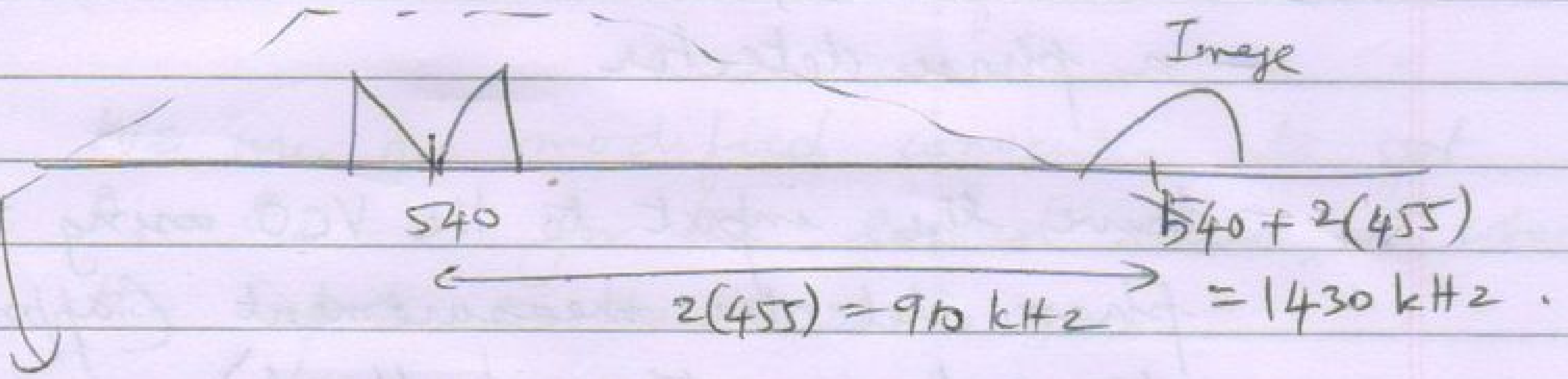
Suppose we want to tune to 540 kHz.



Need $B_{RF} < 910\,\text{kHz}$.

(FM example also given in the book, p 120.)

$\boxed{\text{Lecture 23: (4 Mar 2016)}}$

<u>Phase-locked loop (PLL):</u>

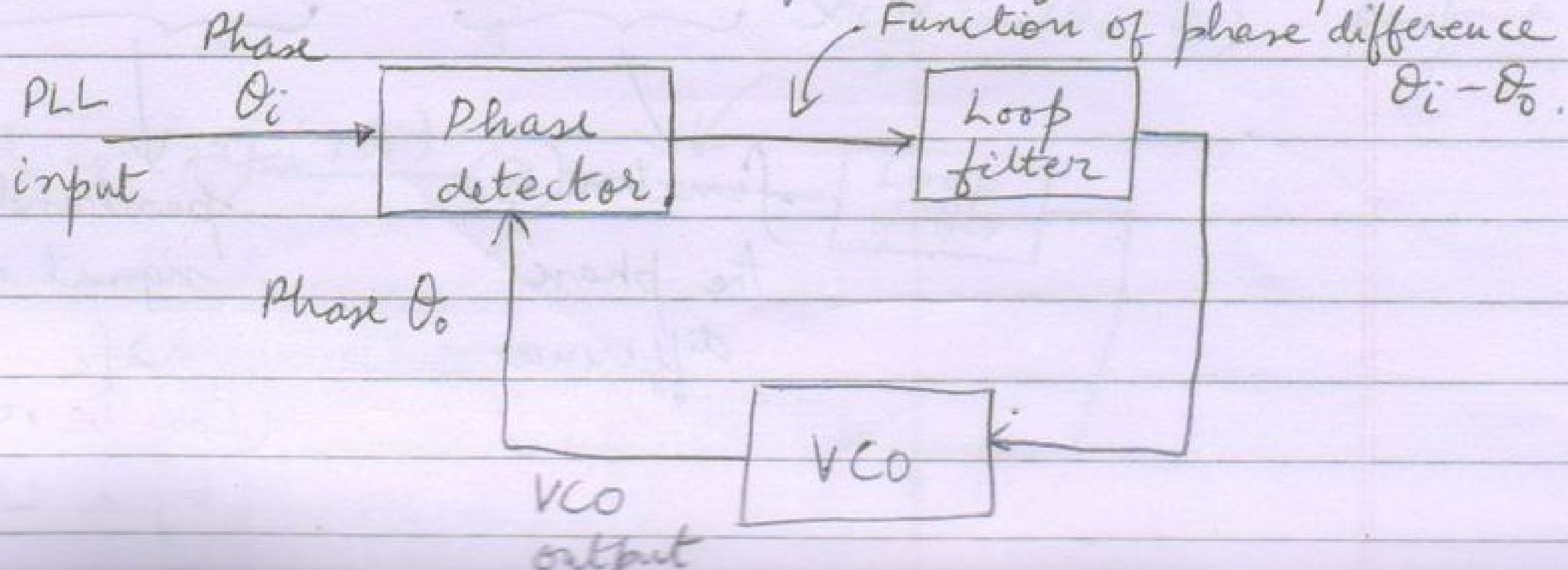
* In the context of the current discussion, the PLL is a <u>demodulator for FM</u>.

* Has broader applications in frequency synthesis and synchronization.

* A <u>negative feedback</u> system

Goal:

To lock on to the phase of the input.



We want the phase of the output of the VCO to be equal to the PLL input phase (ideally). To achieve this, we need:

- a phase detector
- drive the input to the VCO using the phase detector measurement (appropriately to get negative feedback)
- a loop filter may be needed to reduce noise and change the order of the PLL to achieve desired response.

First, we will discuss the phase detector.

(1) Mixer (multiplier) as a phase detector

* Suppose we want to align the phases of two sinusoids.

$$\cos(2\pi f_c t + \theta_1) \ \& \ \cos(2\pi f_c t + \theta_2)$$

The product is

$$\cos(2\pi f_c t + \theta_1)\cos(2\pi f_c t + \theta_2)$$

$$= \frac{1}{2}\Big[\underbrace{\cos(\theta_1 - \theta_2)}_{\text{function of the phase difference}} + \underbrace{\cos(4\pi f_c t + \theta_1 + \theta_2)}_{\text{passband signal at } 2f_c \text{ (can be removed by a LPF)}}\Big]$$

The input driving the VCO is zero when $\cos(\theta_1 - \theta_2) = 0$ (or) $\theta_1 - \theta_2 = \pi/2$. This will be the 'locked' state of the PLL.

* We use a modified convention to get $\theta_1 - \theta_2 = 0$ for the locked state, i.e. when the input to the VCO is zero.

PLL input : $A_c \cos(2\pi f_c t + \theta_i(t))$

VCO output : $A_v \cos(2\pi f_c t + \theta_0(t) + \pi/2)$

$$= -A_v \sin(2\pi f_c t + \theta_0(t)).$$

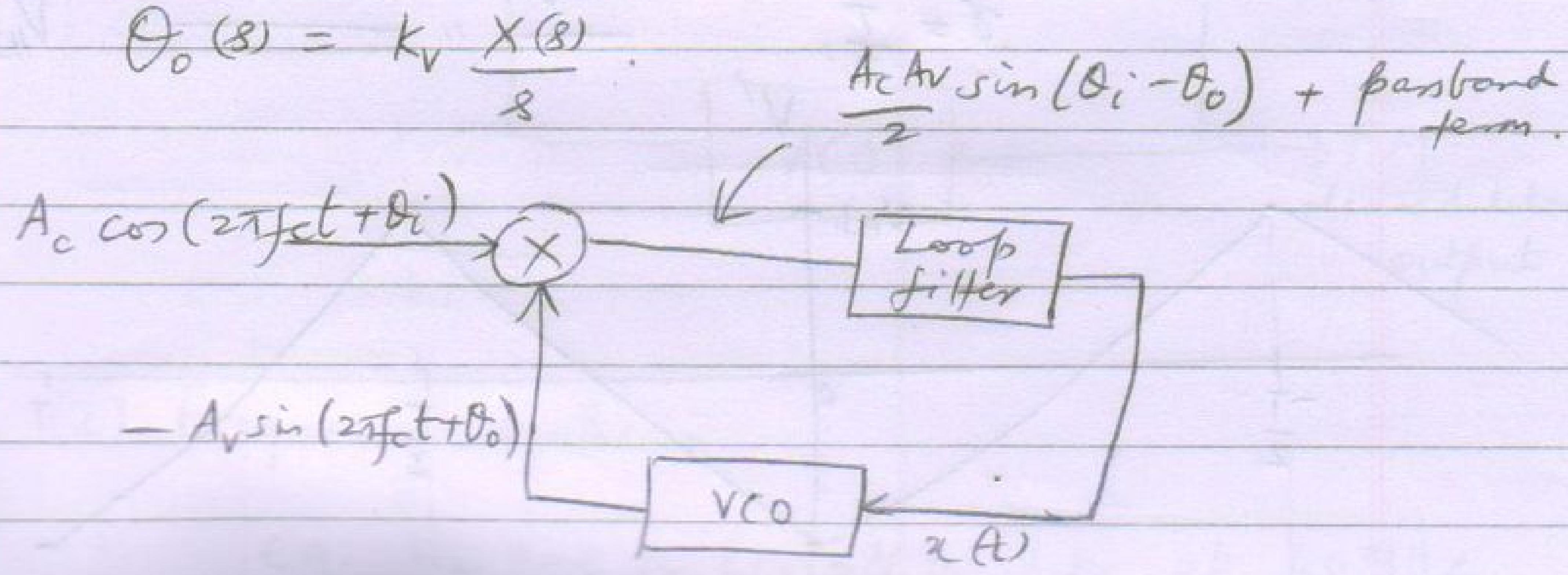
$$\left(A_c \cos(2\pi f_c t + \theta_i(t))\right)\left(-A_v \sin(2\pi f_c t + \theta_0(t))\right)$$

$$= \frac{A_c A_v}{2} \sin(\theta_i(t) - \theta_0(t)) - \frac{A_c A_v}{2} \underbrace{\sin(4\pi f_c t + \theta_i(t) + \theta_0(t))}_{\text{filtered by LPF.}}$$

zero when $\theta_i(t) = \theta_0(t)$ ('Locked' state)

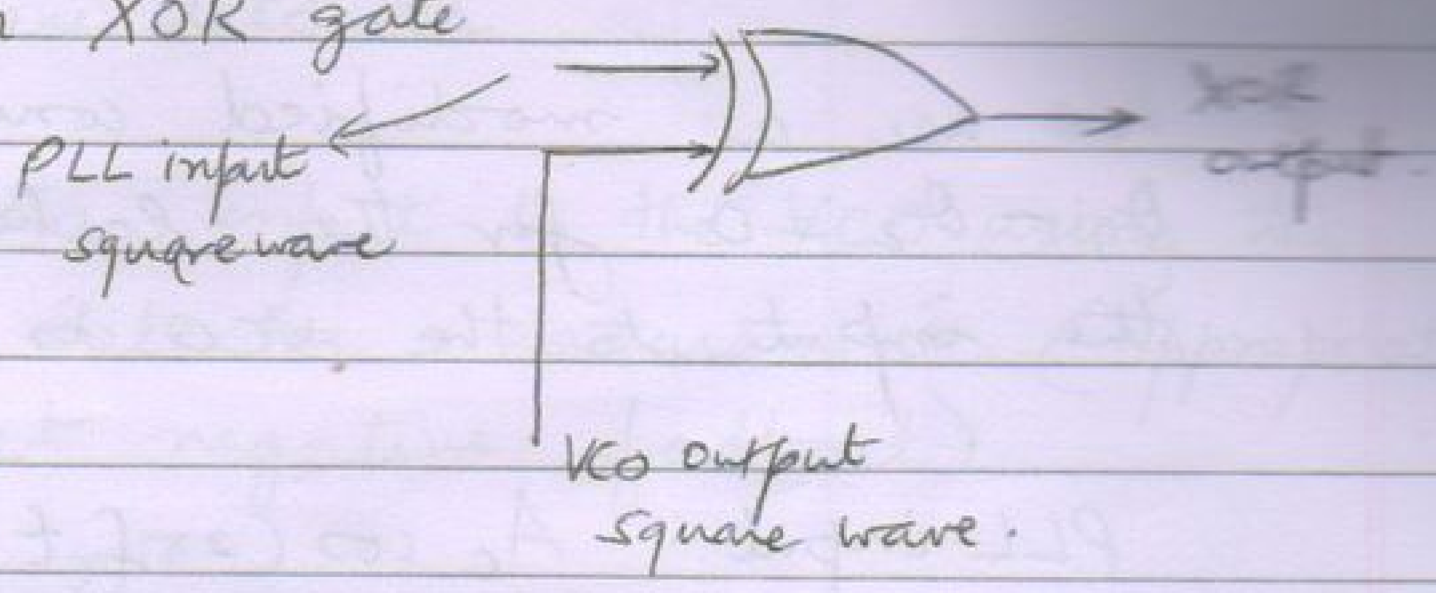
* $\theta_0(t) = k_v \int_0^t x(\tau)\,d\tau$

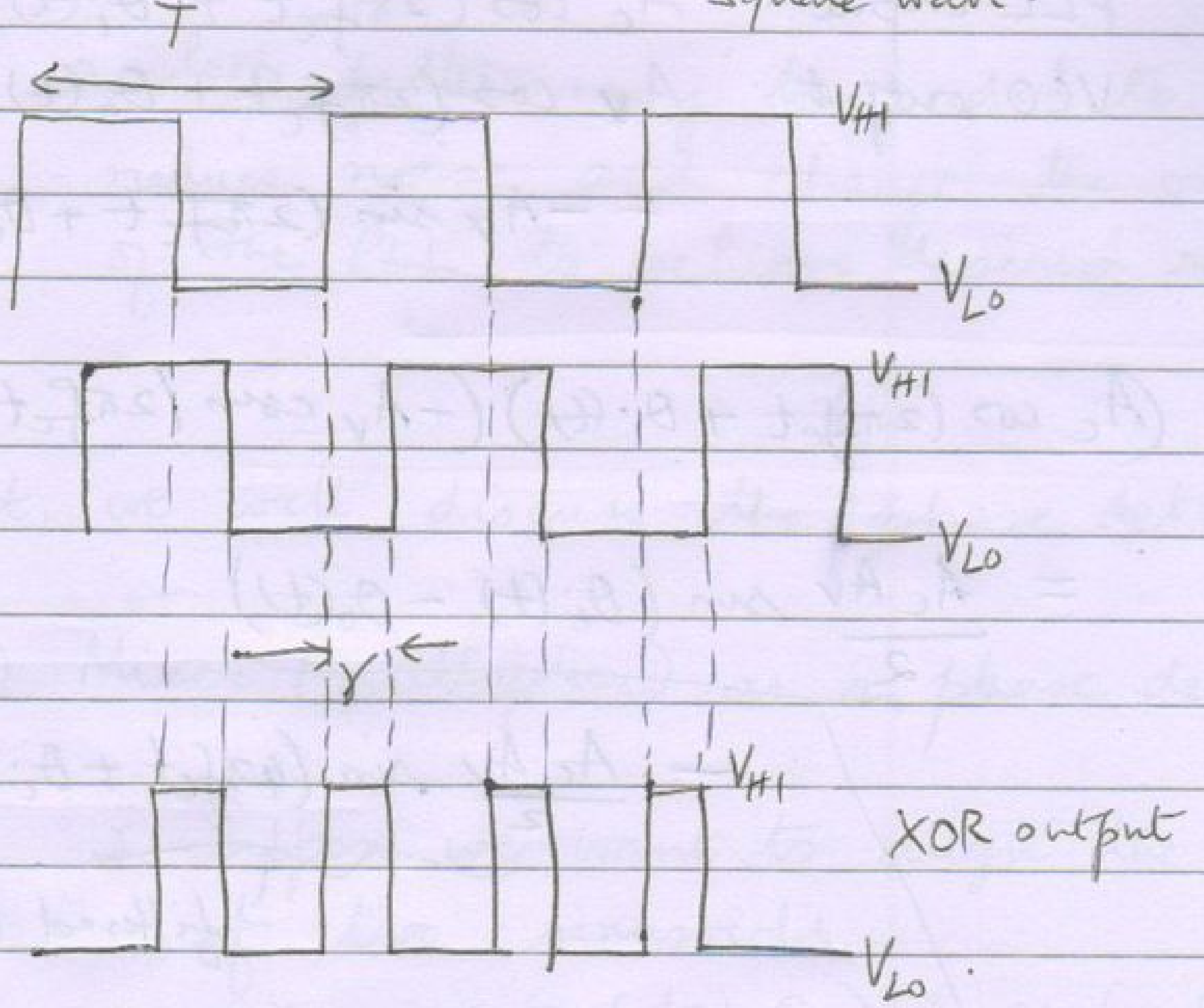
$\Theta_0(s) = k_v \dfrac{X(s)}{s}$.



(2) Mixed-signal phase detector
(Useful when digital waveform are involved)

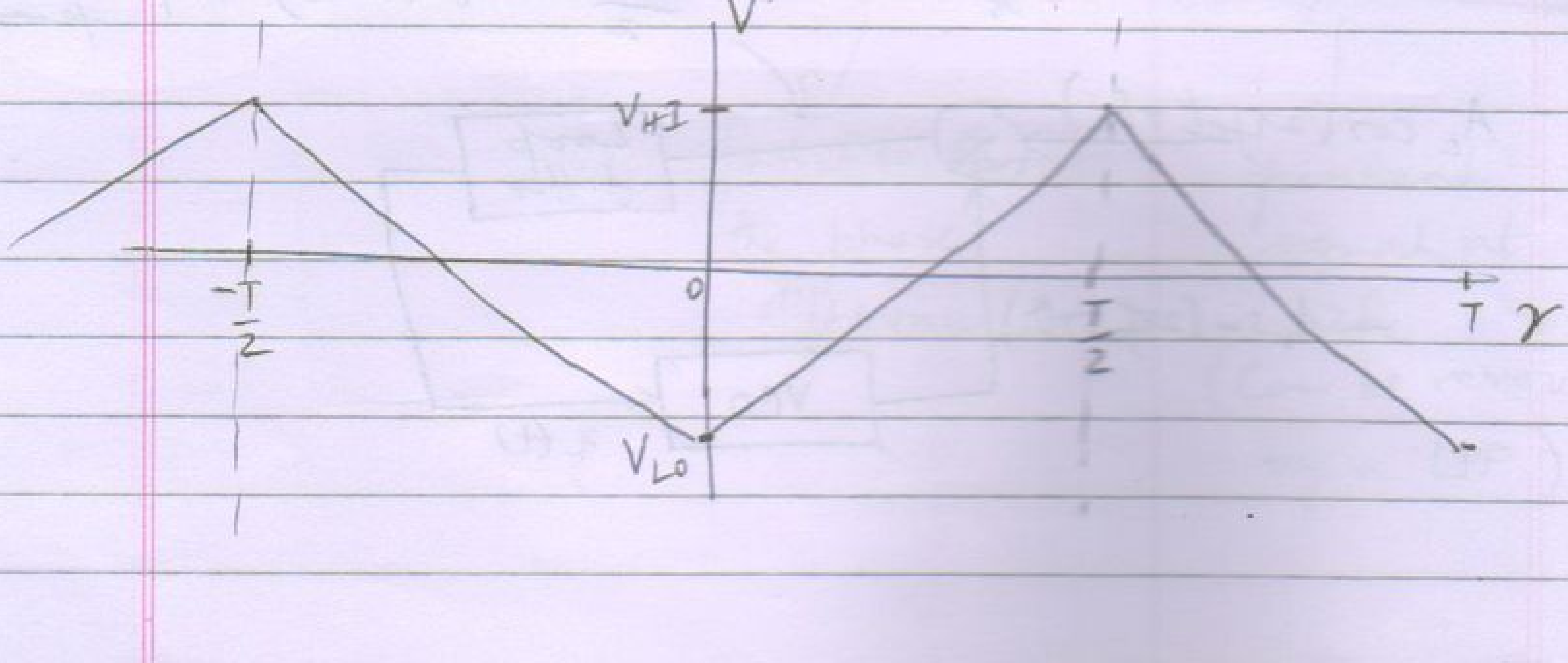
An XOR gate





$V' \triangleq$ Average of the XOR output $\propto \gamma$.

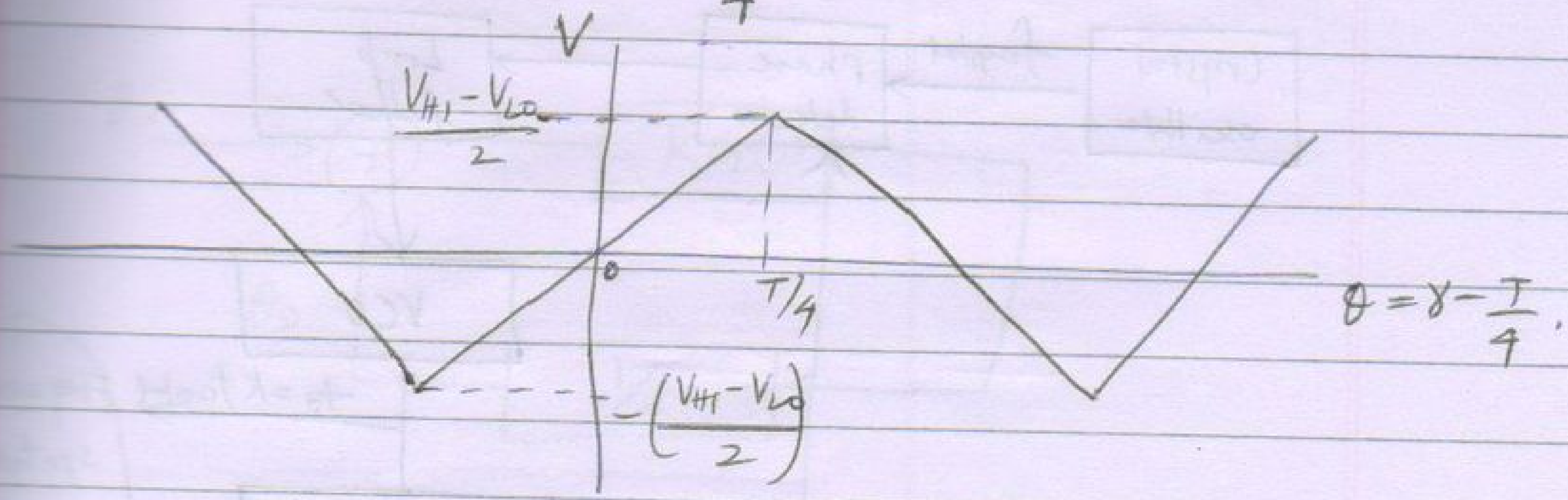
For $\gamma = 0$, Avg of XOR output is V_{LO}.
$\gamma = \frac{T}{2}$, ——— " ——— V_{HI}.



To get the phase error to zero when the VCO input is zero, define

$$V = V' - \frac{(V_{LO} + V_{HI})}{2}$$

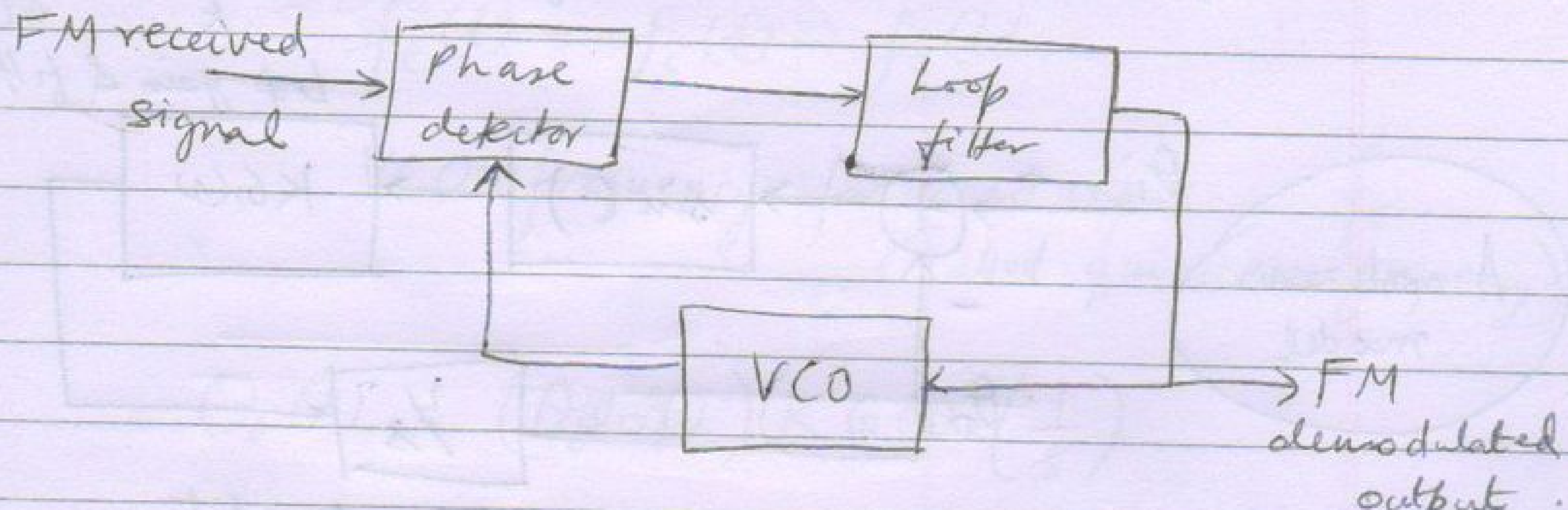
and $\theta = \gamma - \frac{T}{4}$.



$\boxed{\text{Lecture 24 : (8 Mar 2016)}}$

Suppose we have a working PLL, we can use it for

(1) FM demodulation



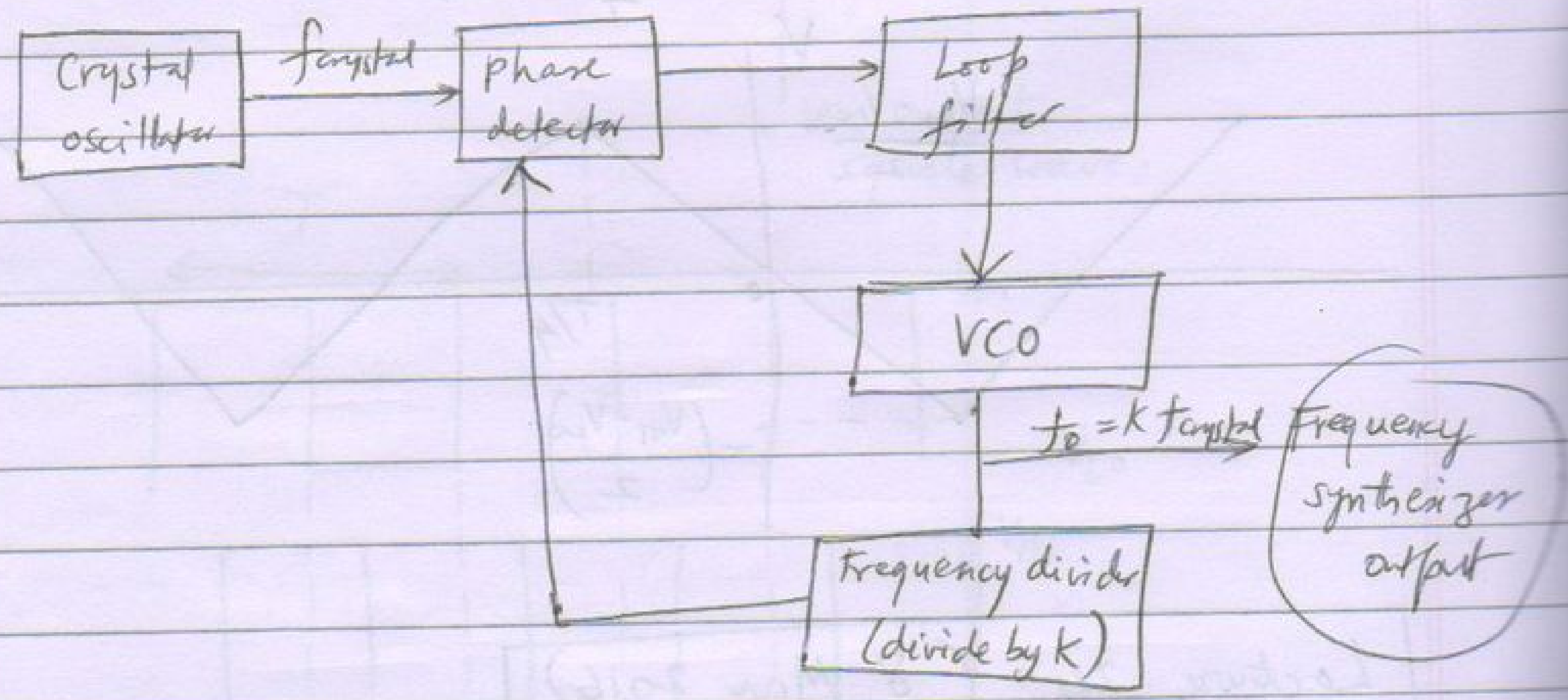
(2) Frequency synthesis

e.g. We have a crystal oscillator at 40 MHz.
& need a oscillator at 1 GHz.

$f_{crystal} = 40\,MHz$

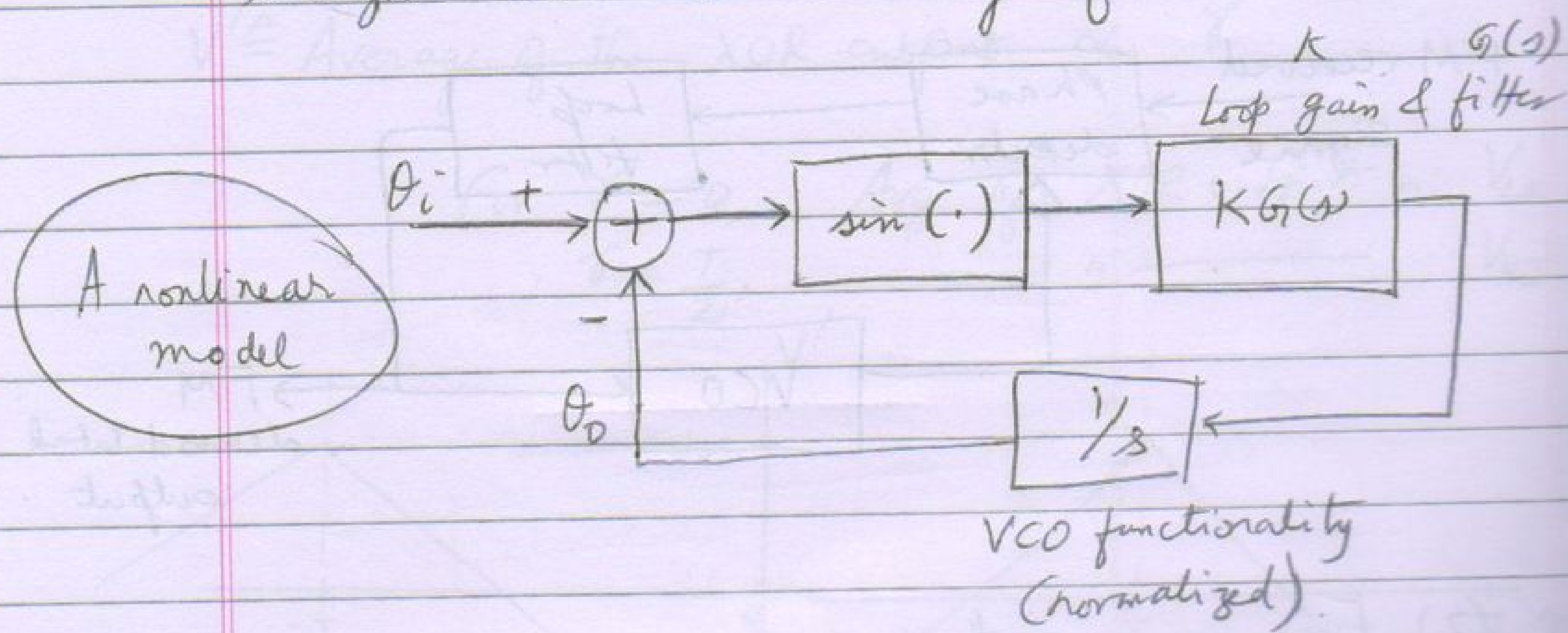
$f_o = 1\,GHz$

Note $\dfrac{f_o}{f_{crystal}} = K = 25$ (an integer).



PLL model:

Now, we will develop a model for the PLL to proceed with analysis and gain some insight on the working of a PLL.

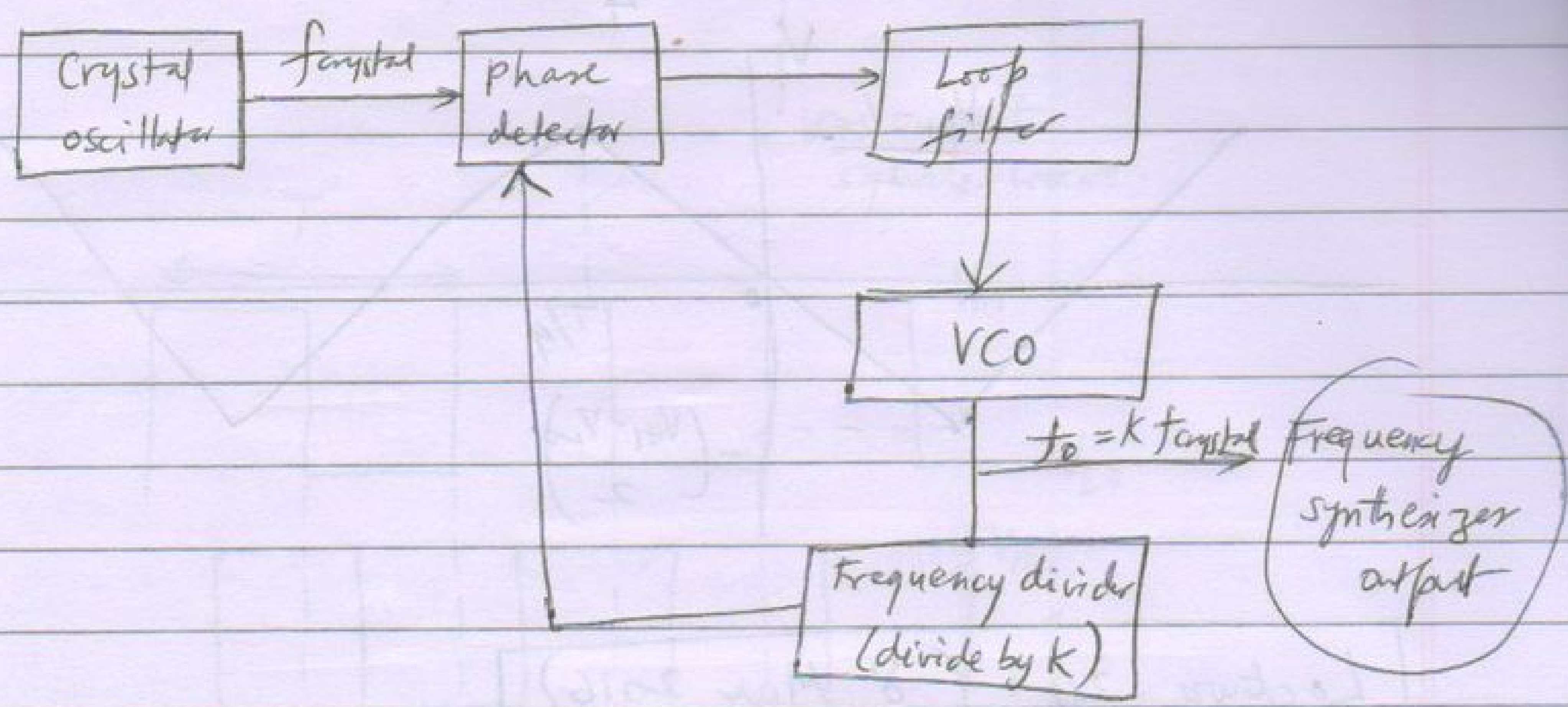


– Ignores the $2f_c$ term at the output of the phase detector

$f_{crystal} = 40\,\text{MHz}$

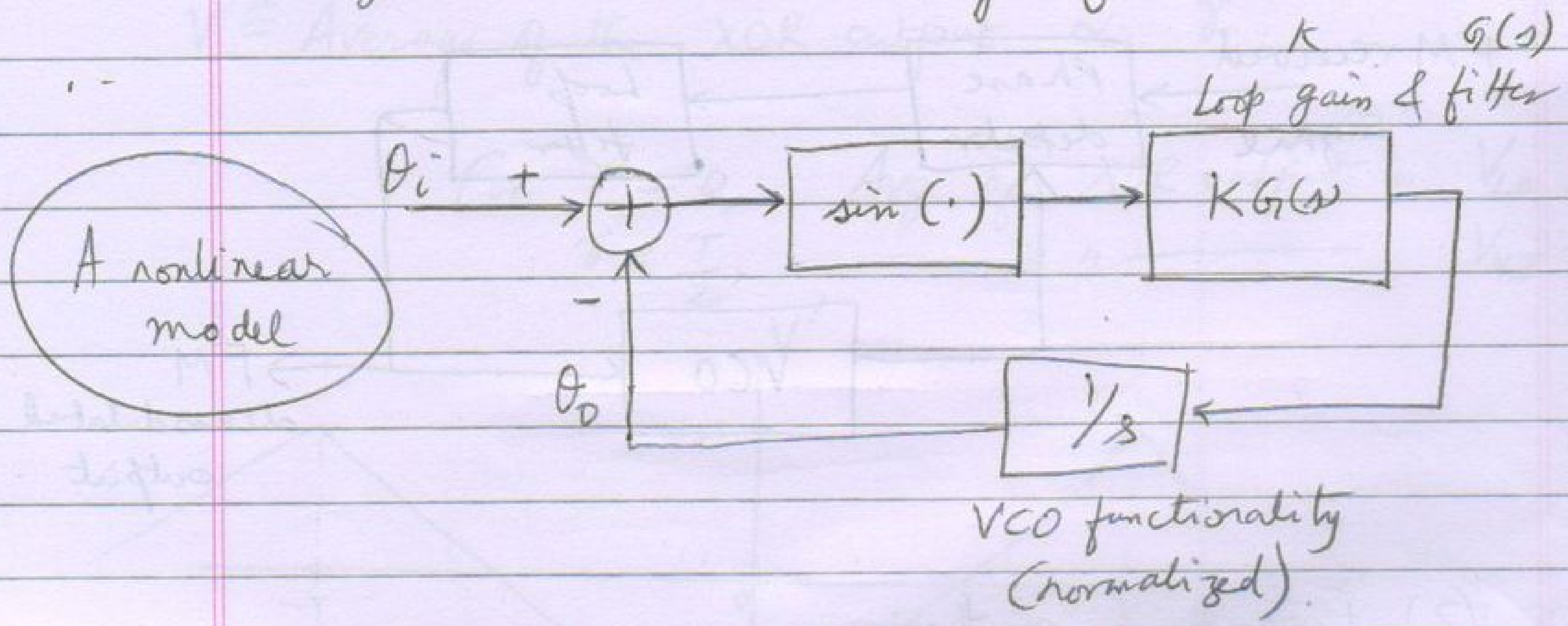
$f_o = 1\,\text{GHz}$

Note $\dfrac{f_o}{f_{crystal}} = K = 25$ (an integer)



PLL model:

Now, we will develop a model for the PLL to proceed with analysis and gain some insight on the working of a PLL.

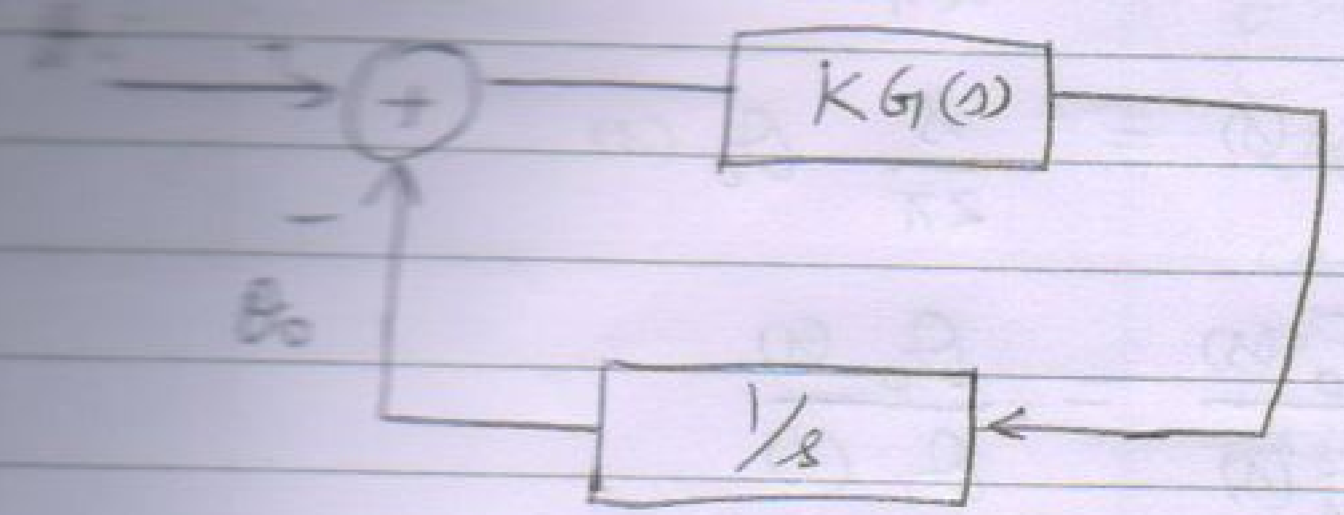


- Ignores the $2f_c$ term at the output of the phase detector
- $K = \frac{1}{2}\underbrace{A_c A_v}_{\text{defined in previous lecture}} K_v$. (Loop gain)

Linearized model

When $\theta_i - \theta_o$ is close to zero (PLL is in tracking mode)

$$\sin(\theta_i - \theta_o) \approx \theta_i - \theta_o .$$



This is an LTI system and can be analyzed using the Laplace transform.

Define $\theta_e(t) = \theta_i(t) - \theta_o(t)$

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

$$f_o(t) = \frac{1}{2\pi}\frac{d\theta_o(t)}{dt}$$

$$f_e(t) = f_i(t) - f_o(t).$$

VCO frequency for zero input : f_c.
(called quiescent frequency)

$$\Theta_o(s) = \left(\Theta_i(s) - \Theta_o(s)\right)\left(K\,G(s)\right)\left(\frac{1}{s}\right)$$

$$\Rightarrow \quad \Theta_o(s)\left[1 + \frac{KG(s)}{s}\right] = \Theta_i(s)\left[\frac{KG(s)}{s}\right]$$

$$H(s) \triangleq \frac{\Theta_o(s)}{\Theta_i(s)} = \frac{KG(s)/s}{1 + \frac{KG(s)}{s}} = \frac{KG(s)}{s + KG(s)}.$$

$$H_e(s) \triangleq \frac{\theta_e(s)}{\theta_i(s)} = \frac{\theta_i(s) - \theta_o(s)}{\theta_i(s)} =$$ [illegible]

$$= \frac{s}{s + KG(s)}.$$

[Note : $F_i(s) = \dfrac{s}{2\pi}\,\theta_i(s)$

$$F_o(s) = \frac{s}{2\pi}\,\theta_o(s)$$

$$\Rightarrow \quad \frac{F_o(s)}{F_i(s)} = \frac{\theta_o(s)}{\theta_i(s)}$$

$$\frac{F_e(s)}{F_i(s)} = \frac{\theta_e(s)}{\theta_i(s)} = \frac{s}{s + KG(s)}$$].

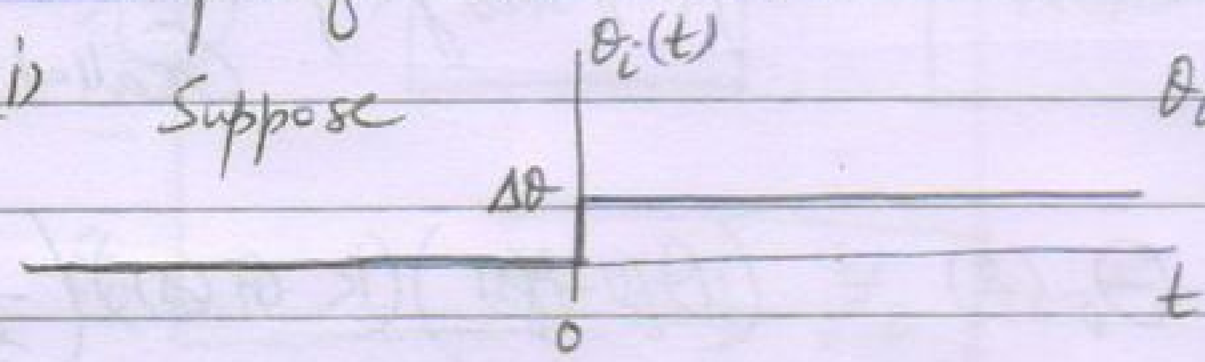
<u>Case 1</u> : (First-order PLL) $\underline{\underline{G(s) = 1}}$.

$$H(s) = \frac{K}{s+K} \; ; \quad H_e(s) = \frac{s}{s+K}.$$

Stable response for $K > 0$. (pole at $-K$)

Some specific scenarios.

(i) Suppose



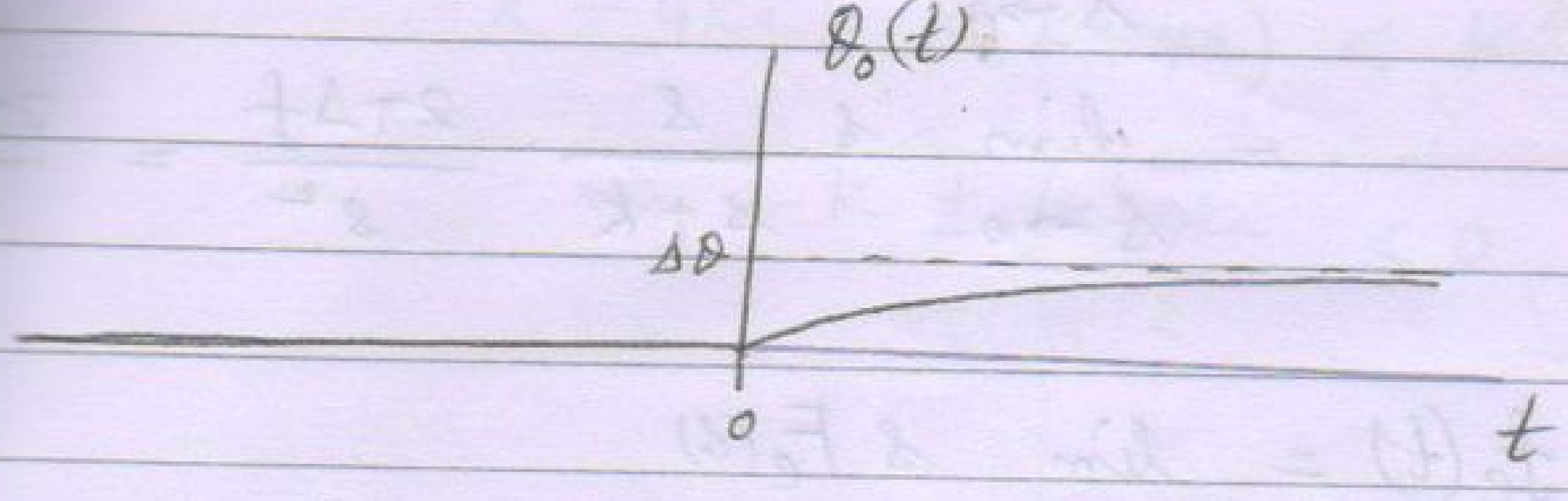
$$\theta_i(t) = \Delta\theta \, I_{[0,\infty)}(t)$$

$$\theta_i(s) = \frac{\Delta\theta}{s}.$$

$$\theta_o(s) = H(s)\,\theta_i(s)$$

$$= \frac{K}{s+K} \cdot \frac{\Delta\theta}{s} = \frac{\Delta\theta}{s} - \frac{\Delta\theta}{s+K}.$$

$$\Rightarrow \theta_o(t) = \Delta\theta\left(1 - e^{-Kt}\right) I_{[0,\infty)}(t).$$



$$\lim_{t\to\infty} \theta_o(t) = \Delta\theta \quad (\text{or}) \quad \underbrace{\lim_{t\to\infty} \theta_e(t) = 0}.$$

The first-order PLL can track a sudden change in phase at the input

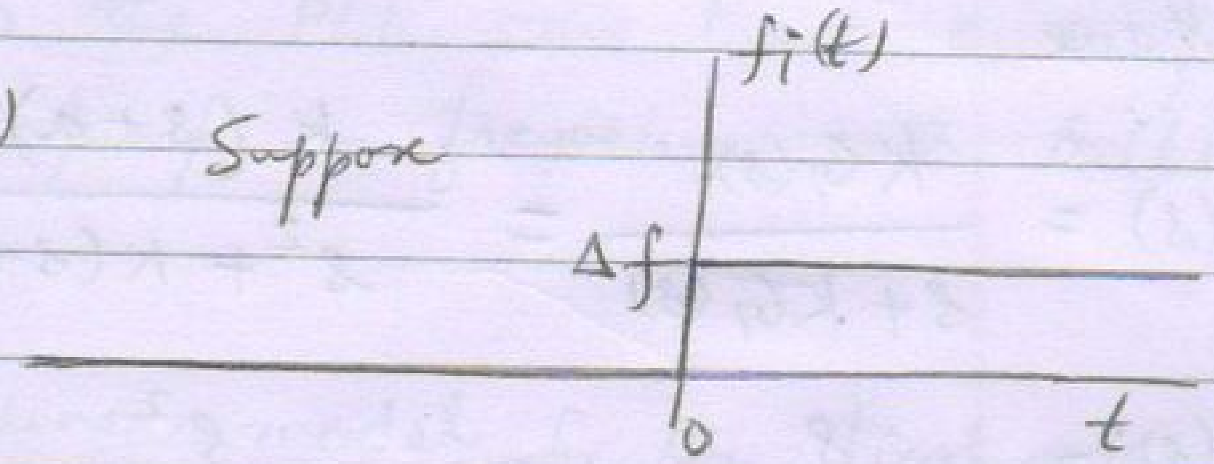
Further, $\theta_e(t) \to 0$ exponentially fast (time constant $\frac{1}{K}$).

Also note $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$.

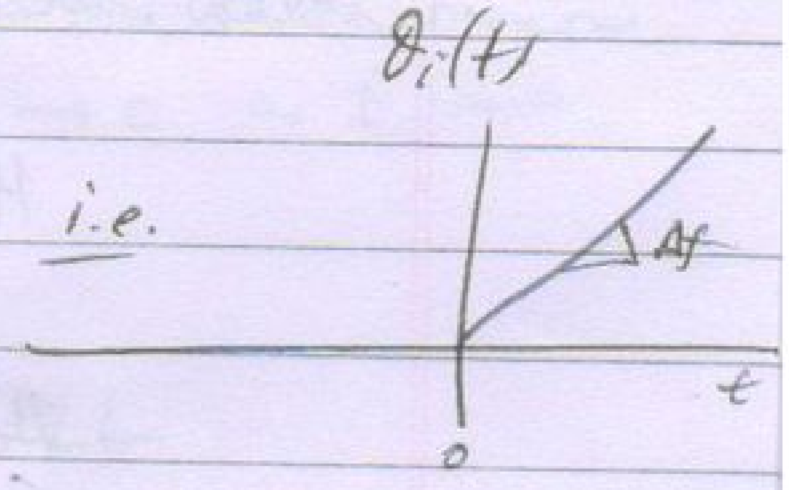
$$\lim_{s\to 0} s\,\Theta_e(s) = \lim_{s\to 0} s\, H_e(s)\,\Theta_i(s)$$

$$= \lim_{s\to 0} s\,\frac{s}{s+K}\cdot\frac{\Delta\theta}{s} = 0.$$

(ii) Suppose



i.e.



$$F_i(s) = \frac{\Delta f}{s}.$$

$$\theta_i(t) = 2\pi \int_0^t f_i(\tau)\,d\tau$$

$$\Theta_i(s) = 2\pi\,\frac{F_i(s)}{s} = \frac{2\pi\,\Delta f}{s^2}.$$

$$\lim_{t \to \infty} \theta_e(t) = \lim_{s \to 0} s\,\theta_e(s)$$

$$= \lim_{s \to 0} s \cdot \frac{s}{s+K} \cdot \frac{2\pi \Delta f}{s^2} = \frac{2\pi \Delta f}{K} \neq 0$$

$$\lim_{t \to \infty} f_e(t) = \lim_{s \to 0} s\,F_e(s)$$

$$= \lim_{s \to 0} s \cdot \frac{s}{s+K} \cdot \frac{\Delta f}{s} = 0.$$

$\Rightarrow$ the PLL can track the jump in instantaneous frequency.
($f_e(t) \to 0$)
But there is a phase offset at steady state equal to $\frac{2\pi \Delta f}{K}$.

$\boxed{\text{Lecture 25: (10 Mar 2016)}}$

<u>case 2</u>: (Second-order PLL) $G(s) = 1 + \frac{a}{s}$. $(a > 0)$

Proportional feedback

Integral feedback

Now, we have

$$H(s) = \frac{KG(s)}{s + KG(s)} = \frac{K(s+a)}{s^2 + K(s+a)}$$

$$H_e(s) = \frac{s}{s + KG(s)} = \frac{s^2}{s^2 + K(s+a)}$$

Poles are at $\frac{-K \pm \sqrt{K^2 - 4Ka}}{2}$

When $K^2 - 4Ka < 0$, both poles are on the LHS if $K > 0$.

when $K^2 - 4Ka > 0$ (or) $K > 4a$

$$K^2 - 4Ka = K(K-4a) < K^2$$

$\Rightarrow \dfrac{-K \pm \sqrt{K^2-4Ka}}{2} < 0$, i.e. both poles are on the LHS for $\underline{K>0}$.

$\Rightarrow$ Stable $H(s)$, $H_e(s)$ for $K > 0$.

(i) Suppose $\theta_i(t) = \Delta\theta \, I_{[0,\infty)}(t)$.

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} s\,\Theta_e(s)$$

$$= \lim_{s\to 0} s \cdot \frac{s^2}{s^2 + K(s+a)} \cdot \frac{\Delta\theta}{s} = 0.$$

(ii) Suppose $f_i(t) = \Delta f \, I_{[0,\infty)}(t)$

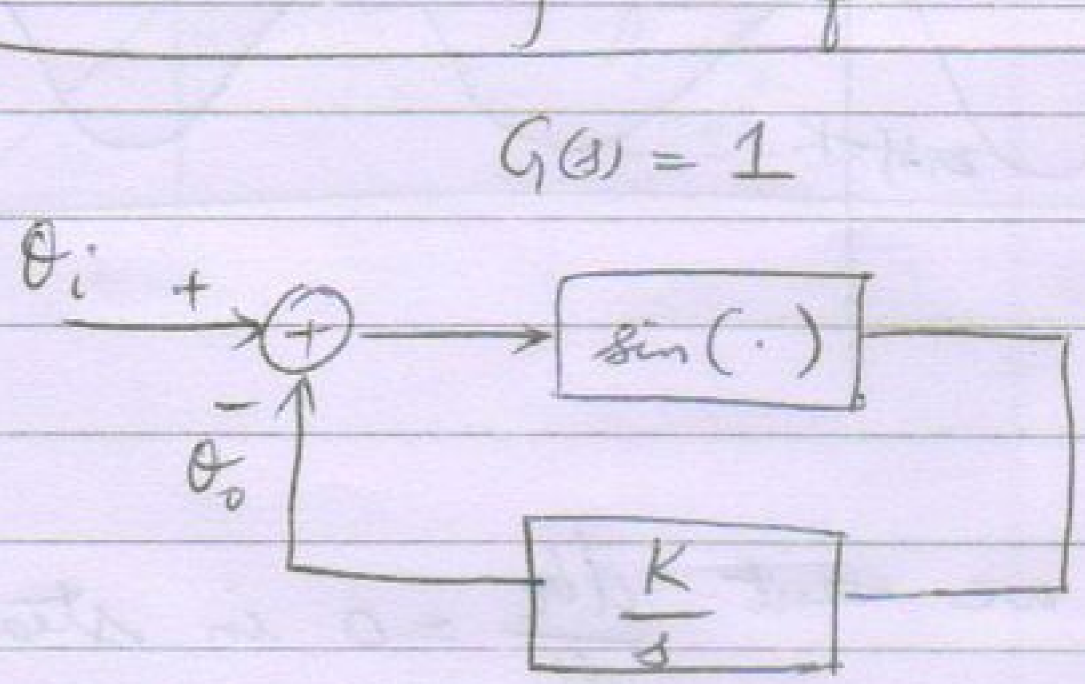
$$\lim_{t\to\infty} f_e(t) = \lim_{s\to 0} sF_e(s) = \lim_{s\to 0} s\,\frac{\cancel{K}(s^2\cancel{+a})}{s^2+K(s+a)} \cdot \frac{\Delta f}{s} = 0$$

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} s\,\Theta_e(s) = \lim_{s\to 0} s\,\frac{s^2}{s^2+K(s+a)}\,\frac{\Delta f}{s^2} = 0.$$

$\Rightarrow$ the PLL can track even for the instantaneous jump in frequency with $\theta_e(t) \to 0$ as $t \to \infty$.

Nonlinear model for the first order PLL

$$G(s) = 1$$



The time domain model can be written as

$$\theta_o(t) = \int_0^t K \sin(\theta_e(\tau))d\tau$$

& $\theta_o(t) = \theta_i(t) - \theta_e(t)$

Together, we can write it as

$$\int_0^t K \sin \theta_e(\tau)\, d\tau = \theta_i(t) - \theta_e(t)$$

(or)

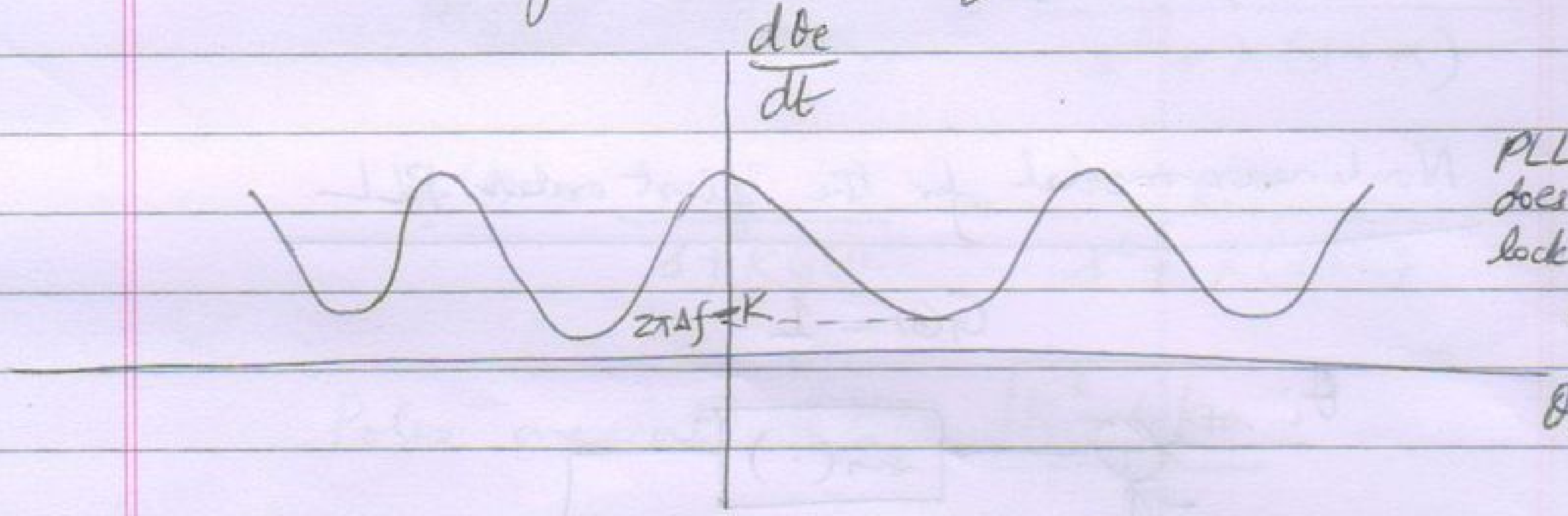
$$\boxed{K \sin \theta_e(t) = \frac{d\theta_i(t)}{dt} - \frac{d\theta_e(t)}{dt}}$$

* Let us consider the case where

$$\frac{d\theta_i}{dt} = 2\pi\Delta f \quad (\text{step frequency input})$$

$$K \sin \theta_e(t) = 2\pi\Delta f - \frac{d\theta_e(t)}{dt}$$

The above differential eqn cannot be solved analytically. We can plot $\frac{d\theta_e}{dt}$ against θ_e to gain some insight

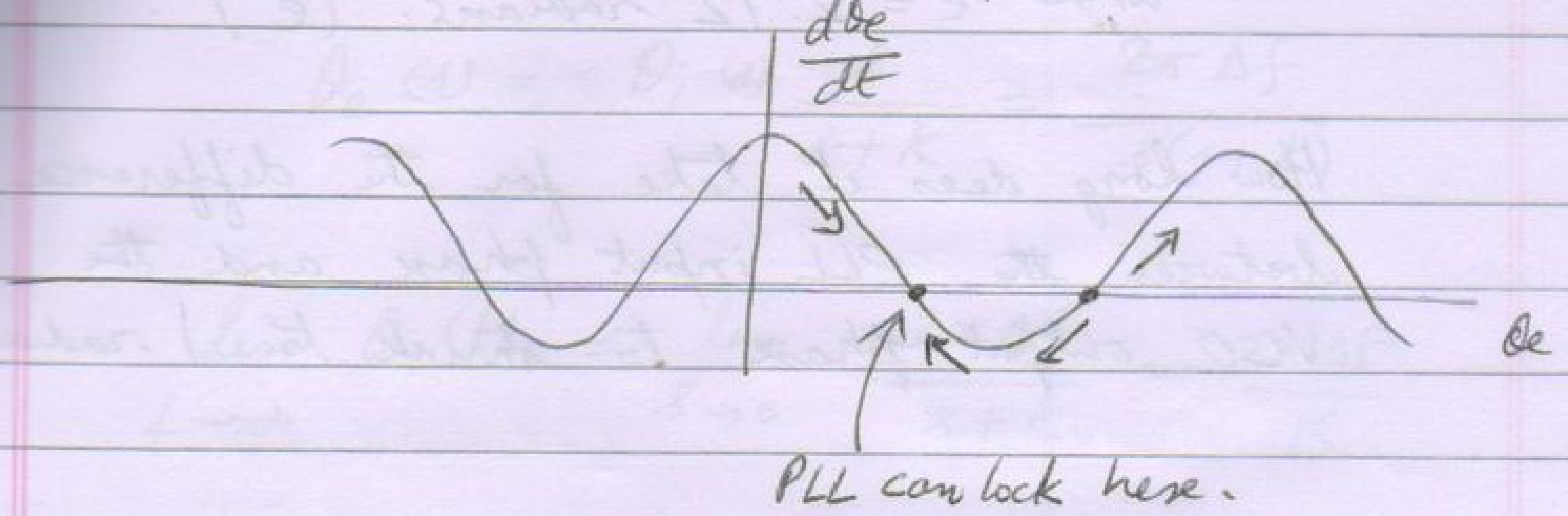


PLL does not lock in the case.

Note that we want $\frac{d\theta_e}{dt} = 0$ in steady state.

This is not possible if $2\pi\Delta f - K > 0$.

In this case, the PLL does not lock.



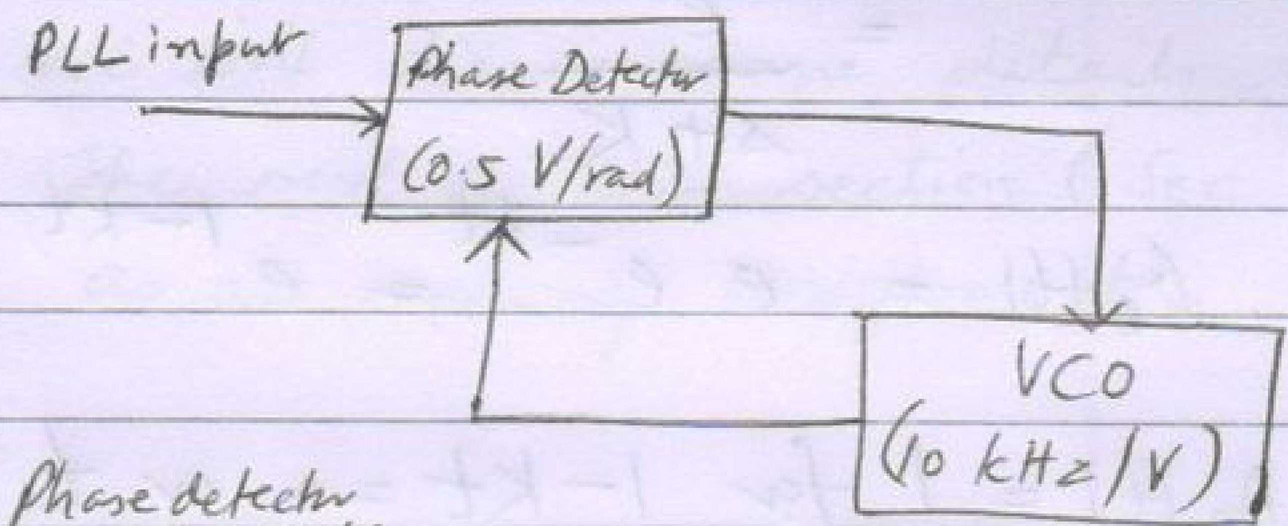
Need $2\pi\Delta f - K < 0$. (or) $\Delta f < \frac{K}{2\pi}$.

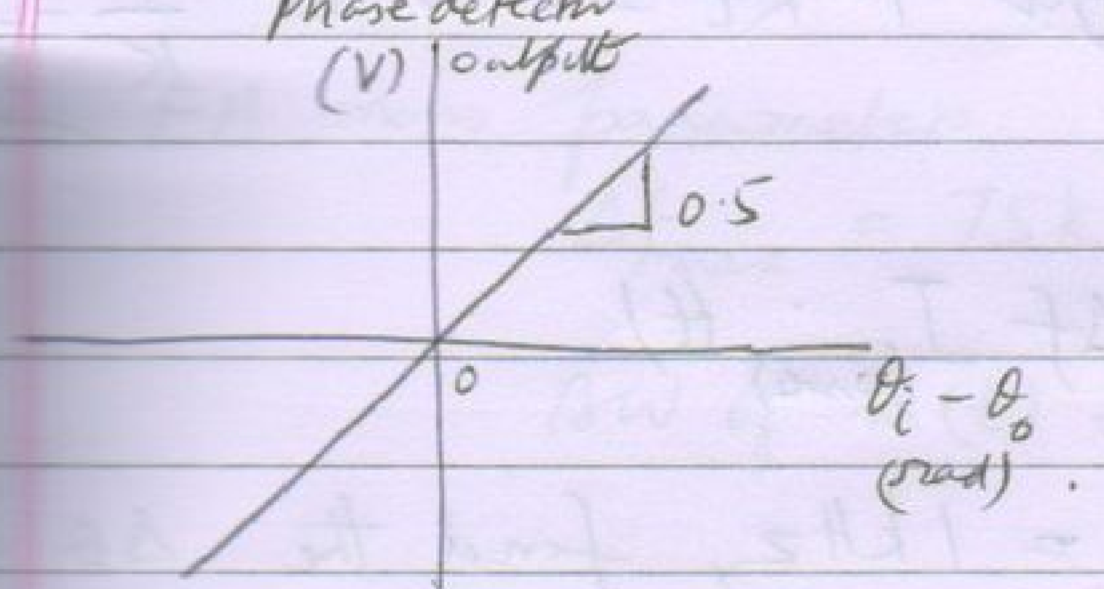
Phase error at lock $= \sin^{-1}\left(\frac{2\pi\Delta f}{K}\right)$

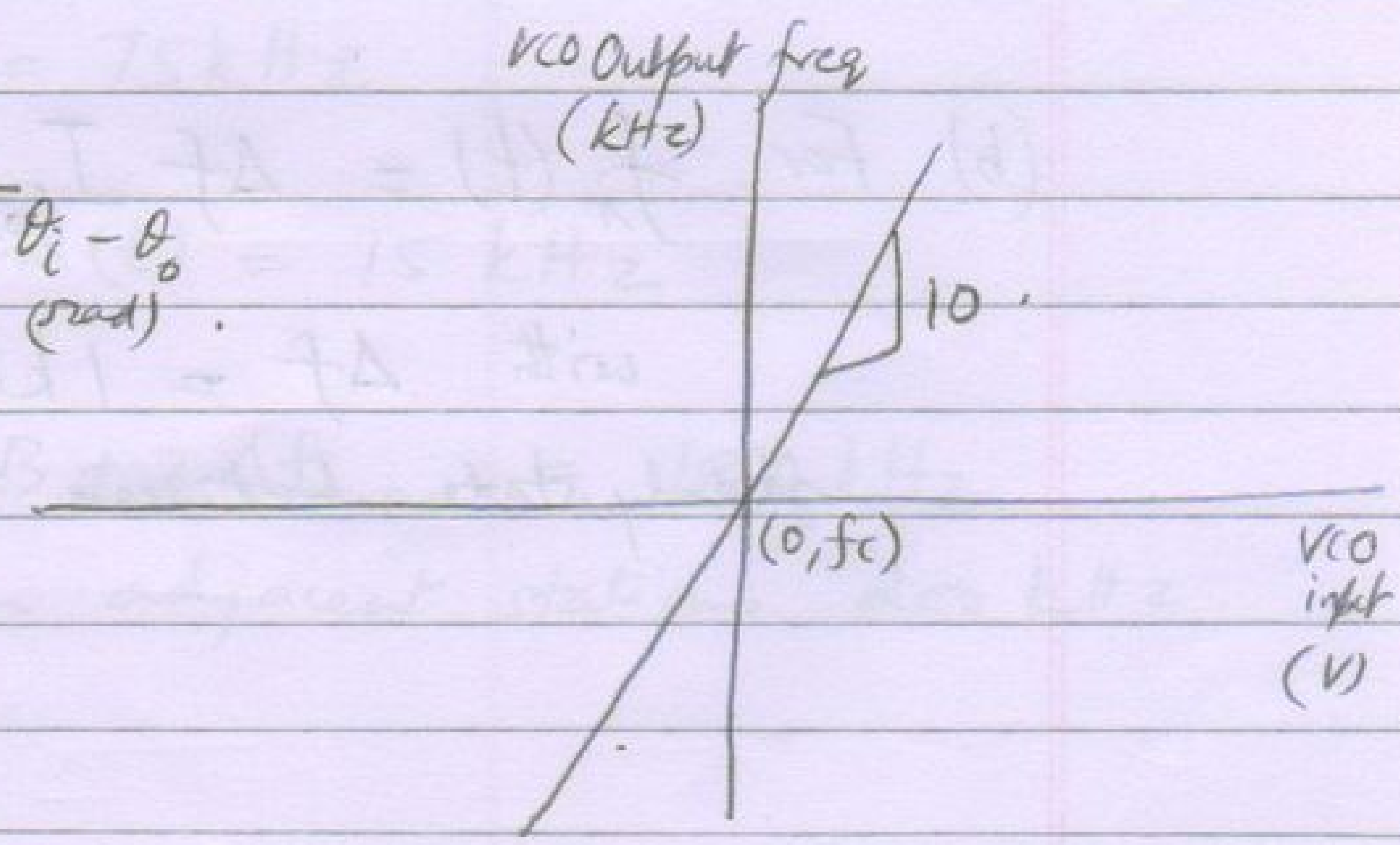
$$\left(\text{solve} \quad K \sin\theta_e(t) = 2\pi\Delta f - 0\right).$$

$\boxed{\text{Lecture 26: (11 Mar 2016)}}$

<u>Example</u> (3.5.1 in the book)







(a) $\theta_i(t) = e \, I_{[0,\infty)}(t)$

where $e = 2.72$ radians. (e°).

How long does it take for the difference between the PLL input phase and the VCO output phase to shrink to 1 radian?

$$\theta_e(t) = \theta_i(t) - \theta_o(t)$$

$$\theta_i(s) - \frac{K}{s}\theta_e(s) = \theta_e(s)$$

$$\frac{\theta_e(s)}{\theta_i(s)} = \frac{s}{s+K}$$

$$\theta_e(s) = \frac{s}{s+K} \cdot \frac{e}{s}$$

$$= \frac{e}{s+K}$$

$$\theta_e(t) = e\, e^{-Kt} = e^{1-Kt}$$

$\theta_e(t) = 1$ for $1 - Kt = 0$ or $t = \frac{1}{K} = \frac{1}{5}$ ms

(b) For $f_i(t) = \Delta f \, I_{[0,\infty)}(t)$

with $\Delta f = 1\text{kHz}$, find the $\Delta\theta$ in steady state between the PLL input & VCO output

$$\theta_i(s) = \frac{2\pi\Delta f}{s^2}$$

$$\theta_e(s) = \theta_i(s)\frac{s}{s+K} = \frac{2\pi\Delta f}{s(s+K)}$$

$$\lim_{t\to\infty}\theta_e(t) = \lim_{s\to 0}\frac{2\pi\Delta f}{s+K} = \frac{2\pi\Delta f}{K}$$

$$= \frac{2\pi}{5} \text{ radians.}$$

Some real-world analog communication systems.

(1) AM/FM radio
(2) Analog broadcast TV
(3) Audiotapes/Videotapes
....

We will discuss some details of FM radio here. The rest of this section (Sec. 3.6) is left as a reading assignment.

FM mono parameters:

$$\Delta f_{max} = 75\,kHz$$

$$BW \text{ of } m(t) = 15\,kHz$$

$$BW_{FM} \approx 2B + 2\Delta f_{max} = 180\,kHz$$

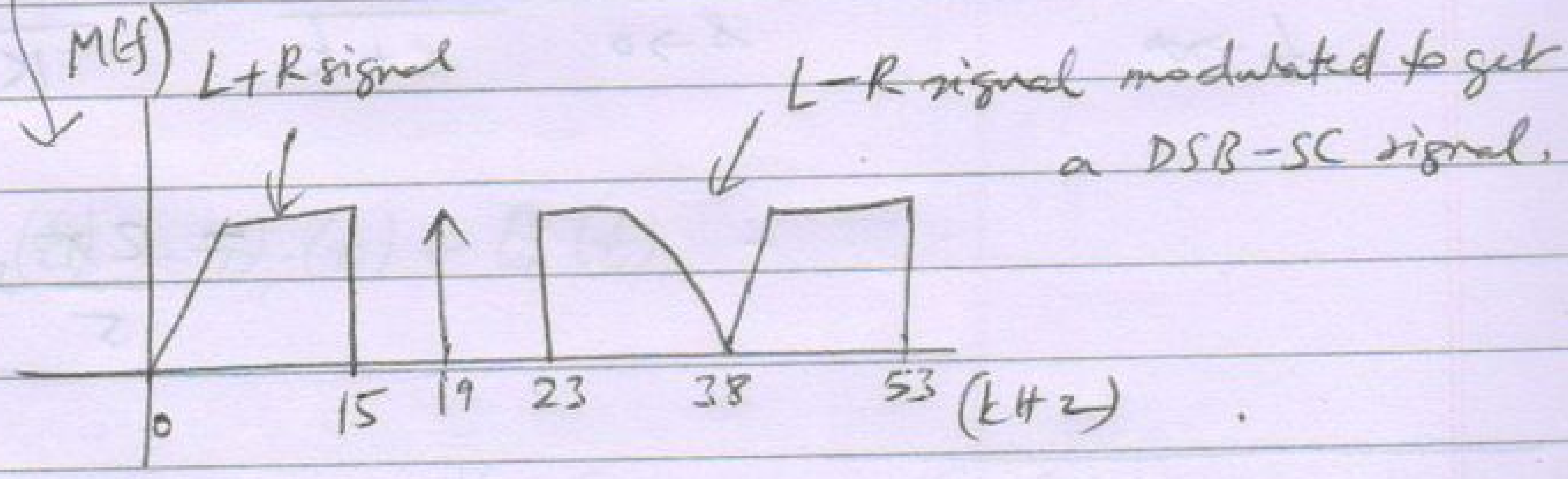
Space between adjacent stations 200 kHz.

FM stereo parameters:

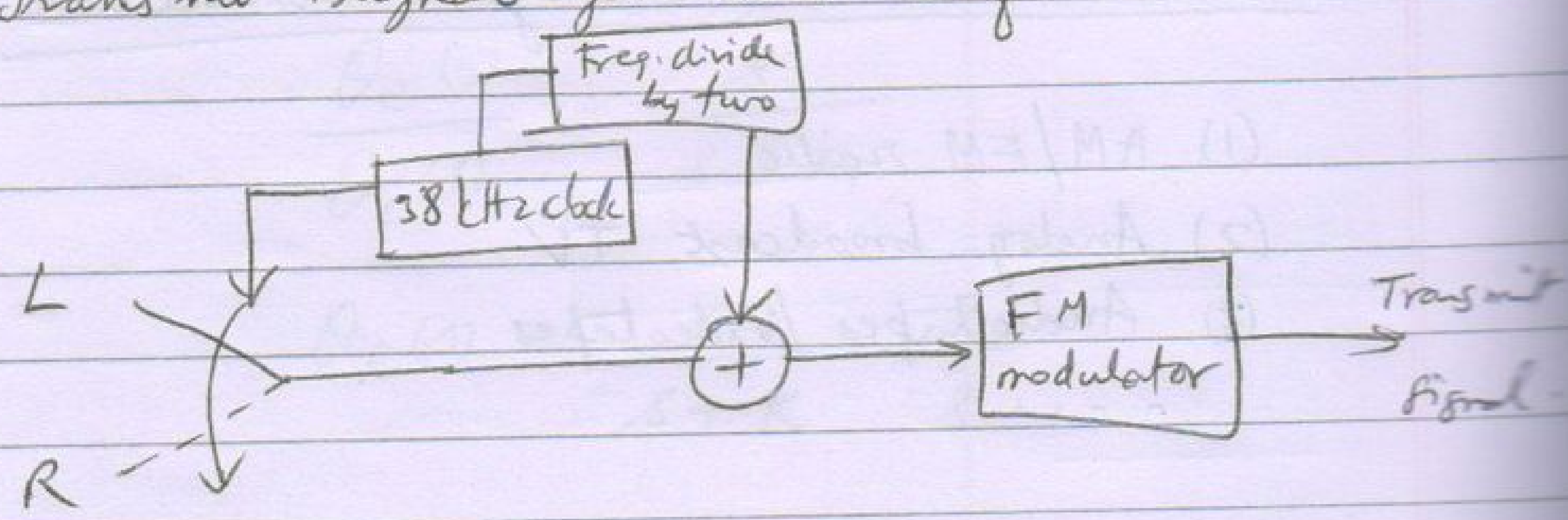
$$\Delta f_{max} = 75 \text{ kHz}$$

$$\text{BW of } m(t) = 53 \text{ kHz}.$$

$$BW_{FM} \approx 2(53+75) = 256 \text{ kHz}.$$



Transmit signal generated as follows.



(Exercise: Show that the above system generates the required transmit signal).