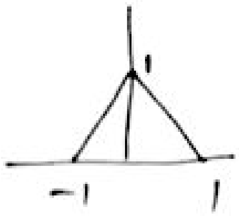
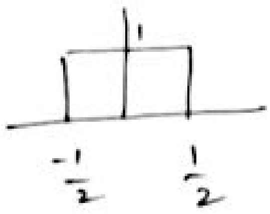
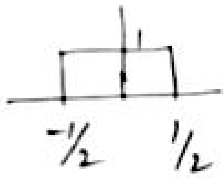
1 (a) $x(t) =$



$\equiv$



$*$



$$\Big\downarrow \text{FT}$$

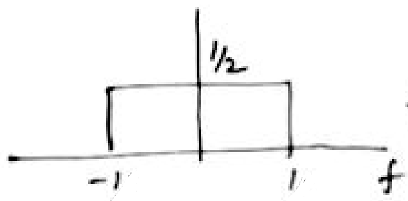
$$= \text{sinc}(f) \cdot \text{sinc}(f)$$

$$= \text{sinc}^2(f) \qquad \left(\text{where } \text{sinc}(f) = \frac{\sin(\pi f)}{\pi f}\right)$$

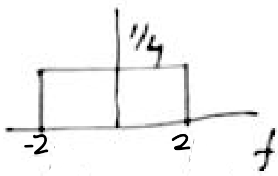
1 (b) From duality,

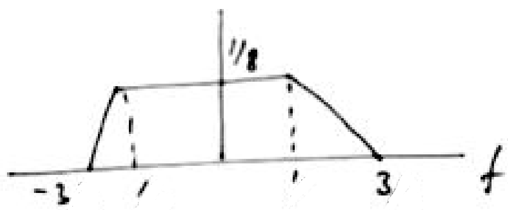
$$\text{sinc}(at) \xrightarrow{FT} \frac{1}{|a|} \text{rect}\left(\frac{f}{a}\right)$$

$$\text{sinc}(2t) \cdot \text{sinc}(4t) \xrightarrow{FT} \frac{1}{2} \text{rect}\left(\frac{f}{2}\right) * \frac{1}{4} \text{rect}\left(\frac{f}{4}\right)$$



$*$





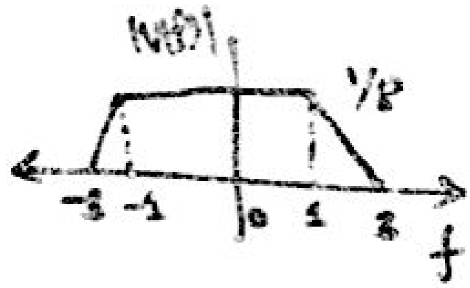
1

(c.) $s(t) = v(t)\cos(200\pi t)$

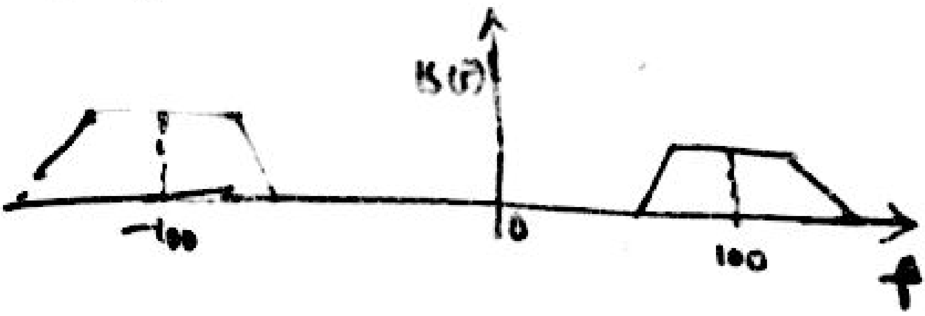
$$= V(f) \circledast \left[\delta(f-100) + \delta(f+100)\right]$$

$\because$ Multiplication in time domain corres convolution in the frequency domain

$V(f) =$



$S(f) =$



(d.)

(a.) → Baseband

(b.) → Baseband

(c.) → Passband

Tutorial 2 : Question 2

Question: Compute the following integrals using Parsevals Theorem.

a) $\int_0^\infty sinc^2(2t)dt$

b) $\int_0^\infty sinc(t)sinc(2t)dt$

Answer: Parsevals theorem states that

$$\int_0^\infty x(t)\bar{y(t)} = \int_0^\infty X(f)\bar{Y(f)}df. \tag{1}$$

Also note that $sinc(t) \xrightarrow{\mathcal{F}} rect(t) = 1$ for $[-0.5, 0.5]$ and $x(at)) \xrightarrow{\mathcal{F}} \frac{1}{a}X(\frac{f}{a})$

a). $x(t) = y(t) = sinc(2t) \implies X(f) = \frac{1}{2}$ for $f \in [-1/4, 1/4]$ and zero else where. Substituting in (1) gives $\int_0^\infty sinc^2(2t)dt = \int_{-0.25}^{0.25} \frac{1}{4}df = \frac{1}{8}$.

b). $x(t) = sinc(t) \implies X(f) = 1$ in $[-0.5, 0.5]$ and $y(t) = sinc(2t) \implies X(f) = 1/2$ in $[-0.25, 0.25]$. $X(f)Y(f) = 1/2$ in $[-0.25, 0.25]$. Hence, $\int_0^\infty sinc(t)sinc(2t)dt = \int_{-0.25}^{0.25} 1/2\, df = 1/4$.

3)

a) $u(t) = \text{sinc}(t)\,\text{sinc}(2t)$

let $x_1(t) = \text{sinc}(t)$

$\Rightarrow X_1(f) = \text{rect}\left(\frac{-1}{2}, \frac{1}{2}\right)$

$x_2(t) = \text{sinc}(2t)$

$\Rightarrow X_2(f) = \frac{1}{2}\,\text{rect}(-1, 1)$

$x_1(t)\, x_2(t) \xleftrightarrow{F.T} X_1(f) * X_2(f)$

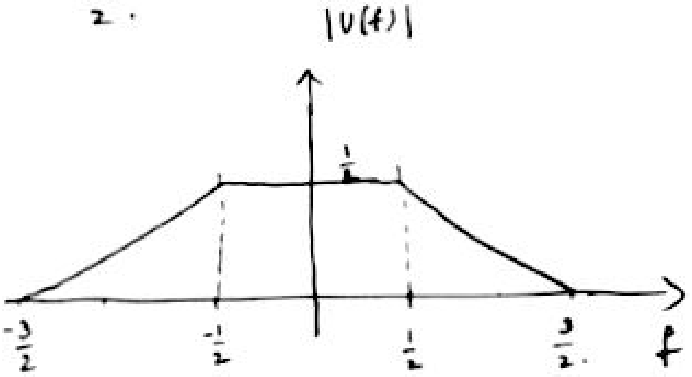
$\Rightarrow U(f) = X_1(f) * X_2(f)$

$$= \int_{-\infty}^{\infty} x_1(\alpha)\, x_2(f-\alpha)\, d\alpha$$

$U(f) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{2}\, d\alpha$; when $\frac{-1}{2} \le f \le \frac{1}{2}$.

$= \frac{1}{2}$

$U(f) = 0$ when $f > \frac{3}{2}$ (or) $f < -\frac{3}{2}$.



$U(f) = \int_{f-1}^{\frac{1}{2}} \frac{1}{2}\, d\alpha$ $\quad \frac{1}{2} \le f \le \frac{3}{2}$.

$= \frac{1}{2}\left[\frac{1}{2} - f + 1\right]$

$= \frac{3}{4} - \frac{f}{2}$, $\frac{1}{2} \le f \le \frac{3}{2}$.

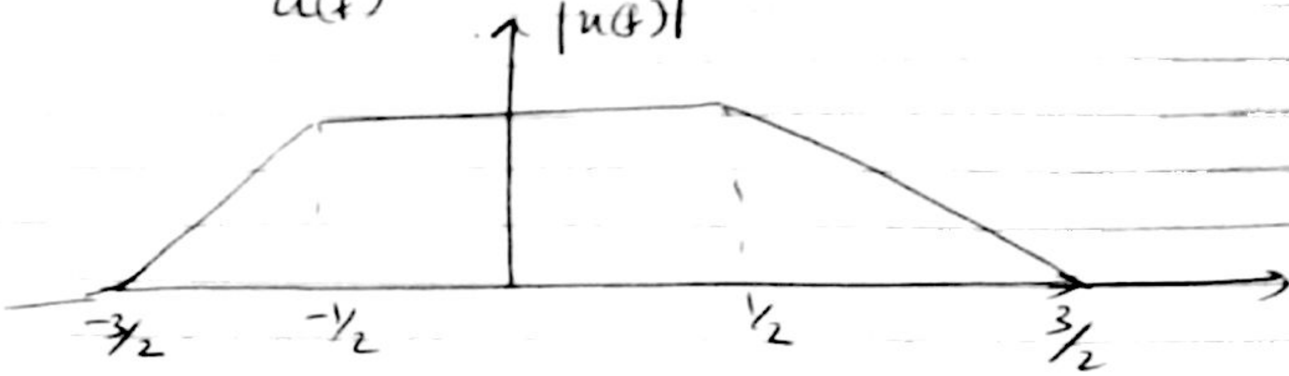
$U(f) = \int_{-\frac{1}{2}}^{1+f} \frac{1}{2}\, d\alpha = \frac{1}{2}\left[1 + f + \frac{1}{2}\right]$; $\frac{-3}{2} \le f \le \frac{-1}{2}$

$= \frac{3}{4} + \frac{f}{2}$. ; $\frac{-3}{2}. \le f \le \frac{-1}{2}$

3(b)

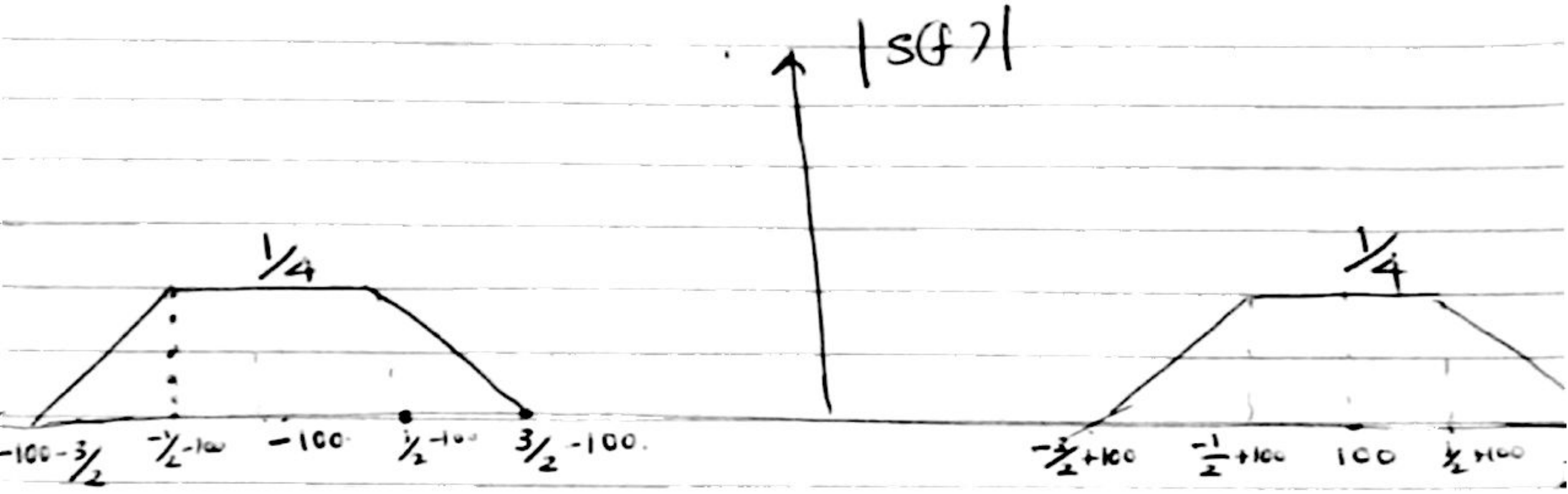
$$u(t)\cos 2\pi f_c t \xleftrightarrow{FT} \frac{U(f-f_c)+U(f+f_c)}{2}$$

$u(f)$

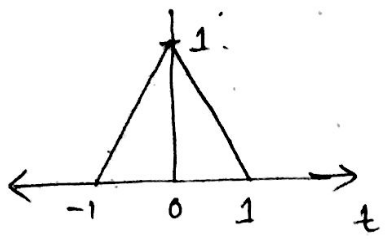


$$s(t) = u(t)\cos(2\pi \times 100 t)$$

$$S(f) = \frac{U(f-f_c)+U(f+f_c)}{2}$$



4(a) $\quad x(t) = (1-|t|)\, I_{[-1,1]}(t).$



$$X(f) = \int_{-\infty}^{\infty} x(t).\, e^{-j2\pi f t}.\, dt$$

$$= \int_{-1}^{0} (1+t)\, e^{-j2\pi f t}.\, dt + \int_{0}^{1} (1-t)\, e^{-j2\pi f t}.\, dt$$

$$= \left[\frac{1+j2\pi f}{4\pi^2 f^2} - \frac{e^{j2\pi f}}{4\pi^2 f^2} \right] - \left[\frac{j2\pi f - 1}{4\pi^2 f^2} + \frac{e^{-j2\pi f}}{4\pi^2 f^2} \right]$$

$$= - \frac{e^{-j2\pi f}\left(e^{j2\pi f} - 1\right)^2}{4\pi^2 f^2}$$

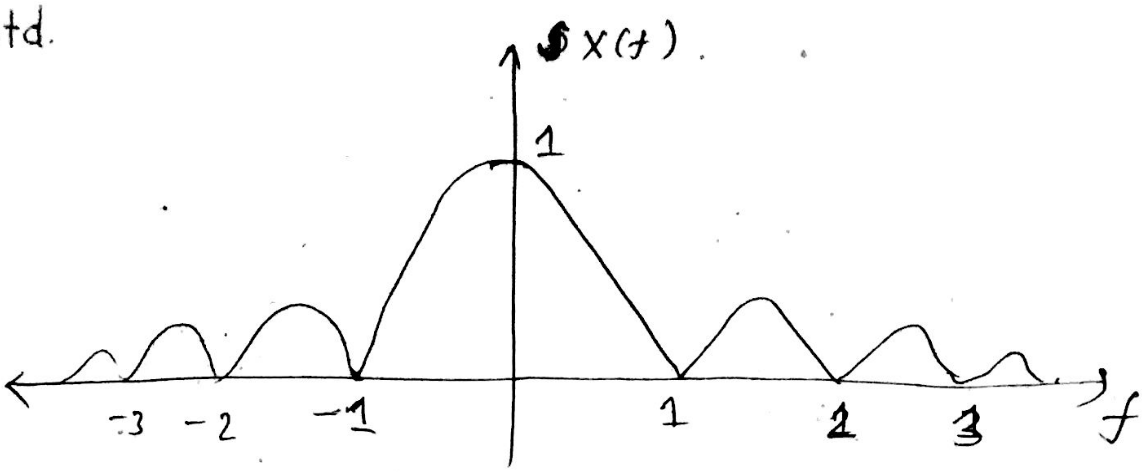
$$= - \frac{e^{-j2\pi f}\left(e^{j\pi f}\left(e^{j\pi f} - e^{-j\pi f}\right)\right)^2}{4\pi^2 f^2}$$

$$= - \frac{e^{-j2\pi f}\, e^{j2\pi f}\, (2i)^2 \sin^2(\pi f)}{4\pi^2 f^2}$$

$$= \left(\frac{\sin(\pi f)}{\pi f}\right)^2 = \underline{\underline{\operatorname{sinc}^2 f}}$$

Scanned by CamScanner

4) a Contd.



4 (b) : We have $\int_{-\infty}^{\infty} \operatorname{sinc}^4(f)\,df = \frac{2}{3}$

We want $\int_{-B}^{B} \operatorname{sinc}^4(f).\,df = 0.99 \times \frac{2}{3}$.

Evaluating Numerically,

$B \approx 2.05$.

But the time steps are here in milliseconds,

$\therefore$ $B \approx$ 2.05 kHz

Communication Systems; Tutorial 2

solution

February 12, 2016

5.a

after low pass filtering, we have

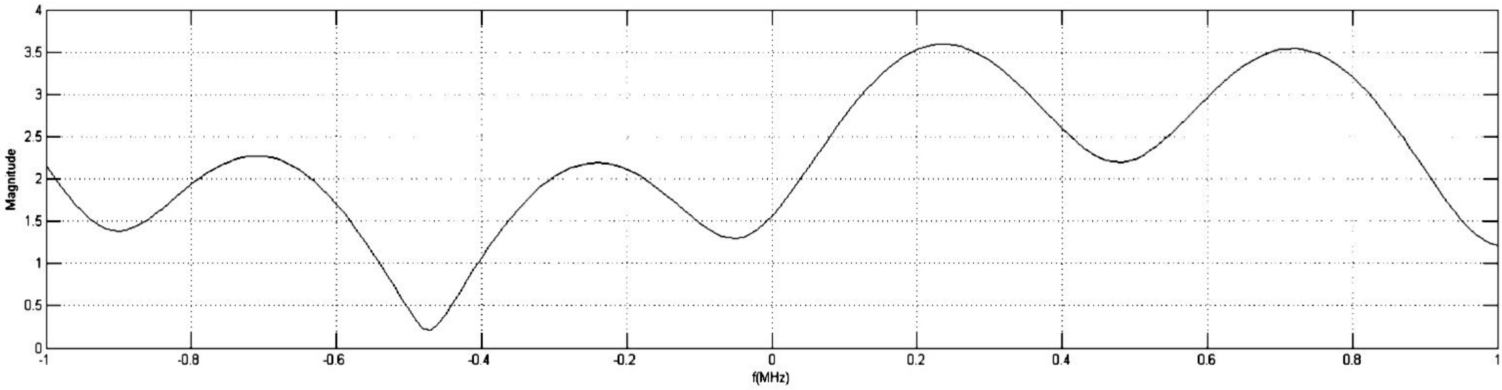
$$Delauspread, T_d = (2.2 - 0.1)ms = 2.1ms$$
$$Coherencebandwidth, B_c = 2.1^-1MHz = 476kHz$$

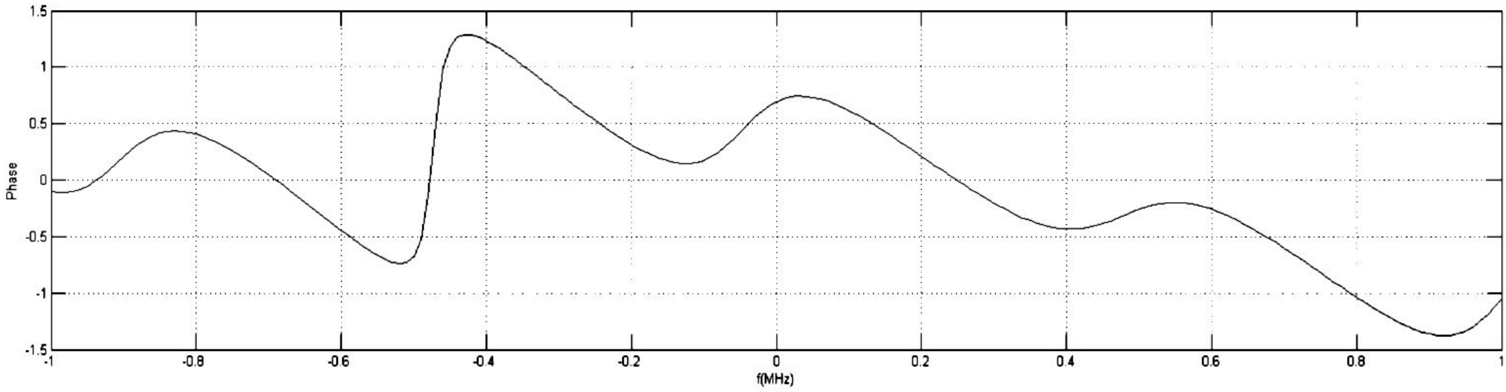
5.b

$$H(f) = 2e^{-0.1j.2\pi f} + je^{-0.64j.2\pi f} - 0.8e^{-2.2j.2\pi f}$$

where, f is in MHz.

We have to plot this in $[-2B_c, 2B_c]$, i.e aproximately from $[-1, 1]$MHz.


Magnitude
f(MHz)


Phase
f(MHz)

Q5) (c) & (d)

$h(t) = 2\delta(t-.1) + j\,\delta(t-.64) - .8\,\delta(t-2.2)$

unit of t : microseconds.

On dropping the first delay,

$H(f) = 2 + j e^{-j2\pi f \times .54} - .8 e^{-j2\pi f \times 2.1}$

fading gain $= 20\log_{10}\left(|H(f)| / |\alpha_1|\right)$

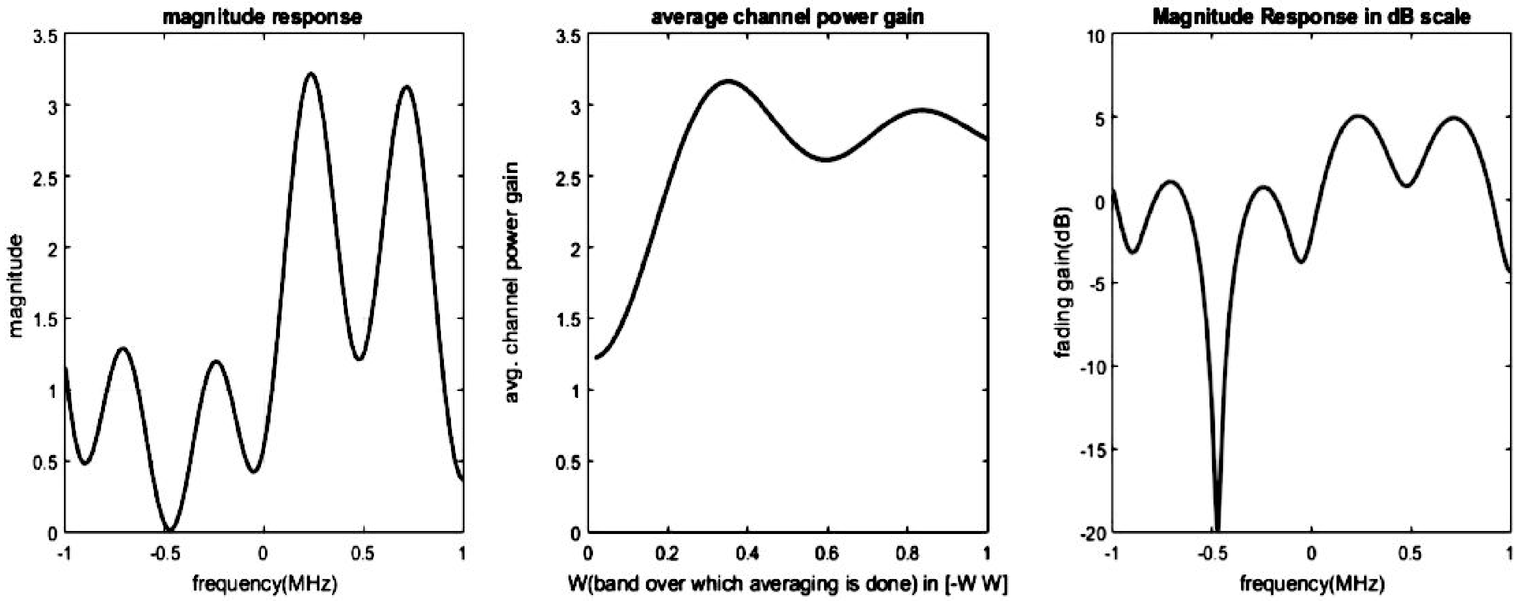
where $\alpha_1 = 2$ (∵ Nominal channel)

From the plot, we see, fading can dip about $-20\,dB$ @ $f \approx .47\,MHz$.

$$B_c = \frac{1}{\tau_d} = \frac{1}{2.1\,\mu s} \approx .5\,MHz$$

The avg. channel power gain over $[-W/2, W/2]$ is plotted.

as $W \uparrow$, we can avg. out fluctuations in $|H(f)|$


magnitude response
magnitude
frequency(MHz)
average channel power gain
avg. channel power gain
W(band over which averaging is done) in [-W W]
Magnitude Response in dB scale
fading gain(dB)
frequency(MHz)