EE3005: Communication Systems

Problem Set 3: Receiver Structure

1. For a signal $s(t)$, the *matched filter* is defined as a filter with impulse response $h(t) = s_{mf}(t) = s^*(-t)$ (we allow signals to be complex valued, since we want to handle complex baseband signals as well as physical real-valued signals).
   (a) Sketch the matched filter impulse response for $s(t) = I_{[1,3]}(t)$.
   (b) Find and sketch the convolution $y(t) = (s * s_{mf})(t)$. This is the output when the signal is passed through its matched filter. Where does the peak of the output occur?
   (c) **(True or False)** $Y(f) \geq 0$ for all f.

2. Consider a pass band signal of the form

$$u_p(t) = a(t) \cos 200\pi t$$

   where $a(t) = \mathrm{sinc(2t)}$, and where the unit of time is in seconds.
   (a) What is the frequency band occupied by $u_p(t)$?
   (b) The signal $u_p(t) \cos(199\pi t)$ is passed through a lowpass filter to obtain an output $b(t)$. Give an explicit expression for $b(t)$, and sketch $B(f)$ (if $B(f)$ is complex-valued, sketch its real and imaginary parts separately).
   (c) The signal $u_p(t) \sin(199\pi t)$ is passed through a lowpass filter to obtain an output $c(t)$. Give an explicit expression for $c(t)$, and sketch $C(f)$ (if $C(f)$ is complex-valued, sketch its real and imaginary parts separately).
   (d) Can you reconstruct $a(t)$ from simple real-valued operations performed on $b(t)$ and $c(t)$? If so, sketch a block diagram for the operations required. If not, say why not.

3. Consider the signal $s(t) = I_{[-1,1]}(t) \cos(400\pi t)$.
   (a) Find and sketch the baseband signal $u(t)$ that results when $s(t)$ is downconverted as shown in the upper branch of Figure 3.1.
   (b) The signal $s(t)$ is passed through the bandpass filter with impulse response $h(t) = I_{[0,1]}(t) \sin(400\pi t + \frac{\pi}{4})$. Find and sketch the baseband signal $v(t)$ that results when the filter output $y(t) = (s * h)(t)$ is downconverted as shown in the lower branch of Figure 3.1.

4. Consider a real-valued passband signal $v_p(t)$ whose Fourier transform for positive frequencies is given by

$$\mathrm{Re}\left(\mathrm{V_p(f)}\right) = \begin{cases} 2, & 30 \leq f \leq 32 \\ 0, & 0 \leq f < 30 \\ 0, & 32 \leq f < \infty \end{cases}$$

$$\mathrm{Im}\left(\mathrm{V_p(f)}\right) = \begin{cases} 1 - |f - 32|, & 31 \leq f \leq 33 \\ 0, & 0 \leq f < 31 \\ 0, & 33 \leq f < \infty \end{cases}$$

   (a) Sketch the real and imaginary parts of $V_p(f)$ for both positive and negative frequencies.
   (b) Specify, in both the time domain and the frequency domain, the waveform that you get when you pass $v_p(t) \cos(60\pi t)$ through a low pass filter.

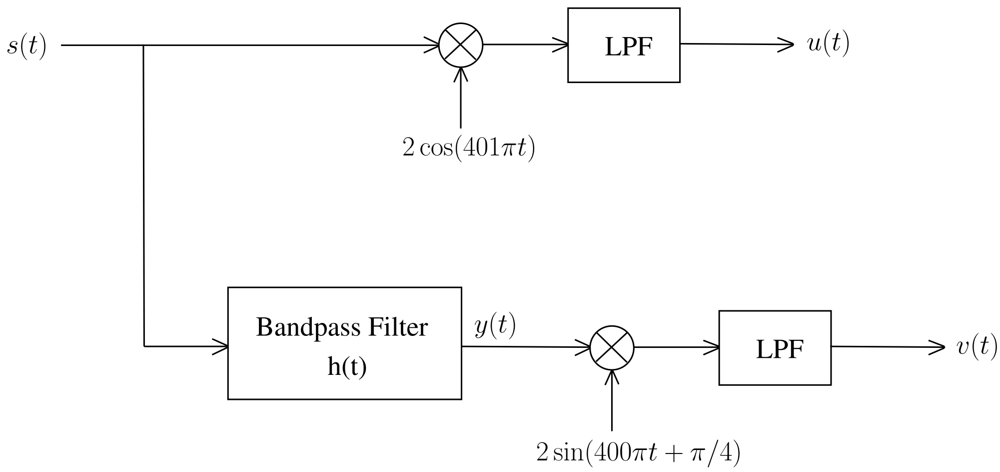

Figure 3.1: Operations involved in Problem 3.