EE3005: Communication Systems

Problem Set 5: Amplitude Modulation

1. The signal $m(t) = 2\cos 20\pi t - \cos 40\pi t$, where the **unit of time is milliseconds**, is amplitude modulated using a carrier frequency f_c of 600 KHz. The AM signal is given by

$$x(t) = 5\cos 2\pi f_c t + m(t)\cos 2\pi f_c t$$

   (a) Sketch the magnitude spectrum of x. What is its bandwidth?

   (b) What is the modulation index?

   (c) The AM signal is passed through an ideal highpass filter with cutoff frequency 595 KHz (i.e., the filter passes all frequencies above 595 KHz, and cuts off all frequencies below 595 KHz). Find an explicit time domain expression for the Q component of the filter output with respect to a 600 KHz frequency reference.

2. Consider a DSB signal corresponding to the message $m(t) = 3\text{sinc}(2\text{t})$ and a carrier frequency f_c which is 100 times larger than the message bandwidth, where the **unit of time is milliseconds**.

   (a) Sketch the magnitude spectrum of the DSB signal $10m(T)\cos 2\pi f_c t)$, specifying the units on the frequency axis.

   (b) Specify a time domain expression for the corresponding LSB signal.

   (c) Now, suppose that the DSB signal is passed through a bandpass filter whose transfer function is given by

$$H_p(f) = \left(f - f_c + \frac{1}{2}\right) I_{[f_c - \frac{1}{2}, [f_c + \frac{1}{2}]} + I_{[f_c + \frac{1}{2}, [f_c + \frac{3}{2}]}, \qquad f > 0$$

   Sketch the magnitude spectrum of the corresponding VSB signal.

   (d) Find a time domain expression for the VSB signal of the form

$$u_c(t)\cos 2\pi f_c t - u_s(t)\sin 2\pi f_c t$$

   carefully specifying u_c and u_s the I and Q components.

3. Figure 5.1 shows a block diagram of Weavers SSB modulator, which works if we choose f_1 , f_2 and the bandwidth of the lowpass filter appropriately. Let us work through these choices for a waveform of the form $m(t) = A_L\cos(2\pi f_L t + \phi_L) + A_H\cos(2\pi f_H t + \phi_H)$, where $f_H > f_L$ (the design choices we obtain will work for any message whose spectrum lies in the band $[f_L, f_H]$).

   (a) For $f_1 = (f_L + f_H)/2$ (i.e., choosing the first LO frequency to be in the middle of the message band), find the time domain waveforms at the outputs of the upper and lower branches after the first mixer.

   (b) Choose the bandwidth of the lowpass filter to be $W = \frac{f_H + 2f_L}{2}$ (assume the lowpass filter is ideal). Find the time domain waveforms at the outputs of the upper and lower branches after the LPF.

   (c) Now, assuming that $f_2 \gg f_H$, find a time domain expression for the output waveform, assuming that the upper and lower branches are added together. Is this an LSB or USB waveform? What is the carrier frequency?

   (d) Repeat (c) when the lower branch is subtracted from the upper branch.

Remark: Weavers modulator does not require bandpass filters with sharp cutoffs, unlike the direct approach to generating SSB waveforms by filtering DSB-SC waveforms. It is also simpler than the Hilbert transform method (the latter requires implementation of a $\pi/2$ phase shift over the entire message band).

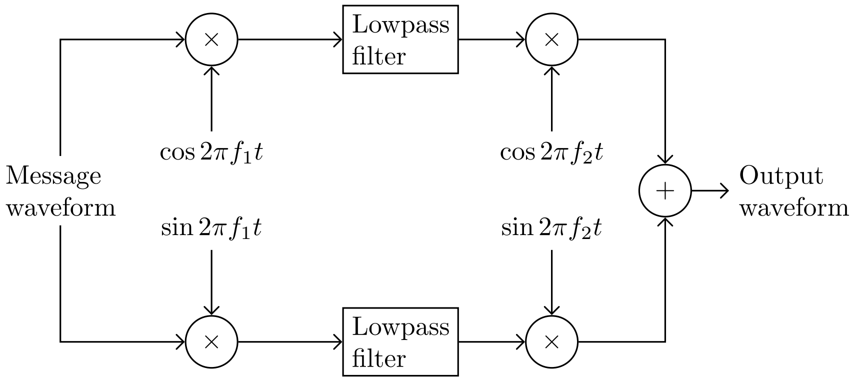

Figure 5.1: Block diagram of Weavers SSB modulator for Problem 3.