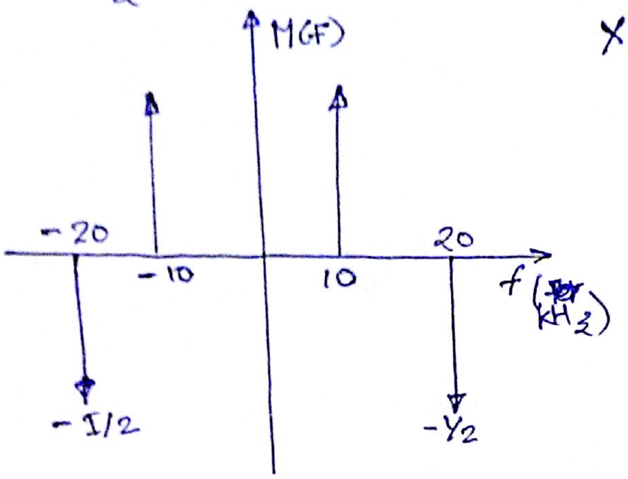
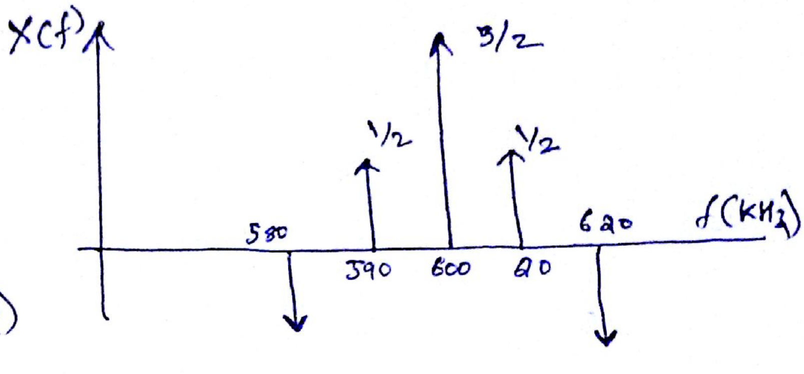
a) The message spectrum $M(f) = \delta(f-10) + \delta(f+10) - \frac{1}{2}\delta(f-20) - \frac{1}{2}\delta(f+20)$. The spectrum of the AM signal is given by

$$X(f) = \frac{5}{2}\delta(f-f_c) + \frac{5}{2}\delta(f+f_c) + \frac{1}{2}M(f-f_c) + \frac{1}{2}M(f+f_c).$$



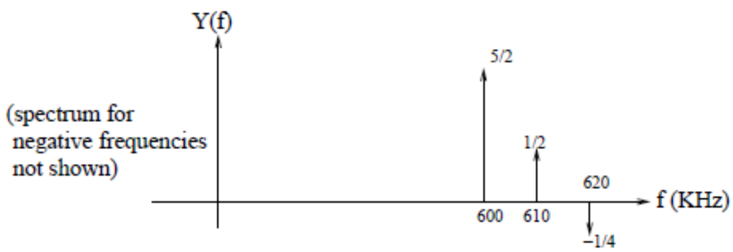
Message Spectrum



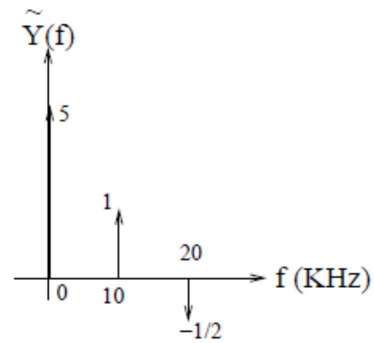
AM Spectrum
(Spectrum for -ve freq not shown).

b) The modulation index $a_{mod} = M_0/A_c$, where $-M_0 = \min_t m(t)$. To simplify notation, let us minimize $g(x) = 2\cos x - \cos 2x$. We can actually do this by inspection: for $x = \pi$, $\cos x = -1$ and $\cos 2x = 1$, so that $\min_x g(x) = -3$. Alternatively, we could set the derivative to zero: $g'(x) = -2\sin x + 2\sin 2x = 2\sin x(-1 + 2\cos x)$ is satisfied if $\sin x = 0$ (ie, $\cos x = \pm 1$) or $\cos x = 1/2$. We can check that the first solution with $\cos x = -1$ minimizes $g(x)$.
Thus, we obtain $M_0 = 3$ and hence $a_{mod} = M_0/A_c = 3/5$ or 60%

1c. It is clear that a highpass filter with cutoff at 595 KHz selects the USB signal plus the carrier. The passband output has spectrum as shown in Figure 3.20(a), and the complex envelope with respect to 600 KHz is shown in Figure 3.20(b)



(a) Passband output of BPF



(b) Complex envelope

Figure 3.20: Passband output of bandpass filter and its complex envelope with respect to 600 KHz reference, for Example 3.2.4. Axes are not to scale.

. Taking the inverse Fouriertransform, the time domain complex envelope is given by

$$\tilde{y}(t) = 5 + e^{j20\pi t} - \frac{1}{2}e^{j40\pi t}$$

$$y_s(t) = \mathrm{Im}\left(\tilde{y}(t)\right) = \sin 20\pi t - \frac{1}{2}\sin 40\pi t$$

where t is in milliseconds. Another approach is to recognize that the Q component is the Qcomponent of the USB signal, which is known to be the Hilbert transform of the message.Yet another approach is to find the Q component in the frequency _ domain using

$$jY_s(f) = \left(\tilde{Y}(f) - \tilde{Y}^*(f)\right)/2$$

and then take inverse Fourier transform. In this particular example, the first approach is probably the simplest.

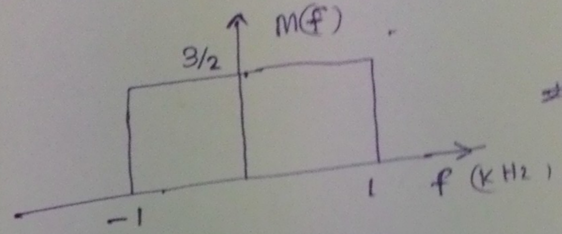
Tutorial 5 :

Question . 2 : a) and b).

$m(t) = 3\,\mathrm{sinc}(2t)$ $\quad$ t is in milliseconds.

f_c is 100 times larger than the message Bandwidth.

a). $M(f)$

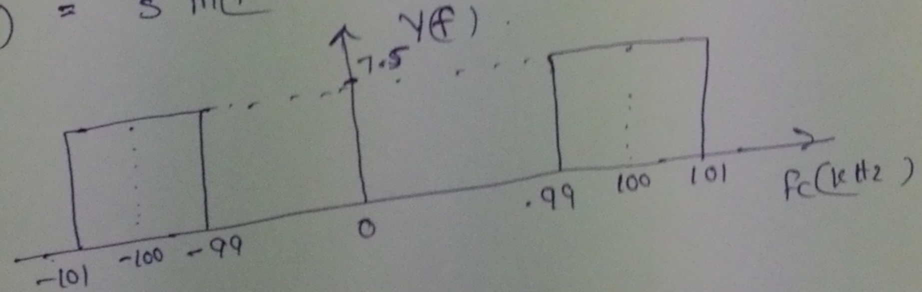


Bandwidth $W = 1$ KHz

$\Rightarrow f_c = 100$ KHz.

spectrum of $y(t) = 10\,m(t)\cos(2\pi f_c t)$.

$$Y(f) = 5\,M(f - f_c) + 5\,M(f + f_c).$$



b). LSB of $y(t) = 10\,m(t)\cos 2\pi f_c t$ is

$\cancel{5\,\hat{m}(t)\cos(2\pi f_c t) + 5\,\hat{m}(t)\sin(2\pi f_c t)}$

$$y_{LSB}(t) = 10\,m(t)\cos(2\pi f_c t) + 10\,\hat{m}(t)\sin(2\pi f_c t).$$

$\hat{m}(t)$ is the Hilbert transform of $m(t)$.

$$H(\mathrm{sinc}(t)) = \mathrm{sinc}\left(\tfrac{t}{2}\right).\sin\left(\tfrac{\pi}{2}t\right).$$

further $H(x(at)) = \mathrm{sgn}(a).\,\hat{x}(at)$.

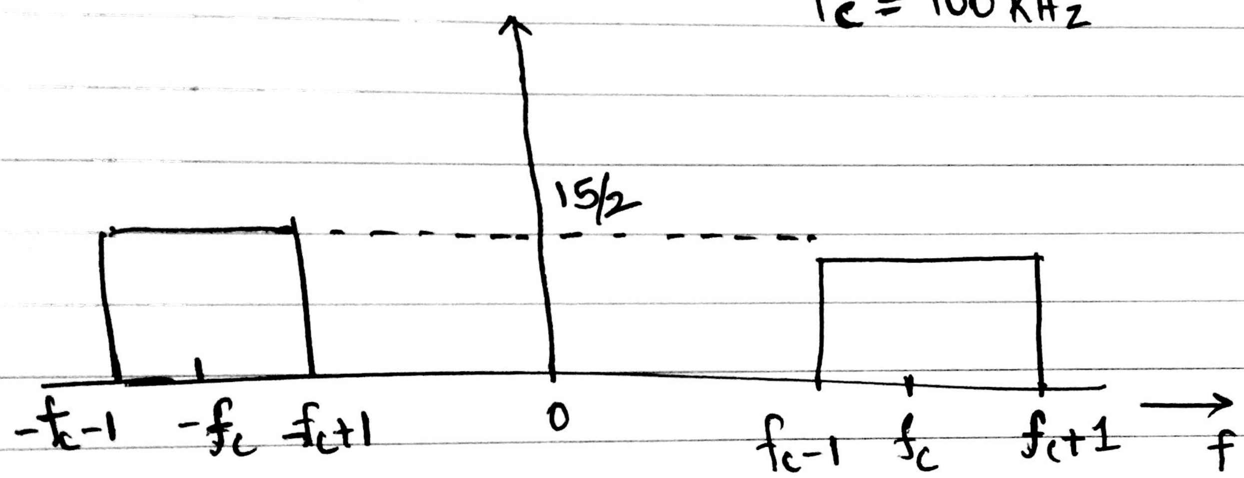
$$\Rightarrow H[\mathrm{sinc}(2t)] = \mathrm{sinc}(t)\sin(\pi t).$$

$$y_{LSB}(t) = 90\,\mathrm{sinc}(2t)\cos(2\pi f_c t) + 90\,\mathrm{sinc}(t)\,\sin(\pi t).\sin(2\pi f_c t).$$

Problem (2) (c)

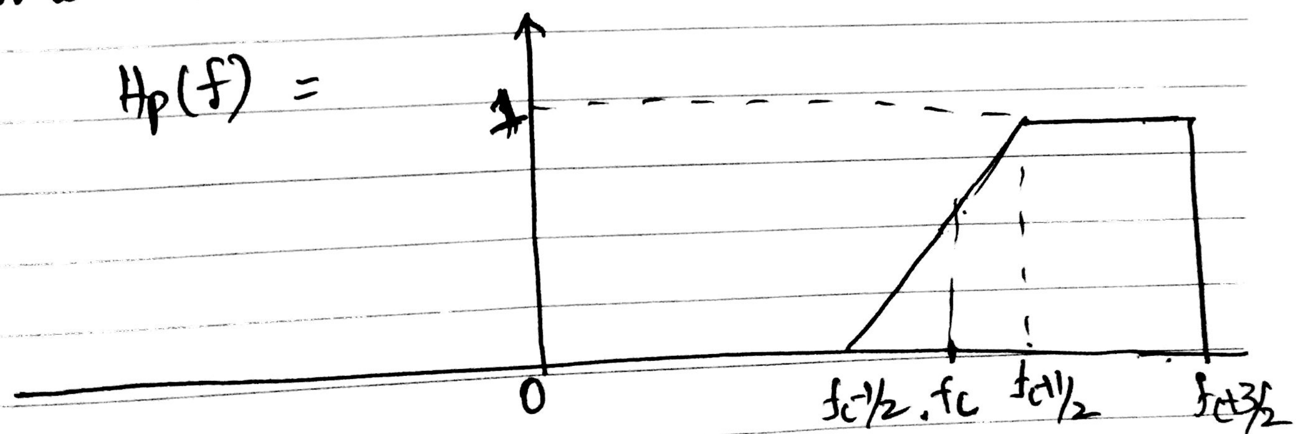
Magnitude spectrum of DSB signal

$f_c = 100\,KHz$

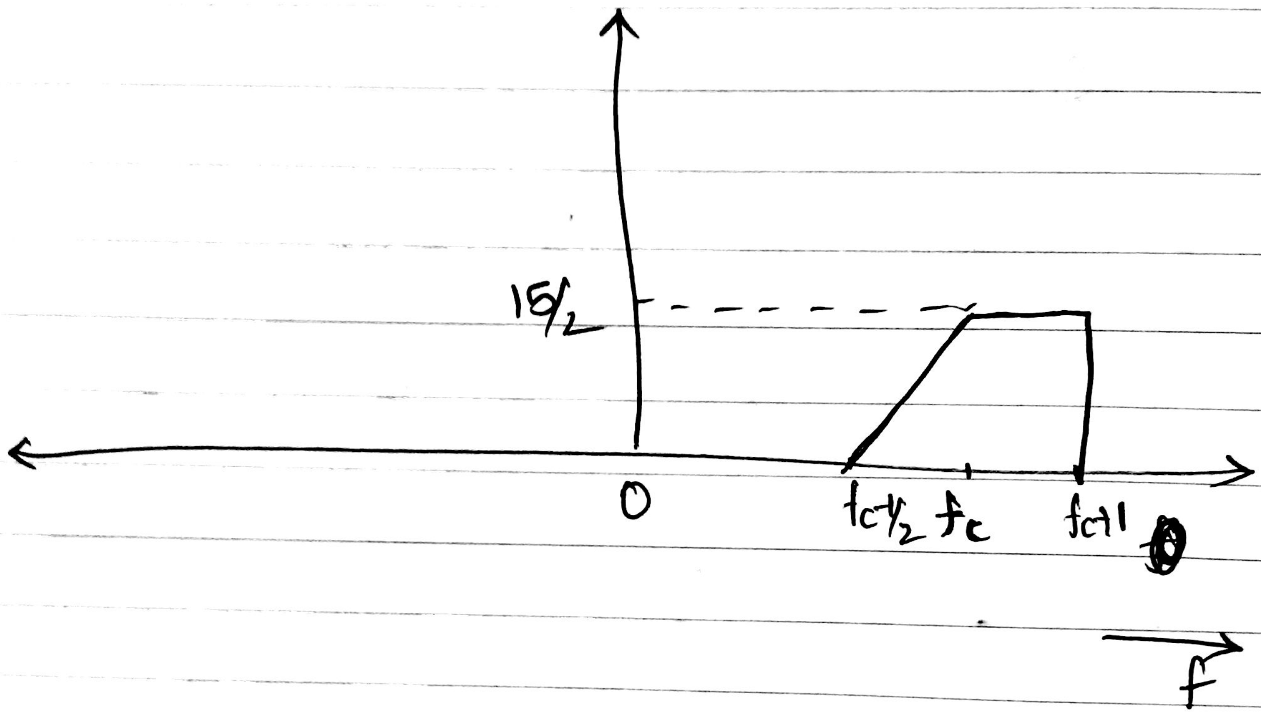


Response $H_p(f)$

$H_p(f) =$

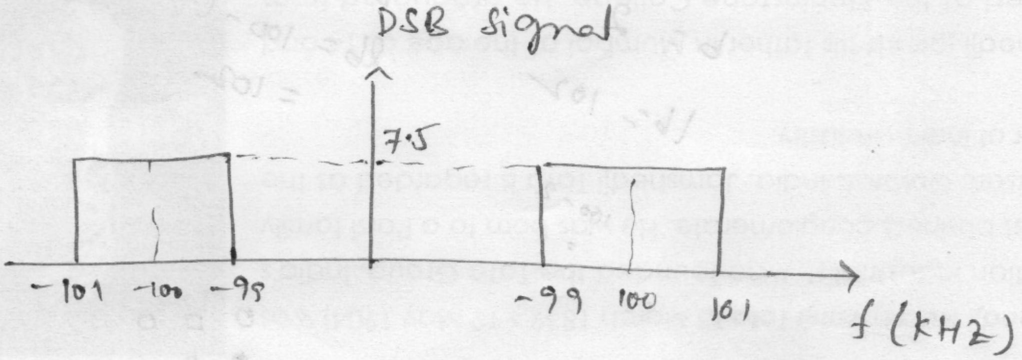


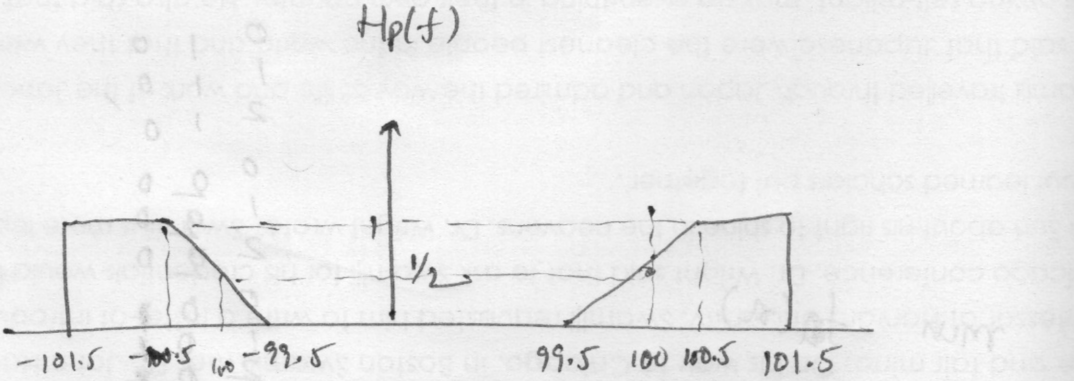
Magnitude spectrum of VSB signal

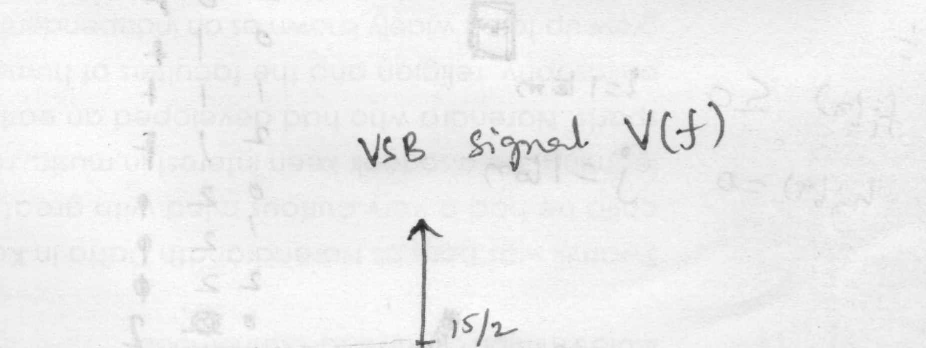


2(d)

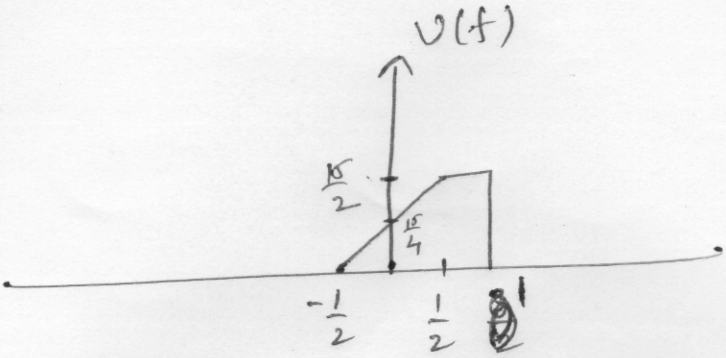
$$U_c(f) = \frac{U(f) + U^*(-f)}{2} \qquad U_s(f) = \frac{U(f) - U^*(-f)}{2j}$$



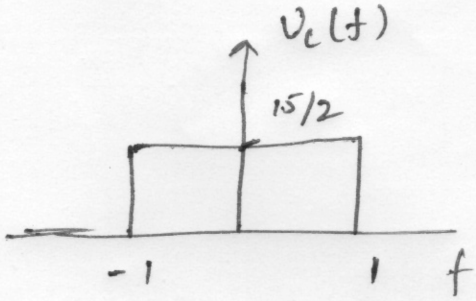




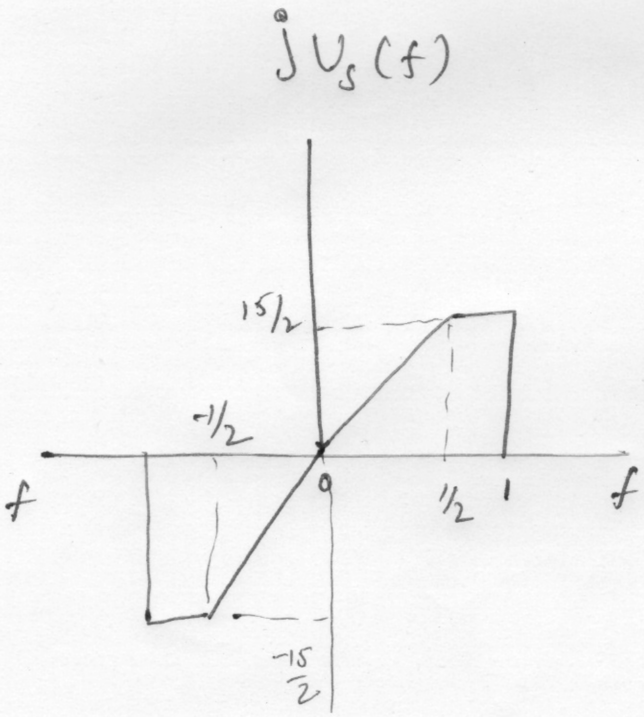
$U(f) = V^{+}(f + f_c)$



$$U_c(f) = \frac{U(f) + U^{*}(-f)}{2}$$



$\Rightarrow u_c(t) = 15\,\mathrm{sinc}(2t)$



$\Rightarrow u_s(t) =$

Q3

a

$$m(t) = A_L \cos(2\pi f_L t + \phi_L) + A_H \cos(2\pi f_H t + \phi_H) \quad ; f_H > f_L$$

$$f_1 = \frac{1}{2}(f_L + f_H)$$

Output from upper mixer,

$$m_u(t) = m(t) \cdot \cos 2\pi f_1 t = [A_L \cos(2\pi f_L t + \phi_L) + A_H \cos(2\pi f_H t + \phi_H)] \cos(2\pi \frac{f_L + f_H}{2} t)$$

$$= A_L \cos(2\pi f_L t + \phi_L) \cdot \cos(2\pi \frac{f_L+f_H}{2} t) + A_H \cos(2\pi f_H t + \phi_H) \cdot \cos 2\pi \frac{f_L+f_H}{2} t$$

$$= A_L (\cos 2\pi f_L t \cdot \cos \phi_L - \sin 2\pi f_L t \cdot \sin \phi_L) \cos(2\pi \frac{f_L+f_H}{2} t)$$

$$+ A_H (\cos 2\pi f_H t \cdot \cos \phi_H - \sin 2\pi f_L t \sin \phi_H) \cos(2\pi \frac{f_L+f_H}{2} t)$$

$$= A_L \cos \phi_L \cos 2\pi f_L t \cdot \cos 2\pi \frac{f_L+f_H}{2} t$$

$$- A_L \sin \phi_L \cdot \sin 2\pi f_L t \cdot \cos 2\pi \frac{f_L+f_H}{2} t$$

$$+ A_H \cos\phi_H \cos 2\pi f_H t \cdot \cos 2\pi \frac{f_L+f_H}{2} t$$

$$- A_H \sin \phi_H \sin 2\pi f_L t \cdot \cos 2\pi \frac{f_L+f_H}{2} t$$

$$= \frac{A_L \cos \phi_L}{2} \left[\cos 2\pi (f_L + \frac{f_L+f_H}{2}) t + \cos 2\pi (f_L - \frac{f_L+f_H}{2}) t\right]$$

$$- \frac{A_L \sin \phi_L}{2} \left[\sin 2\pi (f_L + \frac{f_L+f_H}{2}) t + \sin 2\pi (f_L - \frac{f_L+f_H}{2}) t\right]$$

$$+ \frac{A_H \cos \phi_H}{2} \left[\cos 2\pi (f_H + \frac{f_L+f_H}{2}) t + \cos 2\pi (f_H - \frac{f_L+f_H}{2}) t\right]$$

$$- \frac{A_H \sin \phi_H}{2} \left[\sin 2\pi (f_H + \frac{f_L+f_H}{2}) t + \sin 2\pi (f_H - \frac{f_L+f_H}{2}) t\right]$$

From lower mixer,

$$m_l(t) = m(t) \cdot \sin 2\pi f_1 t$$

$$= A_L \cos \phi_L \cos 2\pi f_L t \cdot \sin 2\pi \frac{f_L+f_H}{2} t - A_L \sin \phi_L \sin 2\pi f_L t \cdot \sin 2\pi \frac{f_L+f_H}{2} t$$

$$+ A_H \cos \phi_H \cos 2\pi f_H t \sin 2\pi \frac{f_L+f_H}{2} t - A_H \sin \phi_H \sin 2\pi f_H t \cdot \sin 2\pi \frac{f_L+f_H}{2} t$$

Pg 2

$$= \frac{A_L \cos\phi_L}{2}\left[\sin 2\pi\left(\frac{f_L+f_H}{2}+f_L\right)t + \sin 2\pi\left(\frac{f_L+f_H}{2}-f_L\right)t\right]$$

$$- \frac{A_L \sin\phi_L}{2}\left[\cos 2\pi\left(\frac{f_L+f_H}{2}-f_L\right)t - \cos 2\pi\left(\frac{f_L+f_H}{2}+f_L\right)t\right]$$

$$+ \frac{A_H \cos\phi_H}{2}\left[\cos 2\pi\left(f_H+\frac{f_L+f_H}{2}\right)t - \cos 2\pi\left(f_H-\frac{f_L+f_H}{2}\right)t\right]$$

$$- \frac{A_H \sin\phi_H}{2}\left[\cos 2\pi\left(f_H-\frac{f_L+f_H}{2}\right)t - \cos 2\pi\left(f_H+\frac{f_L+f_H}{2}\right)t\right]$$

$\boxed{b}$ Output of LPF in upper branch, $W = \frac{f_H+2f_L}{2}$

$$M_{u,LPF}(t) = \frac{A_L \cos\phi_L}{2}\cos 2\pi\left(\frac{f_L-f_H}{2}\right)t$$

$$- \frac{A_L \sin\phi_L}{2}\sin 2\pi\left(\frac{f_L-f_H}{2}\right)t$$

$$+ \frac{A_H \cos\phi_H}{2}\cos 2\pi\left(\frac{f_H-f_L}{2}\right)t$$

$$- \frac{A_H \sin\phi_H}{2}\sin 2\pi\left(\frac{f_H-f_L}{2}\right)t$$

Output of LPF in lower branch, $W = \frac{f_H+2f_L}{2}$

$$M_{l,LPF}(t) = \frac{A_L \cos\phi_L}{2}\sin 2\pi\left(\frac{f_H-f_L}{2}\right)t$$

$$- \frac{A_L \sin\phi_L}{2}\cos 2\pi\left(\frac{f_H-f_L}{2}\right)t$$

$$- \frac{A_H \cos\phi_H}{2}\cos 2\pi\left(\frac{f_H-f_L}{2}\right)t$$

$$- \frac{A_H \sin\phi_H}{2}\cos 2\pi\left(\frac{f_H-f_L}{2}\right)t$$

$\boxed{C}$

$$m_{u,LPF}(t) = \left(\frac{A_L\cos\phi_L}{2}+\frac{A_H\cos\phi_H}{2}\right)\cos 2\pi\left(\frac{f_H-f_L}{2}\right)t$$
$$+\left(\frac{A_L\sin\phi_L}{2}-\frac{A_H\sin\phi_H}{2}\right)\sin 2\pi\left(\frac{f_H-f_L}{2}\right)t$$

$$m_{l,LPF}(t) = \left(\frac{A_L\cos\phi_L}{2}\right)\sin 2\pi\left(\frac{f_H-f_L}{2}\right)t$$
$$-\left(\frac{A_L\sin\phi_L}{2}+\frac{A_H\cos\phi_H}{2}+\frac{A_H\sin\phi_H}{2}\right)\cos 2\pi\left(\frac{f_H-f_L}{2}\right)t$$

Waveform at upper branch after multiplying with f_2,

$$\tilde{m}_u = m_{u,LPF}(t)\cdot\cos 2\pi f_2 t$$
$$= \frac{1}{2}(A_L\cos\phi_L+A_H\cos\phi_H)\cos 2\pi\left(\frac{f_H-f_L}{2}\right)t\cdot\cos 2\pi f_2 t$$
$$+\frac{1}{2}(A_L\sin\phi_L-A_H\sin\phi_H)\sin 2\pi\left(\frac{f_H-f_L}{2}\right)t\cos 2\pi f_2 t$$
$$= \frac{1}{4}(A_L\cos\phi_L+A_H\cos\phi_H)\left[\cos 2\pi\left(f_2+\frac{f_H-f_L}{2}\right)t+\cos 2\pi\left(f_2-\frac{f_H-f_L}{2}\right)t\right]$$
$$+\frac{1}{4}(A_L\sin\phi_L-A_H\sin\phi_H)\left[\sin 2\pi\left(f_2+\frac{f_H-f_L}{2}\right)t+\sin 2\pi\left(f_2-\frac{f_H-f_L}{2}\right)t\right]$$

Similarly, from lower branch,

$$\tilde{m}_l = m_{l,LPF}(t)\cdot\sin 2\pi f_2 t$$
$$= \frac{1}{2}A_L\cos\phi_L\cdot\sin 2\pi\left(\frac{f_H-f_L}{2}\right)t\cdot\sin 2\pi f_2 t$$
$$-\frac{1}{2}(A_L\sin\phi_L+A_H\cos\phi_H+A_H\sin\phi_H)\cos 2\pi\left(\frac{f_H-f_L}{2}\right)t\cdot\sin 2\pi f_2 t$$
$$= \frac{1}{4}(A_L\cos\phi_L)\left[\cos 2\pi\left(f_2-\frac{f_H-f_L}{2}\right)t-\cos 2\pi\left(f_2+\frac{f_H-f_L}{2}\right)t\right]$$
$$-\frac{1}{4}(A_L\sin\phi_L+A_H\cos\phi_H+A_H\sin\phi_H)\left[\sin 2\pi\left(f_2+\frac{f_H-f_L}{2}\right)t - \sin 2\pi\left(f_2-\frac{f_H-f_L}{2}\right)t\right]$$

Final output:

$$m_p(t) = \tilde{m}_u(t) + \tilde{m}_l(t)$$

$$= \frac{1}{4}\left(A_L \cos\phi_L + A_H \cos\phi_H\right) \cos 2\pi\left(f_2 + \frac{f_H - f_L}{2}\right)t$$

$$+ \frac{1}{4}\left(A_L \cos\phi_L + A_H \cos\phi_H\right) \cos 2\pi\left(f_2 - \frac{f_H - f_L}{2}\right)t$$

$$+ \frac{1}{4}\left(A_L \sin\phi_L - A_H \sin\phi_H\right) \sin 2\pi\left(f_2 + \frac{f_H - f_L}{2}\right)t$$

$$+ \frac{1}{4}\left(A_L \sin\phi_L - A_H \sin\phi_H\right) \sin 2\pi\left(f_L - \frac{f_H - f_L}{2}\right)t$$

$$+ \frac{1}{4}\left(A_L \cos\phi_L\right) \cos 2\pi\left(f_2 - \frac{f_H - f_L}{2}\right)t$$

$$\cancel{- \frac{1}{4}\left(A_L \sin\phi_2 + A_H \cos\phi_H + A_H \sin\phi_H\right)}$$

$$- \frac{1}{4}\left(A_L \cos\phi_L\right) \cos 2\pi\left(f_2 - \frac{f_H - f_L}{2}\right)t$$

$$- \frac{1}{4}\left(A_L \sin\phi_L + A_H \cos\phi_H + A_H \sin\phi_H\right) \sin 2\pi\left(f_2 + \frac{f_H - f_L}{2}\right)t$$

$$+ \frac{1}{4}\left(A_L \sin\phi_L + A_H \cos\phi_H + A_H \sin\phi_H\right) \sin 2\pi\left(f_2 - \frac{f_H - f_L}{2}\right)t$$