Q.1

Sol^n: $c^T = [\cancel{8}\ \ 1\ \ 6\ \ -7\ \ 1\ \ 5]$

$A = \begin{bmatrix} 1 & -3/4 & 2 & -1/4 & 0 \\ 0 & 1/4 & 3 & -3/4 & 1 \end{bmatrix}$ $\quad B^{-1} = \begin{array}{cc} x_1 & x_5 \end{array} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$b = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$

$\underline{I_0}$

			RHS
z	0	0	26
x_1	1	0	5
x_5	0	1	5

$\cancel{x_4}$

x_3
24
2
3

Here $w = (\cancel{1}, \cancel{5})$. Noting that $z_j - c_j = w a_j - c_j$, we get

$z_2 - c_2 = -2$, $z_3 - c_3 = 2\cancel{4}$, $z_4 - c_4 = -20/4$ 5

$k = 3$, x_3 enters

$y_3 = B^{-1} a_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Insert the vector

$\begin{bmatrix} z_3 - c_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 2 \\ 3 \end{bmatrix}$

$\underline{I_1}$

z'	0	-8	-30/3
x_1	1	-2/3	5/3
x_3	0	1/3	5/3

1
1/4
-1/4

$z_j - c_j$: $z_2 - c_2 = -6$, $z_4 - c_4 = 1$, $z_5 - c_5 = -8$

$k = 4$ & x_4 enters & x_1 leaves

z	-4	-16/3	-50/3
x_4	4	-8/3	20/3
x_3	1	-1/3	10/3

Optimal

$z_j - c_j$: $z_1 - c_1 = -4$, $z_2 - c_2 = -11/3$, $z_5 - c_5 = -16/3$

Q.2 (1)

Sol: Product form of Inverse

$\bar{b} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$ $x_B = \begin{bmatrix} x_{B_1} \\ x_{B_2} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$ $x_N = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$w = c_B = (1, 5)$

$z_j - c_j = wa_j - c_j$

$z_2 - c_2 = -2$ $z_3 - c_3 = 24$ ✓ $z_4 - c_4 = 20/4$

$k = 3$, x_3 enters

$y_3 = a_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

x_{B_r} : $\min \left\{ 5/2, \; 5/3 \right\} = 5/3$

x_5 leaves the basis & x_3 enters

$y = \begin{bmatrix} -\frac{y_{13}}{y_{23}} \\ \frac{1}{y_{23}} \end{bmatrix} = \begin{bmatrix} -2/3 \\ 1/3 \end{bmatrix} \Rightarrow E_1 = \begin{bmatrix} y & e \end{bmatrix}$

update $\bar{b}$

$\bar{b} = E_1 \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} -2/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 5/3 \\ 5/3 \end{bmatrix}$

$z = -10$, $w = c_B E_1 = (1, -7) E_1 = (1, -3)$

$wa_j - c_j$

$z_2 - c_2 = -6$, $z_5 - c_5 = -8$, $z_4 - c_4 = 1$ ✓

$k = 4$, x_4 enters

$y_4 = E_1 a_1 = E_1 \begin{bmatrix} -1/4 \\ -3/4 \end{bmatrix} = \begin{bmatrix} 0 \\ -3/4 \end{bmatrix} - 1/4 \begin{bmatrix} -2/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 1/4 \\ -1/4 \end{bmatrix}$

x_4 enters & x_1 leaves

$$g = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \Rightarrow E_4 = \begin{bmatrix} g \\ 1 \end{bmatrix}$$

$$\bar{b} = E_2 \begin{bmatrix} 5/3 \\ 5/3 \end{bmatrix} = \begin{bmatrix} 0 \\ 5/3 \end{bmatrix} + 5/3 \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 20/3 \\ 10/3 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_{B_1} \\ x_{B_2} \end{bmatrix} = \begin{bmatrix} x_4 \\ x_3 \end{bmatrix} = \begin{bmatrix} 20/3 \\ 10/3 \end{bmatrix} \quad x_N = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$z = -10 - 20/3(1) = -50/3$$

$$w = c_B E_2 E_1 = (1, -7) E_2 E_1$$

$$= [-3 \quad -1/3]$$

$z_1 - c_1 = -4$

$z_2 - c_2 = -11/3$

$z_5 - c_5 = -16/3$

as all $z_j - c_j \leq 0$, optimal is reached

$x_4 = 20/3$, $x_3 = 10/3$

Q-3

Max $2x_1 + x_2 + 3x_3 \Rightarrow \min \; -2x_1 - x_2 - 3x_3$

s.t $3x_1 + x_2 + x_3 \le 12 \Rightarrow 3x_1 + x_2 + x_3 + x_4 = 12$

$-x_1 + x_2 \le 5 \Rightarrow -x_1 + x_2 + x_5 = 5$

$x_2 + 2x_3 \le 8 \Rightarrow x_2 + 2x_3 + x_6 = 8$

$0 \le x_1 \le 3, \quad 0 \le x_2 \le 6, \quad 0 \le x_3 \le 4$

Sol^n: → x_4, x_5 & x_6 as basic variables with bound 0 to ∞. Setting non basic variables at their lower bound, $\underline{x_4} = x_2 = x_3 = 0$, objective is 0, we will set the initial table

$\underline{P_0}$

	Z	x_1 (l)	x_2 (l)	x_3 (l) ↓	x_4	x_5	x_6	RHS
Z	1	2	1	3	0	0	0	0
x_4	0	3	1	1	1	0	0	12
x_5	0	−1	1	0	0	1	0	5
x_6	0	0	1	2	0	0	1	8

$\gamma_1 = \min(12, -, 4)$, $\gamma_2 = \{\min(\infty)$

$u_3 - l_3 = 4 - 0 = 4$

$\Delta_3 = \min\{4, \infty, 4\}$, x_6 will leave basis

R.H.S

objective $= 0 - (z_3 - c_3)\Delta_3 = -3 \times 4 = -12$

$$\begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 12 \\ 5 \\ 8 \end{bmatrix} - y_3\Delta_3 = \begin{bmatrix} 12 \\ 5 \\ 8 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \times 4 = \begin{bmatrix} 8 \\ 5 \\ 0 \end{bmatrix}$$

& value of $x_3 = l_3 + \Delta_3 = 0 + 4 = 4$

P_1:

	Z	$x_1^l \downarrow$	x_2^l	x_3	x_4	x_5	x_6^u	RHS
	1	2	$-1/2$	0	0	0	$-1/2$	-12
x_4	0	3	$1/2$	0	1	0	$-1/2$	8
x_5	0	-1	1	0	0	1	0	5
x_3	0	0	$1/2$	1	0	0	$1/2$	4

$\gamma_1 = 8/3$ ✓, $\gamma_2 = \infty$ $\quad u_1 - l_1 = 3$

$\Delta_1 = \min \{ 8/3 ✓, \infty, 3 \}$

x_1 enters & x_4 leaves the basis

RHS:

obj value $= -12 - 2 \times 8/3 = -52/3$

$$\begin{bmatrix} x_4 \\ x_5 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} 8/3 = \begin{bmatrix} 0 \\ 23/3 \\ 4 \end{bmatrix}$$

$x_1 = l_1 + \Delta_1 = 8/3$

	Z	x_1	x_2^l	x_3	x_4^l	x_5	x_6^l	RHS
Z	1	0	$-1/6$	0	$-2/3$	0	$-1/6$	$-52/3$
x_1	0	1	$1/6$	0	$1/3$	0	$-1/6$	$8/3$
x_5	0	0	$7/6$	0	$1/3$	1	0	$23/3$
x_3	0	0	$1/2$	1	0	0	$1/2$	4

As all $z_j - c_j$ at lower bound are $-$ve, optimal sol^n

$x_1 = 8/3$, $x_2 = 0$, $x_3 = 4$

$Z = -52/3 = -17.33$