, 1-2-3-4 with length 1 is a shorter path, but we fail to find this because Dijkstra's method works only when all the lengths are non-negative.

Q3 We have to first formulate the problem as a shortest path problem. The underlying graph has 8 nodes, labeled 1 to 8. An arc (i,j) exists whenever i<j. A path from 1 to 8 corresponds to a solution of the original problem. For instance, the path ((1,2),(2,4),(4,8)) means that 400 type-1 boxes, 300+500=800 type-2 boxes, 700+200+400+200=1,500 type-4 boxes are produced. The costs of the arcs are given in the following table:

1	14,200	24,100	40,600	63,700	70,300	83,500	90,100
2		10,000	25,000	46,000	52,000	64,000	70,000
3			14,000	32,200	37,400	47,800	53,000
4				17,800	22,600	32,200	37,000
5					4,800	12,400	16,200
6						8,200	11,800
7							4,400

For example, c36 = 1000 + 26(500+700+200) = \$37,400.

We now use the Dijkstra's Algorithm to solve the shortest path problem. (Alternatively, you can use the transportation simplex method to solve it. In that case, you just need to add a cost 0 to every route (i,i), and a cost M (huge constant) to every route (i,j) such that i>j. Also, you need to assign a supply or demand of 1 to each row and each column.)

0	∞	∞	∞	∞	∞	∞	∞
0*	14,200	24,100	40,600	63,700	70,300	83,500	90,100
	1 ->	1 ->	1 ->	1 ->	1 ->	1 ->	1 ->
0*	14,200*	24,100	39,200	60,200	66,200	78,200	84,200
	1 ->	1 ->	2 ->	2 ->	2 ->	2 ->	2 ->
0*	14,200*	24,100*	38,100	56,300	61,500	71,900	77,100
	1 ->	1 ->	3 ->	3 ->	3 ->	3 ->	3 ->
0*	14,200*	24,100*	38,100*	55,900	60,700	70,300	75,100
	1 ->	1 ->	3 ->	4 ->	4 ->	4 ->	4 ->
0*	14,200*	24,100*	38,100*	55,900*	60,700	68,300	72,100
	1 ->	1 ->	3 ->	4 ->	4 or 5 ->	5 ->	5 ->
0*	14,200*	24,100*	38,100*	55,900*	60,700*	68,300	72,100
	1 ->	1 ->	3 ->	4 ->	4 or 5 ->	5 ->	5 ->
0*	14,200*	24,100*	38,100*	55,900*	60,700*	68,300*	72,100
	1 ->	1 ->	3 ->	4 ->	4 or 5 ->	5 ->	5 ->
0*	14,200*	24,100*	38,100*	55,900*	60,700*	68,300*	72,100*
	1 ->	1 ->	3 ->	4 ->	4 or 5 ->	5 ->	5 ->

We find that the shortest path is 1-3-4-5-8. The minimum total cost of $72,100 is attained by using a size 33 box for size 33 and 30 demand, a size 26 box for size 26 demand, a size 24 box for size 24 demand, and a size 19 box for the remaining demand

Q4 max $z = x_0$

s. t. $x_{so,1} \leq 2$, $x_{12} \leq 4$, $x_{1,si} \leq 3$, $x_{2,si} \leq 2$, $x_{23} \leq 1$, $x_{3,si} \leq 2$, $x_{so,3} \leq 1$

$x_0 = x_{so,1} + x_{so,3}$ (Node so)

$x_{so,1} = x_{1,si} + x_{12}$ (Node 1)

$x_{12} = x_{23} + x_{2,si}$ (Node 2)

$x_{23} + x_{so,3} = x_{3,si}$ (Node 3)

$x_{1,si} + x_{2,si} + x_{3,si} = x_0$ (Node si)

All variables ≥ 0

We begin with a flow of 0 through each arc. Label the sink by (so, 1)-(1, 2)-(2, 3)-(3, si). We can increase the flow on each of these arcs by one unit, obtaining the following feasible flow:

Arc	Flow
(so,1)	1
(so,3)	0
(1,2)	1
(1,si)	0
(2,3)	1
(2,si)	0
(3,si)	1
Flow to Sink	1

We next label the sink by (so, 1)-(1, 2)-(2, si) and increase the flow in each of these arcs by 1 unit. We obtain the following feasible flow:

Arc	Flow
(so,1)	2
(so,3)	0
(1,2)	2
(1,si)	0

(2,3)	1
(2,si)	1
(3,si)	1
Flow to Sink	2

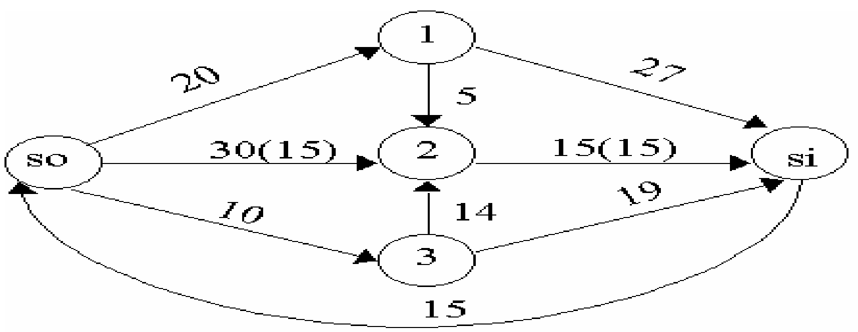
Now we label the sink by (so, 3)-(3, si), and increase the flow in each of these arcs by 1 unit. We obtain the following feasible flow:

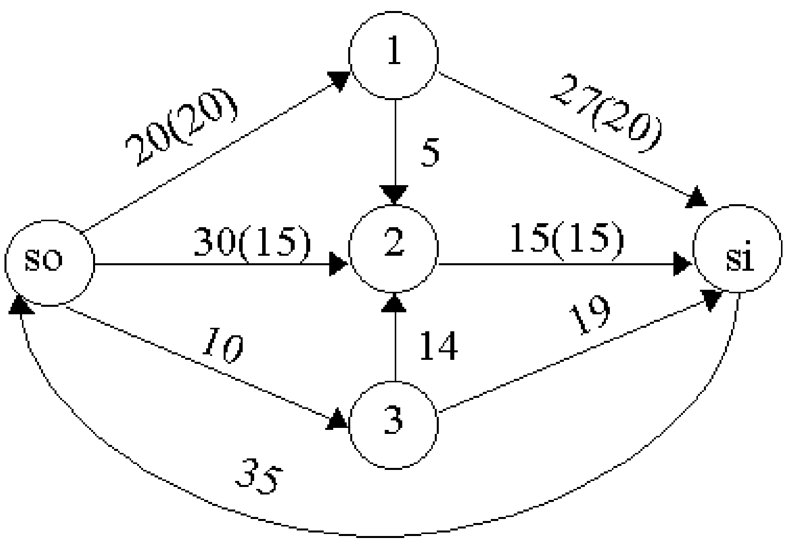
Arc	Flow
(so,1)	2
(so,3)	1
(1,2)	2
(1,si)	0
(2,3)	1
(2,si)	1
(3,si)	2
Flow to sink	3

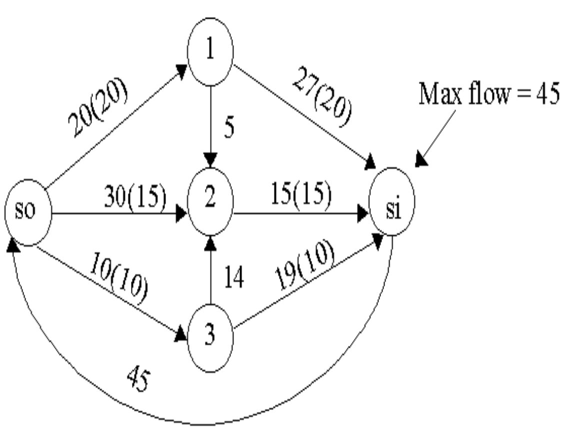
Now the sink cannot be labeled, so we have obtained a maximal flow. V' = {1, 2, 3, si} yields the minimal cut (arcs (so, 1) and (so, 3)) with a capacity of 2+1=3 = maximal flow.

Q5 Maximum flow is 45. Min Cut Set = {1, 3, and si}.

Capacity of Cut Set = 20 + 15 + 10 = 45. See Figure.







Q6 All arcs from month i workers to Project j have a capacity of 6. All projects can be completed if and only if the maximum flow from source to sink equals 30.