EE3005: Communication Systems

Problem Set 1: LTI Systems

1. Prove that the time average of a complex periodic signal equals the average over a single period.
2. A system with input $x(t)$ has output given by

$$y(t) = \int_{-\infty}^{t} e^{u-t} x(u) du$$

   (a) Show that the system is LTI and find its impulse response.
   (b) Find the transfer function $H(f)$ and plot $|H(f)|$.
   (c) If the input $x(t) = 2$ sinc(2t), find the energy of the output.

3. Find and sketch $y = x_1 * x_2$ for the following.

(a) $x_1(t) = e^{-t} I_{[0,\infty]}(t), x_2(t) = x_1(-t)$.

(b) $x_1(t) = I_{[0,2]}(t) - 3I_{[1,4]}(t), x_2(t) = I_{[0,1]}(t)$. (*Hint:* In (b), use LTI property.)

4. A digital circuit generates the following periodic waveform with period 0.5:

$$u_t = \begin{cases} 1, & 0 \le t \le 0.1 \\ 0, & 1 \le t \le 0.5 \end{cases}$$

   where **the unit of time is microseconds** throughout this problem.

   (a) Find the complex exponential Fourier series for $\dfrac{du}{dt}$.
   (b) Find the complex exponential Fourier series for $u(t)$, using the results of (a).
   (c) Find an explicit time-domain expression for the output when $u(t)$ is passed through an ideal lowpass filter of bandwidth 100 kHz.
   (d) Repeat (c) when the filter band width is increased to 300 kHz.