Course EE6417: Allied Topics in Control Systems
Assignment 2
Submission Deadline:10-02-2016

1. Find the $rank(A)$ and $nullity(A)$ given

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

2. Prove the Cayley-Hamilton theorem which states that "Every matrix A is a root of its characterstic polynomial".
Let

$$A = \begin{bmatrix} 1 & -2 \\ 4 & 2 \end{bmatrix}$$

Find $f(A)$, where (1) $f(t) = t^2 - 3t + 7$ (2) $f(t) = t^2 - 6t + 13$.

3. Consider the system whose state space equation is given by

$$A(t) = \begin{bmatrix} 1 & e^{-t} \\ 0 & -1 \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Is this system controllable at $t = 0$?. If so, find the minimum-energy control to drive it from $\mathbf{x}(0) = \mathbf{0}$ to $\mathbf{x}^1 = [1 \;\; 1]^T$ at $t = 1$.

4. Consider the following state space systems

$$\text{System 1} \quad \begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

$$\text{System 2} \quad \begin{aligned} \dot{w} &= -A^T w + C^T v \\ z &= B^T w + D^T v \end{aligned}$$

Then show that System 1 is totally controllable (observable) if and only if System 2 is totally observable (controllable).

5. Given the system

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 1 & -4 & 3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T u$$

and $y = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \mathbf{x}$. Find the controllable/uncontrollable states and observable/unobservable states of the system.

6. Consider an $n \times n$ matrix A and a $1 \times n$ row vector c, Consider an $n \times n$ matrix defined by $M = \begin{bmatrix} c & cA & cA^2 \dots cA^{n-1} \end{bmatrix}^T$.
Show that if a vector $x \in \mathbb{R}^n$ belongs to the null space of M then Ax also belongs to the null space of M.

7. Consider the following system

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \mathbf{x}$$

and $y = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \mathbf{x}$, where c_1, c_2, c_3 are unknown scalars.
1) Provide an example of values for c_1, c_2, c_3 for which the system is not observable.
2) Provide an example of values for c_1, c_2, c_3 for which the system is observable.
3) Provide a necessary and sufficient condition on the c_i so that the system is observable.
4) Generalize the above result for an arbitrary system with a single output and diagonal matrix A.