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Discrete-Time Switched Systems: Pole Location and Structural Constrained Control

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Abstract

This paper addresses the problem of state feedback control of discrete-time switched systems with linear modes of operation. Sufficient linear matrix inequality conditions are given for the existence of a state feedback control law assuring: *i)* pole location inside a circle for each linear mode of operation; *ii)* overall stability for any arbitrarily fast switching sequence. Two feedback control laws are investigated: the first one with a fixed gain for all linear modes and the second using a switched gain. Moreover, structural constraints such as decentralization can be easily imposed to the feedback gains. Simulation results show that with the switched gain more stringent design specifications can be imposed to the closed-loop system, providing closed-loop performances that cannot be produced by means of a constant feedback gain.

Key-words: Discrete-time switched systems; Pole location; Structural constrained control; Parameter dependent Lyapunov function; Linear matrix inequalities.

1 Introduction

Switching among different system structures is an essential feature of many engineering control applications. Examples include gain scheduling, switched power converters, pulse width modulation control [1], [2] and hybrid systems in general [3], [4]. This paper is focused on an important class of hybrid systems: switched systems with N linear modes of operation and arbitrary switched functions.

One of the most useful tools for the stability analysis of arbitrarily fast time-varying systems, which includes the switched systems under consideration here, is the quadratic stability condition [5], based on a fixed Lyapunov function. This approach allows to determine a fixed state feedback control gain for the stabilizability problem through a simple variable transformation [6], [7]. Although the quadratic approach is numerically simple and admits a linear matrix inequality (LMI) formulation, many times it can lead to conservative results, since the same fixed Lyapunov function is used to guarantee the closed-loop stability. Less conservative conditions for analysis and synthesis, based on piecewise quadratic Lyapunov functions have appeared in [8], [9], [10], in general related to piecewise linear systems (a linear mode of operation for each region of the state space). Necessary and sufficient LMI conditions for the stability and stabilizability of discrete-time switched systems with linear modes of operation have been given in [11], extending previous results for discrete time-varying systems based on parameter dependent Lyapunov functions [12].

In order to guarantee that each linear mode of the switched system will exhibit a desired performance level, some specific pole location can be imposed [13]. If the switching sequence is slow enough, the system dynamics response can be significantly improved by fixing the closed-loop pole location for each mode. Many kind of regions could be chosen for pole location (see, for instance, [14]) but, as discussed in [15], the choice of a circle with appropriate radius inside the unit circle in the complex plane can guarantee the desired rate of asymptotic damping for a linear system. By specifying a pole location to all the N linear modes of a discrete-time switched system, the closed-loop dynamics of each linear mode can be adequately chosen. Note that the overall stability of the switched system must also be assured for all possible transitions, since the local stability of each linear mode does not imply on the global stability.

This paper presents sufficient conditions for the existence of a state feedback control gain assuring to each linear mode of a discrete-time switched system a closed-loop pole location inside a circle contained in the unit circle of the complex plane, or at least guaranteeing its stability under any arbitrarily fast switching sequence. It can be viewed as an extension of [11], [13], to cope with pole location inside a circle and structural constrained control, and also as an extension of [16] in the context of switched systems. The conditions are formulated as a set of LMIs described in terms of the system modes. Numerical evaluations show that the switched control gain can provide more stringent design specifications than the ones obtained through constant control gains.

2 Preliminaries

2.1 Stability analysis

Consider the autonomous discrete-time switched system given by

$$x_{k+1} = A_{\alpha(k)} x_k \tag{1}$$

with state $x_k \in \mathbb{R}^n$ and dynamic matrix $A_{\alpha(k)} \in \mathbb{R}^{n \times n}$. The time-varying parameter $\alpha(k)$ describes a switching rule, defined by $\alpha(k) : \mathbb{N} \longrightarrow I$, with $I = \{1, \ldots, N\}$. Such systems are called switched systems and belong to the class of hybrid systems [4].

The quadratic stability [5], largely used in the analysis of time-varying uncertain systems, is a sufficient condition for the asymptotic stability of the system (1). If there exists $P = P' > 0$ such that the linear matrix inequality

$$A_j' P A_j - P < 0 \;;\; j = 1, \ldots, N \tag{2}$$

has a solution, the autonomous switched system (1) is asymptotically stable. Equation (2) is a sufficient condition for the stability of any operation mode j of the system and of any possible transition $j \to i$ with $i, j \in I$. However, in many times, this condition can lead to conservative results.

In [12], parameter-dependent Lyapunov functions have been used to verify the robust stability of time-varying uncertain systems in polytopic domains. A sufficient condition for the stability of system (1) under any switching rule $\alpha(k)$ is given by the existence of $P_j > 0$, $j = 1, \ldots, N$, such that

$$A_j' P_i A_j - P_j < 0 \;;\; \forall (i, j) \in I \times I \tag{3}$$

Following the ideas from [17], the above condition has been rewritten in an equivalent way in [11], using Schur complement and including new matrix variables. Then, the autonomous system (1) is stable if there exist N symmetric positive definite matrices S_j, $j = 1, \ldots, N$ and N matrices G_j, $j = 1, \ldots, N$ such that

$$\begin{bmatrix} G_j + G_j' - S_j & G_j' A_j' \\ A_j G_j & S_i \end{bmatrix} > 0; \; \forall (i, j) \in I \times I \tag{4}$$

Moreover, the use of condition (4) with $G_j = G$, $j = 1, \ldots, N$, can provide a state feedback gain obtained independently of matrices S_i, S_j used to ensure the stability. A switched state feedback control can be obtained from (4), and in this last case with gains independent of matrices S_i, S_j, which can represent an important degree of freedom that can be used, for instance, in decentralized control design.

2.2 Stabilizability

Consider now the discrete-time switched system

$$x_{k+1} = A_{\alpha(k)} x_k + B_{\alpha(k)} u_k \tag{5}$$

with state $x_k \in \mathbb{R}^n$ and control input $u_k \in \mathbb{R}^m$. Matrices $A_{\alpha(k)} \in \mathbb{R}^{n \times n}$ and $B_{\alpha(k)} \in \mathbb{R}^{n \times m}$ define respectively the open-loop dynamics and the input. Again, $\alpha(k)$ is assumed as being a switching rule, given by $\alpha(k) : \mathbb{N} \longrightarrow I$, with $I = \{1, \ldots, N\}$.

The problem of state feedback stabilizability is as follows: find, if possible, a state feedback control law u_k such that the closed-loop discrete-time switched system given by (5) becomes stable.

A simple solution for that is given through the quadratic stabilizability condition, that is, if there exist a symmetric positive definite matrix $S \in \mathbb{R}^{n \times n}$ and $Z \in \mathbb{R}^{m \times n}$ such that

$$\begin{bmatrix} S & SA_j' + Z'B_j' \\ A_j S + B_j Z & S \end{bmatrix} > 0; \; j = 1, \ldots, N \tag{6}$$

then $u_k = K x_k$, with $K = Z S^{-1}$, stabilizes the switched system (5). This result comes from (2) by applying Schur complement, pre and post multiplying by *block diag*$\{P^{-1}, P^{-1}\}$, replacing A_j by $A_j + B_j K$ and making the usual change of variables $S = P^{-1}$ and $KS = Z$ [7].

Using similar ideas, a switched gain $K_j = Z_j S_j^{-1}$ can be obtained from condition (3). The same applies for condition (4) in which case the gain is given by $K_j = Z_j G_j^{-1}$, as presented in [11] in the context of static output feedback design. Note that, by fixing $G_j = G$, $Z_j = Z$ one has a constant state feedback design which is less conservative than the one based on quadratic stability, in the sense that a stabilizing gain can be obtained in situations where there is no quadratically stabilizing control gain.

In next section, an extension of these results is presented, improving locally the dynamics of the switched system by imposing a specific pole location for each linear mode.

3 Pole location

The aim here is to determine a state feedback control law

$$u_k = K_{\alpha(k)} x_k \tag{7}$$

with $K_{\alpha(k)} \in \mathbb{R}^{m \times n}$ such that the modes of the closed-loop switched system, given by $A_j + B_j K_j$, $j = 1, \ldots, N$ have all the eigenvalues inside the circle with center at $(\sigma, 0)$ and radius r in the unit circle, as shown in Figure 1. This strategy assures to each linear mode of the system the desired rate of asymptotic damping and frequency.

The pole specification given in Figure 1 can be introduced by simply replacing the dynamic matrix A by $(A - \sigma\mathbf{I})/r$ in the stability analysis conditions [15], [18]. In this sense, an autonomous switched system given by (1) has all its eigenvalues for each linear mode in the region depicted in Figure 1 if and only if there exist $P_j > 0$, $P_i > 0$, $i, j = 1, \ldots, N$,

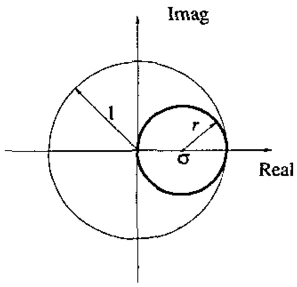

Figure 1: Circular region for pole location.

such that

$$\frac{(A_j-\sigma\mathbf{I})'}{r}P_i\frac{(A_j-\sigma\mathbf{I})}{r}-P_j<0 \ ; \ \forall(i,j)\in I\times I \quad (8)$$

The coincidence $i=j$ guarantees the pole location for each linear mode. Moreover, if (8) holds for $|\sigma|+r\le 1$, $|\sigma|<1$, then $P_j>0$, $P_i>0$ satisfying (8) are such that (3) holds, thus assuring stability.

The following lemmas present sufficient conditions for the existence of a state feedback control law (both fixed and switched gains) such that: i) if the discrete-time switched system varies slowly enough to allow the accommodation of each linear mode, then the system will have the dynamic response specified by the pole location in Figure 1; ii) in any case (arbitrarily time-varying switching sequences), the closed-loop stability is assured.

Lemma 1 If there exist a symmetric positive definite matrix $S\in\mathbb{R}^{n\times n}$ and a matrix $Z\in\mathbb{R}^{m\times n}$ such that

$$\begin{bmatrix} rS & SA_j'+Z'B_j'-\sigma S \\ A_jS+B_jZ-\sigma S & rS \end{bmatrix}>0;\ j=1,\ldots,N \quad (9)$$

then $u_k=Kx_k$, with $K=ZS^{-1}$, stabilizes the switched system (5) and imposes to any local mode the pole location given in Figure 1.

Proof: Straightforward, by replacing A_j by $(A_j-\sigma\mathbf{I})/r$ and B_j by B_j/r in (6). Note that the adequate choice of σ and r, placing the inner circle inside the unit circle, guarantees the overall stability of the switched system.

Lemma 2 If there exist N positive definite matrices $S_j\in\mathbb{R}^{n\times n}$, $j=1,\ldots,N$, matrices $F\in\mathbb{R}^{m\times n}$ and $G\in\mathbb{R}^{n\times n}$ such that

$$\begin{bmatrix} r(G+G'-S_j) & G'A_j'+F'B_j'-\sigma G' \\ A_jG+B_jF-\sigma G & rS_i \end{bmatrix}>0$$
$$\forall(i,j)\in I\times I \quad (10)$$

then $u_k=Kx_k$, with $K=FG^{-1}$, stabilizes the switched system (5) and imposes to any local mode the pole location given in Figure 1.

Proof: Comes from equation (4) adapted for the desired pole location through state feedback (i.e. with $(A_j+B_jK-\sigma\mathbf{I})/r$ replacing A_j), $G_j=G$ and the change of variables $KG=F$. Note that the control gain is obtained independently from the Lyapunov matrices S_i, S_j, $j,i=1,\ldots,N$.

Lemma 2 allows to determine a fixed state feedback gain which provides the desired pole location for each linear mode of operation of the switched system. Since a set of Lyapunov matrices is used, the results are less conservative than the ones obtained with quadratic stability (Lemma 1). Next lemmas use similar manipulations to provided a stabilizing switched feedback gain that imposes the desired pole location.

Lemma 3 If there exist N positive definite matrices $S_j\in\mathbb{R}^{n\times n}$ and matrices $Z_j\in\mathbb{R}^{m\times n}$, $j=1,\ldots,N$ such that

$$\begin{bmatrix} rS_j & S_jA_j'+Z_j'B_j'-\sigma S_j \\ A_jS_j+B_jZ_j-\sigma S_j & rS_i \end{bmatrix}>0$$
$$\forall(i,j)\in I\times I \quad (11)$$

then the switched control law $u_k=K_jx_k$, with $K_j=Z_jS_j^{-1}$, $j=1,\ldots,N$ stabilizes the switched system (5) and imposes to any local mode the pole location given in Figure 1.

Proof: Replacing A_j by $(A_j+B_jK_j-\sigma\mathbf{I})/r$ and making $G_j=G_j'=S_j$ in (4), and multiplying by r, one gets (11).

Lemma 4 If there exist N positive definite matrices $S_j\in\mathbb{R}^{n\times n}$ and matrices $F_j\in\mathbb{R}^{m\times n}$, $j=1,\ldots,N$ such that

$$\begin{bmatrix} r(G_j+G_j'-S_j) & G_j'A_j'+F_j'B_j'-\sigma G_j' \\ A_jG_j+B_jF_j-\sigma G_j & rS_i \end{bmatrix}>0$$
$$\forall(i,j)\in I\times I \quad (12)$$

then the switched control law $u_k=K_jx_k$, with $K_j=F_jG_j^{-1}$, $j=1,\ldots,N$ stabilizes the switched system (5) and imposes to any local mode the pole location given in Figure 1.

Proof: Similar to the proof of Lemma 3.

Using lemmas 3 and 4, two alternative ways to compute a switched feedback gain are given. Although the results in terms of stabilizability are equivalent, the use of Lemma 4 allows one to impose structural constraints in the switched gain as, for instance, decentralization, without any additional constraint on the Lyapunov matrices S_j. This represents an important degree of freedom for design purposes.

Finally, it is worth mentioning that the conditions of lemmas 1–4 are valid for discrete-time switching systems with arbitrary switching functions (not known a priori). For instance, if not all transitions $i\rightarrow j$ are allowed, the number of LMIs to be tested can be reduced.

3.1 Structurally constrained control

Imposing some structural constraints on the matrix variables, sufficient conditions for the existence of decentralized or static output feedback control are easily obtained, as presented in Corollaries 1 and 2 as extensions of Lemma 4.

Corollary 1 Consider matrices G_j and F_j in Lemma 4 with a block diagonal structure

$$G_{jD} = block\ diag\{G_j^1, \ldots, G_j^M\} \tag{13}$$

$$F_{jD} = block\ diag\{F_j^1, \ldots, F_j^M\} \tag{14}$$

with M being the number of subsystems. If a feasible solution exists, the control gain given by $K_{jD} = F_{jD}G_{jD}^{-1}$ is such that

$$K_{jD} = block\ diag\{K_j^1, \ldots, K_j^M\} \tag{15}$$

Suppose now that[1] only a subset of the states of the system (1) is available for feedback; in other words, $y_k \in \mathbb{R}^p$ is an output given by

$$y_k = \left[\begin{array}{c|c} \mathbf{I}_p & \mathbf{0} \end{array}\right] x_k \tag{16}$$

In this case, the static output feedback control problem can be formulated as the search of a state feedback gain $K_j \in \mathbb{R}^{m \times n}$ with the following structure

$$K_j = \left[\begin{array}{c|c} K_{jO} & 0 \end{array}\right] \tag{17}$$

with $K_{jO} \in \mathbb{R}^{m \times p}$. A sufficient condition for the existence of such gain is easily obtained as follows (see [16], [20])

Corollary 2 A static output feedback control gain can be obtained from the previous results by imposing to matrices G_j and F_j in Lemma 4 the structure constraints

$$G_{jO} = \left[\begin{array}{c|c} G_{jO}^{11} & \mathbf{0} \\ \hline \mathbf{0} & G_{jO}^{22} \end{array}\right] \tag{18}$$

with $G_{jO}^{11} \in \mathbb{R}^{p \times p}$ and $G_{jO}^{22} \in \mathbb{R}^{(n-p)\times(n-p)}$ and

$$F_{jO} = \left[\begin{array}{c|c} F_{jO}^1 & \mathbf{0} \end{array}\right] \tag{19}$$

with $F_{jO}^1 \in \mathbb{R}^{m \times p}$. If a feasible solution exists, the control gain is given by $K_j = F_{jO}G_{jO}^{-1}$ such that (17) holds.

4 Examples

Some examples are given in order to provide a numerical evaluation of the conditions for pole placement presented in the paper.

The first one is a discrete-time switched system in the form (5), with three linear modes of operation, described by

$$A_1 = \begin{bmatrix} 0.7582 & 0.6802 \\ -0.6919 & -1.0725 \end{bmatrix};\ B_1 = \begin{bmatrix} 0.6234 \\ 0.6859 \end{bmatrix} \tag{20}$$

$$A_2 = \begin{bmatrix} 0.8998 & 0.2847 \\ -2.1231 & -0.7333 \end{bmatrix};\ B_2 = \begin{bmatrix} 0.6773 \\ 0.8768 \end{bmatrix} \tag{21}$$

$$A_3 = \begin{bmatrix} -0.7734 & -0.3368 \\ 0.1518 & 0.9708 \end{bmatrix};\ B_3 = \begin{bmatrix} 0.0129 \\ 0.3104 \end{bmatrix} \tag{22}$$

[1]There always exists a similarity transformation that allows the output of a linear system to be written as in (16) [19].

Although the vertices A_1, A_2, and A_3 are stable, this system is not quadratically stabilizable, i.e. the conditions of Lemma 1 are not able to find a stabilizing state feedback control gain for this system, not even for $r = 1$ and $\sigma = 0$ (unit circle). However, using conditions of Lemma 2, this system can be stabilized through the constant feedback gain

$$K = \begin{bmatrix} 0.3524 & -0.0888 \end{bmatrix} \tag{23}$$

with matrices G and F given by

$$G = \begin{bmatrix} 48.9397 & -42.0100 \\ -43.4477 & 119.8579 \end{bmatrix};\ F' = \begin{bmatrix} 21.1029 \\ -25.4457 \end{bmatrix}$$

Both lemmas 3 and 4 are able to stabilize the system (5) with vertices (20)–(22) by means of switched gains K_i, with $i = 1, \ldots, 3$. Moreover, they can guarantee that the closed-loop system poles are inside the circle of $\sigma = 0$ and $r = 0.52$ which represents a pole location that cannot be attained by fixed gains, yielding an improvement in the system dynamic response. For instance, using Lemma 3 one can find

$$K_1 = \begin{bmatrix} -0.3135 & -0.0142 \end{bmatrix}; K_2 = \begin{bmatrix} 0.3415 & 0.1394 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 3.2548 & -1.3176 \end{bmatrix}$$

and using Lemma 4, the switched gains are

$$K_1 = \begin{bmatrix} -0.3138 & -0.0144 \end{bmatrix}; K_2 = \begin{bmatrix} 0.3413 & 0.1392 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 3.2388 & -1.3244 \end{bmatrix}$$

Note that for this particular example the switched gains obtained with Lemma 3 and 4 are practically the same.

Consider now the switched system (5) with

$$A_1 = \begin{bmatrix} 0 & 1.00 \\ -2.50 & -1.00 \end{bmatrix};\ B_1 = \begin{bmatrix} 0 \\ 1.00 \end{bmatrix} \tag{24}$$

$$A_2 = \begin{bmatrix} 0 & 1.00 \\ 0.11 & 1.00 \end{bmatrix};\ B_2 = \begin{bmatrix} 1.00 \\ 1.00 \end{bmatrix} \tag{25}$$

This system has unstable vertices A_1 and A_2 (eigenvalues are respectively $-0.50 \pm 1.50i$ and -0.10, 1.10). It is not possible to find a stabilizing constant feedback gain through lemmas 1 and 2, not even for $r = 1$ and $\sigma = 0$ (unit circle). Both lemmas 3 and 4 are able to stabilize this switched system and also to provide the pole specifications given by Figure 1 with $r = 0.5$ and $\sigma = 0.5$. For example, using Lemma 4, a switched control law is obtained in such a way that, for each one of the modes $i = 1, 2$, the gains are given by

$$K_1 = \begin{bmatrix} 2.025 & 2.450 \end{bmatrix}; K_2 = \begin{bmatrix} 0.661 & -1.132 \end{bmatrix} \tag{26}$$

The time response of the system (5) with (24)-(25) and switched gains (26) is shown in Figure 2. The symbol $(\cdot)$ indicates the trajectories of the states x_1 and x_2 for $A_1 + B_1K_1$ whereas the symbol $(*)$ is used for $A_2 + B_2K_2$. The trajectories of the switched system are represented by solid lines. In this simulation, a change in the dynamical mode $(1 \rightleftharpoons 2)$ occurs at each sample instant.

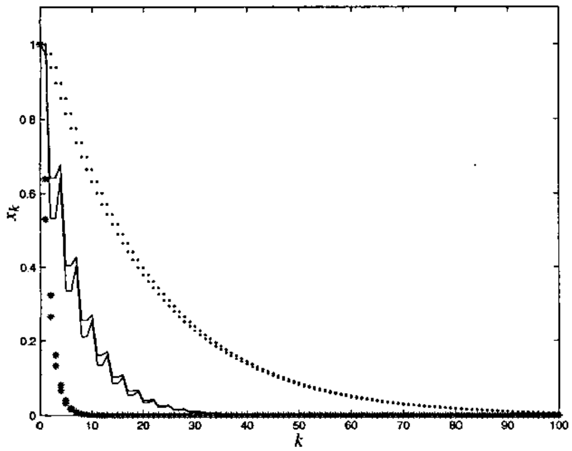

Figure 2: State trajectories for the controlled discrete-time switched system (5) with (24)-(25) and switched gains (26). The symbol $(\cdot)$ indicates the trajectories of the states x_1 and x_2 for $A_1+B_1K_1$ whereas the symbol $(*)$ is used for $A_2+B_2K_2$. The trajectories of the switched system are represented by solid lines.

A well known strategy for discrete-time systems control is to make its transient response as fast as possible, by means of an almost deadbeat controller. Fixing $\sigma=0$ and reducing the radius till $r=0.15$, lemmas 3 and 4 remain feasible. From Lemma 4, one can get the switched state feedback control given by

$$K_1=\begin{bmatrix} 2.500 & 0.994 \end{bmatrix};K_2=\begin{bmatrix} -0.104 & -1.000 \end{bmatrix} \quad (27)$$

Figure 3 shows the time response (almost deadbeat) for the controlled switched system (5) with (24)-(25) and switched gains (27) starting in mode 1, with $x_0=[1\ -1]'$, and changing the modes at each sample instant (a zero order hold has been included).

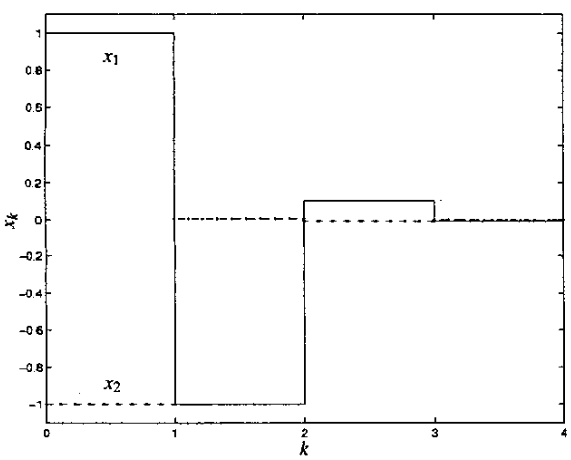

Figure 3: State trajectories for the controlled discrete-time switched system (5) with (24)-(25) and switched gains (27) (almost deadbeat response).

The third example illustrates how a constrained structure for the control gain can be imposed. Consider the system (5) with vertices given by

$$A_1=\begin{bmatrix} 0.5960 & 0.0336 \\ 0.7818 & 0.8570 \end{bmatrix};\ B_1=\begin{bmatrix} 0.4720 \\ 0.0881 \end{bmatrix} \quad (28)$$

$$A_2=\begin{bmatrix} 0.6826 & 0.9177 \\ 0.8019 & 0.3881 \end{bmatrix};\ B_2=\begin{bmatrix} 0.9251 \\ 0.6785 \end{bmatrix} \quad (29)$$

Although a constant stabilizing gain could be achieved from the results of Lemma 1 and 2 with full state feedback information, no feasible solution is obtained when the following structural constraints

$$K_j=\begin{bmatrix} k_{11}^j & 0 \end{bmatrix}\ ;\ \text{j=1,2}$$

are imposed, meaning that the information available for the synthesis of the control signal u is only x_1. It is not possible to find a switched stabilizing gain through Lemma 3 as well. Only Lemma 4 has a feasible solution for this problem, with $\sigma=0$ and $r=1$, given by

$$K_1=\begin{bmatrix} -2.3439 & 0 \end{bmatrix};\ K_2=\begin{bmatrix} -0.8736 & 0 \end{bmatrix} \quad (30)$$

Moreover, Lemma 4 can provide a pole location in the circle with $\sigma=0$ and with $r=0.93$, with the control gains

$$K_1=\begin{bmatrix} -3.0085 & 0 \end{bmatrix}\ ;\ K_2=\begin{bmatrix} -0.7989 & 0 \end{bmatrix} \quad (31)$$

Finally, consider the switched system with $N=2$ modes, states x_1, x_2, and x_3, and two control inputs, u_1 and u_2, represented by

$$A_1=\begin{bmatrix} 0.254 & 0.271 & 0.631 \\ 0.307 & 0.749 & 0.417 \\ 0.487 & 0.430 & 0.723 \end{bmatrix};B_1=\begin{bmatrix} 0.054 & 0.442 \\ 0.544 & 0.925 \\ 0.670 & 0.756 \end{bmatrix}.$$

$$A_2=\begin{bmatrix} 0.567 & 0.642 & 0.762 \\ 0.776 & 0.518 & 0.654 \\ 0.810 & 0.494 & 0.267 \end{bmatrix};B_2=\begin{bmatrix} 0.188 & 0.342 \\ 0.106 & 0.149 \\ 0.996 & 0.588 \end{bmatrix}$$

Imposing the block diagonal structure constraint

$$K_j=\begin{bmatrix} k_{11}^j & k_{12}^j & 0 \\ 0 & 0 & k_{23}^j \end{bmatrix}\ ;\ \text{j=1,2} \quad (32)$$

lemmas 1, 2 and 3 fail. Again, only Lemma 4 provides the stabilizing ($\sigma=0$, $r=1$) feedback gain

$$K_1=\begin{bmatrix} -0.7399 & -0.9974 & 0 \\ 0 & 0 & -0.8000 \end{bmatrix}$$

$$K_2=\begin{bmatrix} -1.1511 & -0.8375 & 0 \\ 0 & 0 & -1.3356 \end{bmatrix}$$

If a faster dynamic response is required, a pole location in the circle at $\sigma=0$ and $r=0.76$ is obtained by means of the gains

$$K_1=\begin{bmatrix} -0.8482 & -1.2237 & 0 \\ 0 & 0 & -0.8284 \end{bmatrix}$$

$$K_2=\begin{bmatrix} -1.8575 & -1.4650 & 0 \\ 0 & 0 & -1.7335 \end{bmatrix}$$

5 Conclusion

Sufficient LMI conditions have been given for the state feedback control of discrete-time switched systems with linear modes of operation. The conditions provide constant or switched state feedback gains that can assure a desired pole location for each linear mode of the closed-loop system or, at least, guarantee its overall stability under any arbitrarily fast switching sequence. Moreover, structural constraints can be imposed allowing to address static output feedback and decentralized control problems. Numerical examples have shown that switched gains can provide more stringent design specifications than fixed control laws, at the price of implementing a switching strategy.

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