EE3005: Communication Systems

Problem Set 2: Fourier Analysis

1. Find and sketch the Fourier transforms for the following signals:
   (a) $u(t) = (1 - |t|)I_{[-1,1]}(t)$.
   (b) $v(t) = \text{sinc(2t)sinc(4t)}$.
   (c) $s(t) = v(t)\cos(200\pi t)$.
   (d) Classify each of the signals in (a)-(c) as baseband or passband.

2. Use Parseval's identity to compute the following integrals:
   (a) $\int_{-\infty}^{\infty} \text{sinc}^2\text{(2t)}$.
   (b) $\int_0^{\infty} \text{sinc(t)sinc(2t)}$.

3. (a) For $u(t) = \text{sinc(t)sinc(2t)}$, where t is in microseconds, find and plot the magnitude spectrum $|U(f)|$, carefully labeling the units of frequency on the x axis.
   (b) Now, consider $s(t) = u(t)\cos(200\pi t)$. Plot the magnitude spectrum $|S(f)|$, again labeling the units of frequency and carefully showing the frequency over which spectrum is non-zero.

4. Consider the tent signal $s(t) = (1 - |t|)I_{[-1,1]}(t)$.
   (a) Find and sketch the Fourier transform $S(f)$.
   (b) Compute the 99% energy containment bandwidth in KHz, assuming that the unit of time is milliseconds.

5. A wireless channel has impulse response given by $h(t) = 2\delta(t-0.1) + j\delta(t-0.64) - 0.8\delta(t-2.2)$, where the unit of time is in microseconds.
   (a) What is the delay spread and coherence bandwidth?
   (b) Plot the magnitude and phase of the channel transfer function $H(f)$ over the interval $[-2B_c, 2B_c]$, where B_c denotes the coherence bandwidth computed in (a). Comment on how the phase behaves when $|H(f)|$ is small.
   (c) Express $|H(f)|$ in dB, taking 0 dB as the gain of anominal channel $h_{nom}(t) = 2\delta(t - 0.1)$ corresponding to the first ray alone. what are the fading depths that you see with respect to this nominal?
   Define the average channel power gain over $[-W/2, W/2]$ as

   $$\bar{G}(W) = \frac{1}{W}\int_{-W/2}^{W/2} |H(f)|^2 df$$

   This is a simplified measure of how increasing bandwidth W can help compensate for frequency-selective fading: we hope that, as W gets large, we can average out fluctuations in $|H(f)|$.
   (d) Plot $\bar{G}(W)$ as a function of W/B_c and comment on how large the bandwidth needs to be (as a multiple of B_c) to provide "enough averaging".