EE3005: Communication Systems

Problem Set 6: Angle Modulation

1. The signal $a(t)$ shown in Figure 6.1 is fed to a VCO with quiescent frequency of 5 MHz and frequency deviation of 25 KHz/mV. Denote the output of the VCO by $y(t)$.

   (a) Provide an estimate of the bandwidth of y. Clearly state the assumptions that you make.

   (b) The signal $y(t)$ is passed through an ideal bandpass filter of bandwidth 5 KHz, centered at 5.005 MHz. Provide the simplest possible expression for the power at the filter output (if you can give a numerical answer, do so).

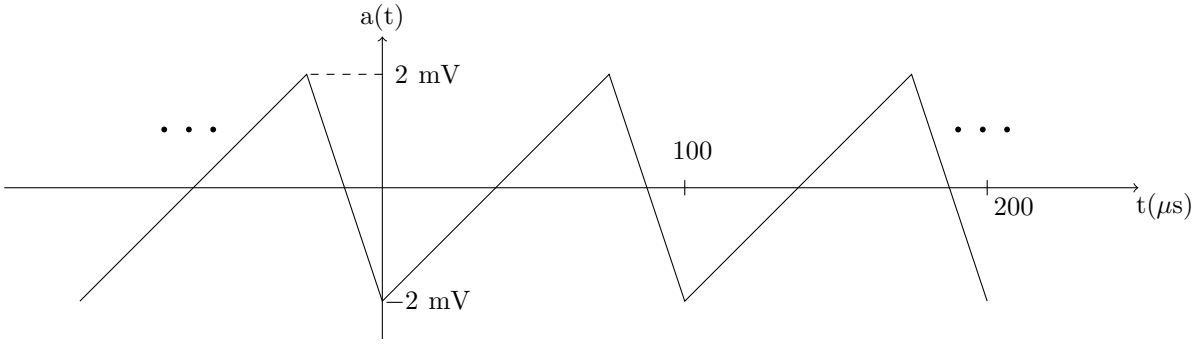

Figure 6.1: Input to the VCO in Problem 1.

2. Consider a message signal $m(t)$ with spectrum $M(f) = I_{[-2,2]}(f)$.

   (a) Sketch the spectrum of the DSB-SC signal $u_{DSB-SC} = 10m(t)\cos 300\pi t$. What is the power and bandwidth of u?

   (b) The signal in (a) is passed through an envelope detector. Sketch the output, and comment on how it is related to the message.

   (c) What is the smallest value of A such that the message can be recovered without distortion from the AM signal $u_{AM} = (A + m(t))\cos 300\pi t$ by envelope detection?

   (d) Give a time-domain expression of the form

   $$u_p(t) = u_c(t)\cos 300\pi t - u_s(t)\sin 300\pi t$$

   obtained by high-pass filtering the DSB signal in (a) so as to let through only frequencies above 150 Hz.

   (e) Consider a VSB signal constructed by passing the signal in (a) through a passband filter with transfer function for positive frequencies specified by:

   $$H_p(f) = \begin{cases} f - 149 & 149 \le f \le 151 \\ 2 & f \ge 151. \end{cases}$$

(you should be able to sketch $H_p(f)$ for both positive and negative frequencies.) Find a time domain expression for the VSB signal of the form

$$u_p(t) = u_c(t)\cos 300\pi t - u_s(t)\sin 300\pi t.$$

3. Figure 6.2 shows, as a function of time, the phase deviation of a bandpass FM signal modulated by a sinusoidal message.

   (a) Find the modulation index (assume that it is an integer multiple of π for your estimate).

   (b) Find the message bandwidth.

   (c) Estimate the bandwidth of the FM signal using Carson's formula.

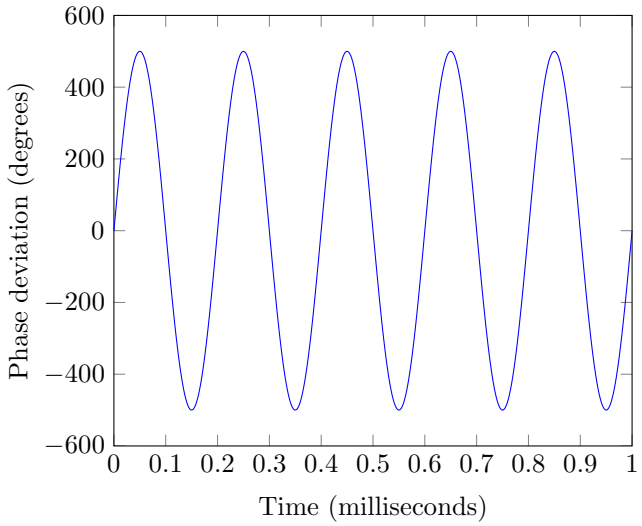

Figure 6.2: Phase deviation of FM signal for Problem 3.

4. The input $m(t)$ to an FM modulator with $k_f = 1$ has Fourier transform

$$M(f) = \begin{cases} j2\pi f & |f| < 1 \\ 0 & \text{else.} \end{cases}$$

The output of the FM modulator is given by

$$u(t) = A\cos(2\pi f_c t + \phi(t))$$

where f_c is the carrier frequency.

   (a) Find an explicit time domain expression for $\phi(t)$ and carefully sketch $\phi(t)$ as a function of time.

   (b) Find the magnitude of the instantaneous frequency deviation from the carrier at time $t = \frac{1}{4}$.

   (c) Using the result from (b) as an approximation for the maximum frequency deviation, estimate the bandwidth of $u(t)$.