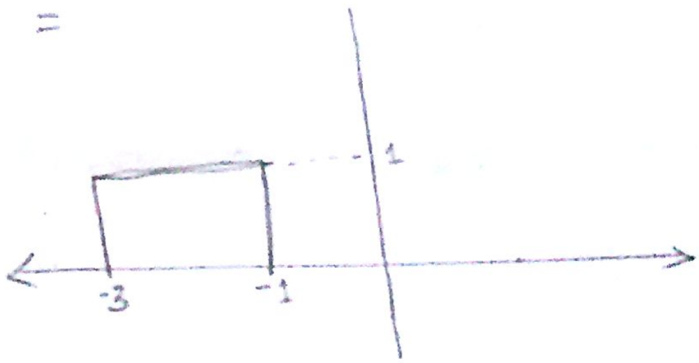
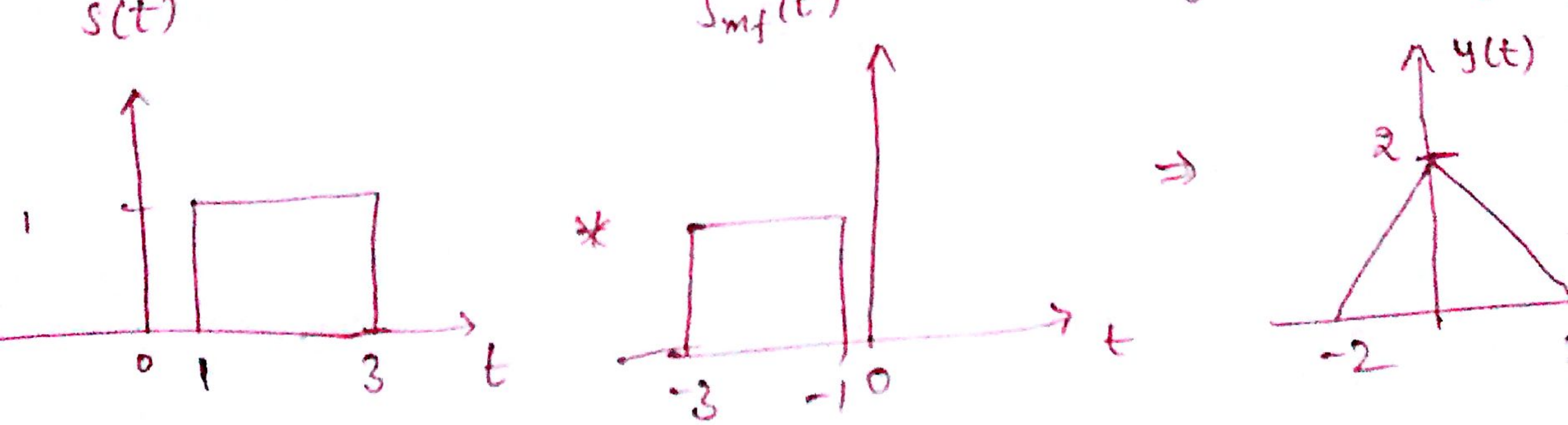
(1) Given $s(t) =$

$S_{mf} = S^*(-t) =$



$y(t) = s(t) * S_{mf}(t)$



$Y(f) \geq 0 \ \forall f$ (TRUE) $\because$ $Y(f)$ is sinc^2 pulse

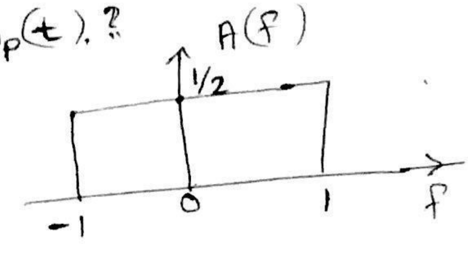
Problem set 3:

Question: 2.A and 2.B.

Given $U_p(t) = a(t)\cos(200\pi t)$, where $a(t) = \text{sinc}(2t)$.

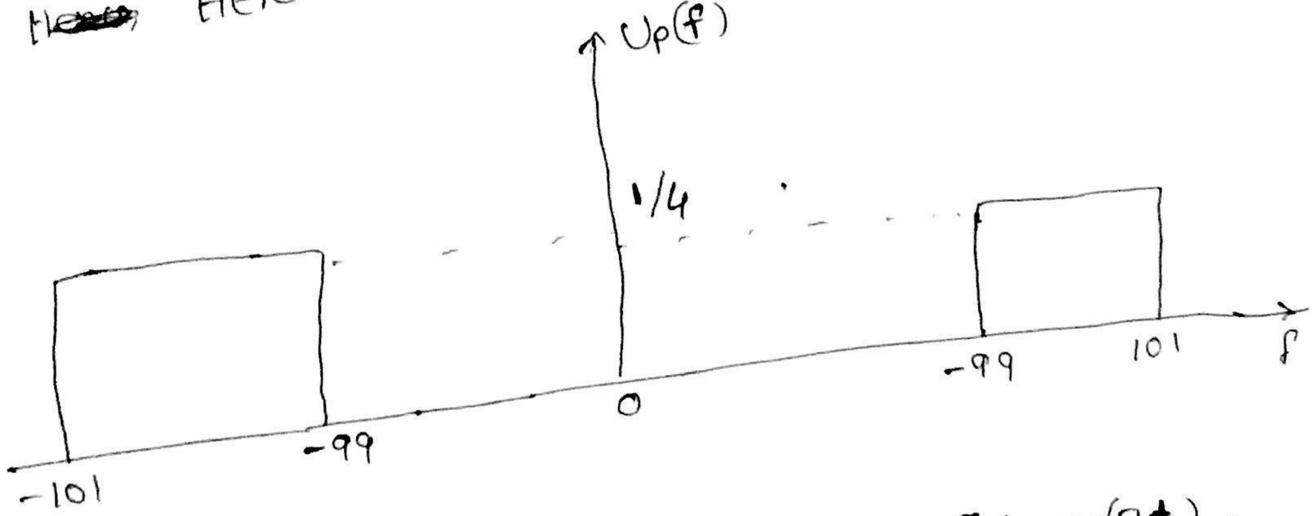
a). Band occupied by $U_p(t)$?

Ans). $a(t) = \text{Sinc}(2t) \xrightarrow{F}$



$$a(t).\cos(2\pi f_0 t) \longrightarrow \frac{A(f-f_0) + A(f+f_0)}{2}$$

~~Hence~~ Here $f_0 = 100$ Hz. Hence $U_p(f)$ is given by.



b) $\cos(200\pi t)\cos(199\pi t) \rightarrow \frac{1}{2}\cos(399\pi t) + \frac{1}{2}\cos(\pi t)$:

Hence the component associated with $\cos(399\pi t)$ will not be present after low pass filtering the signal

$$U_p(t)\cos(199\pi t) = \frac{1}{2}a(t)\cos(399\pi t) + \frac{1}{2}a(t)\cos(\pi t).$$

Thus after low pass filtering we have.

$$b(t) = \frac{1}{2}a(t)\cos(\pi t).$$

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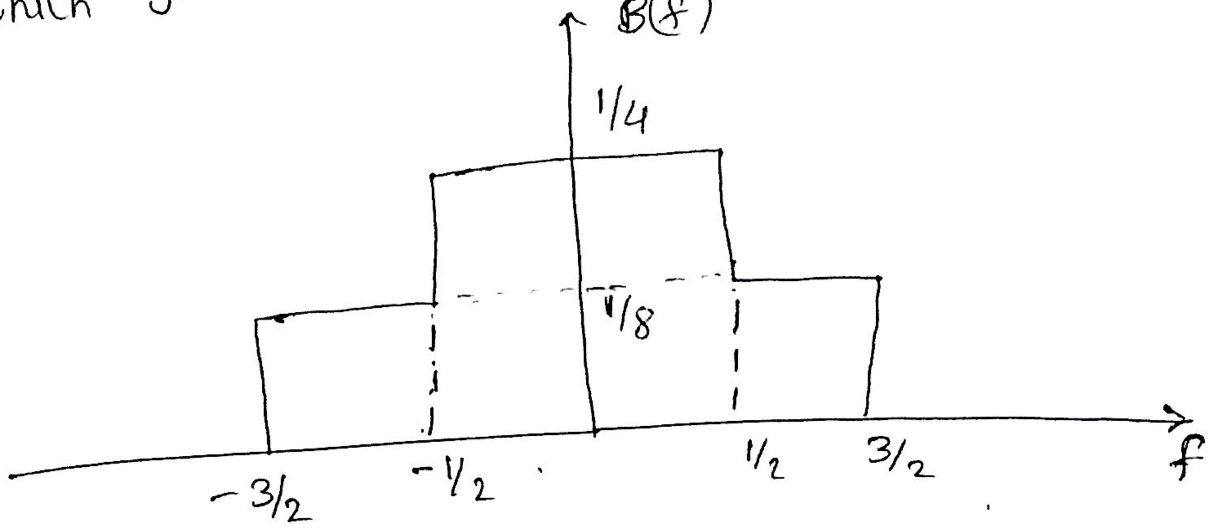
$$b(t) = \frac{1}{2} a(t) \cos(\pi t)$$

$$B(f) = \frac{1}{4} A\left(f - \frac{1}{2}\right) + \frac{1}{4} A\left(f + \frac{1}{2}\right).$$

which gives.



(2c) $u_p(t)\sin(199\pi t) = a(t)\sin(199\pi t)\cos(200\pi t)$

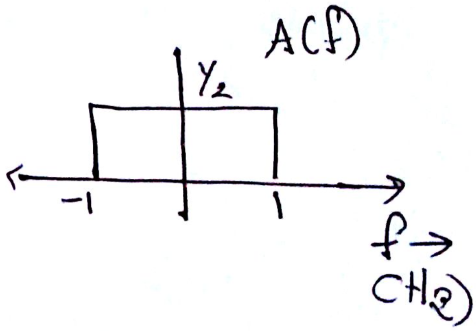
$$= \frac{a(t)}{2}\left[\sin 399\pi t - \sin \pi t\right]$$

This is LP filtered to get $c(t)$.

Since $\frac{a(t)}{2}\left[\underbrace{\sin 399\pi t}_{\text{high freq}} - \underbrace{\sin\pi t}_{\text{low freq}}\right]$ is filtered

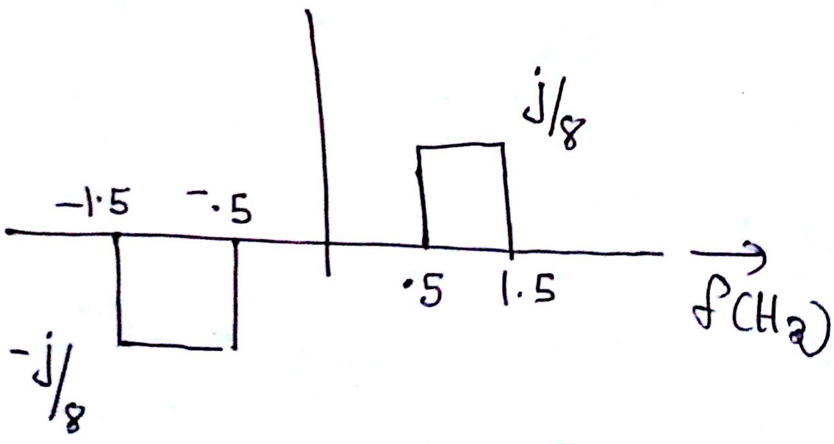
we get $c(t) = -\frac{a(t)}{2}\sin \pi t$

. spectrum of $a(t)$, $A(f)$ is



So

$C(f) =$



note that $C(f)$ is complex valued

$$Re\{C(f)\} = 0 \quad \forall f$$

& $Imag\{C(f)\} = C(f) \quad \forall f$

Communication Systems; Tutorial 3

2.d; solution

February 11, 2016

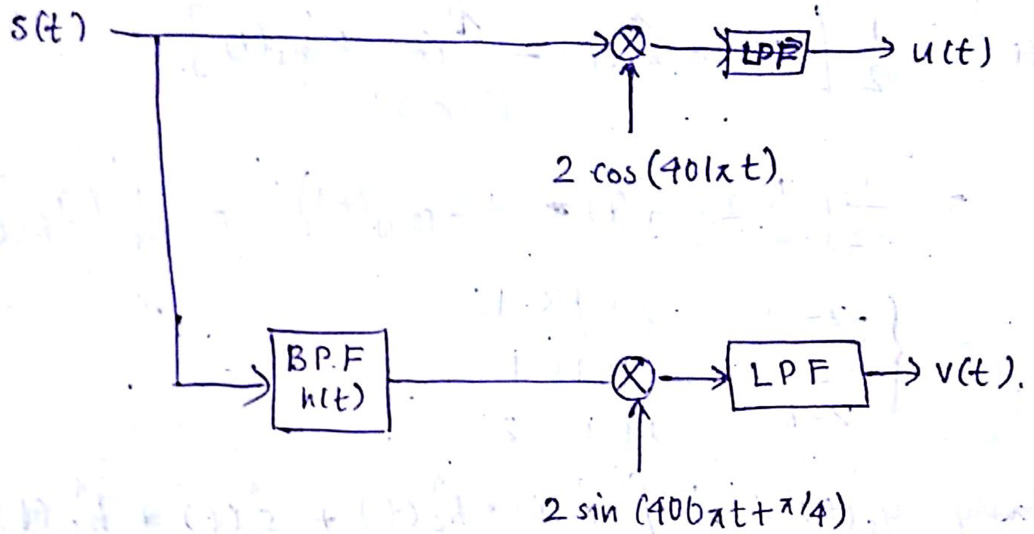
after low pass filtering, we have

$$\begin{aligned} a(t) &= 0.5u_p(t)cos(\pi t) \\ b(t) &= -0.5u_p(t)sin(\pi t) \end{aligned}$$

now multiply with $cos(\pi t)$ and $sin(\pi t)$ respectively and subtract to get

$$\begin{aligned} a(t)cos(\pi t) - b(t)sin(\pi t) &= 0.5u_p(t)cos^2(\pi t) + 0.5u_p(t)sin^2(\pi t) \\ &= 0.5u_p(t) \end{aligned}$$

3a) $s(t) = I_{[-1,1]}(t) \cos(400\pi t)$.



(a) $u(t) = LPF\{ s(t) \cdot 2\cos(401\pi t)\}$

$= LPF\{ I_{[-1,1]}(t) . \cos 400\pi t . 2 . \cos(401\pi t)\}$

$= LPF\{ I_{[-1,1]}(t) (\cos \pi t + \cos 801\pi t)\}$

$= \underline{\underline{I_{[-1,1]}(t) \cos \pi t}}$

3b) $h(t) = I_{[0,1]}(t) \sin\left(400\pi t + \frac{\pi}{4}\right)$

$s(t) = I_{[-1,1]}(t) \cos 400\pi t$

$\Rightarrow \hat{s}_c(t) = \frac{1}{\sqrt{2}} I_{[-1,1]}(t)$ $\hat{s}_s(t) = 0$.

$h(t) = I_{[0,1]} \sin\left(400\pi t + \frac{\pi}{4}\right) = I_{[0,1]} \left[\frac{1}{\sqrt{2}} \sin 400\pi t + \frac{1}{2} \cos 400\pi t \right]$

$$\hat{h}_c(t) = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} I_{[0,1]} = \frac{1}{2} I_{[0,1]}(t).$$

Simillarly, $\hat{h}_s(t) = \frac{1}{2} I_{[0,1]}(t)$

Now

$$y_c(t) = \frac{1}{\sqrt{2}} \left[\hat{s}_c(t) * \hat{h}_c(t) - \underset{(=0)}{\hat{s}_s(t) * \hat{h}_s(t)} \right].$$

$$= \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} I_{[-1,1]}(t) * \frac{1}{2} I_{[0,1]}(t) \right) = \frac{1}{4} \left(I_{[-1,1]}(t) * I_{[0,1]}(t) \right)$$

$$= \begin{cases} -2-t & -2 \le t \le -1 \\ 1 & -1 \le t \le 1 \\ 2-t & 1 \le t \le 2. \end{cases}$$

Simillarly $y_s(t) = \frac{1}{\sqrt{2}} \left[\hat{s}_s(t) * \hat{h}_c(t) + \hat{s}_c(t) * \hat{h}_s(t) \right]$

$$= \frac{1}{4} \left[I_{[-1,1]}(t) * I_{[0,1]}(t) \right] = y_c(t).$$

$$\hat{y}(t) = y_c(t) + j\, y_s(t)$$

$$y(t) = Re\left(\sqrt{2}\, \hat{y}(t)\, e^{j400\pi t} \right)$$

$$= Re\left\{ \sqrt{2}\, \hat{y}(t)\, e^{-j\pi/4} \cdot e^{j(400\pi t + \pi/4)} \right\}$$

from text, Ex 2.5.2,

$$\tilde{y}(t) = \sqrt{2}\, \hat{y}(t)\, e^{-j\pi/4}$$

$$= \sqrt{2}\, \hat{y}(t) \left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} \right) = \left(y_c(t) + j\, y_s(t) \right)(1-j).$$

Now LPF is $-\frac{1}{2} \operatorname{Im}\{\tilde{y}(t)\}$

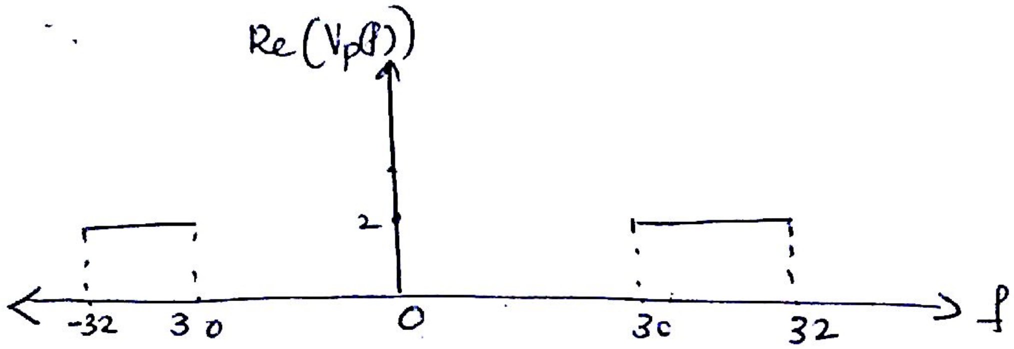
=

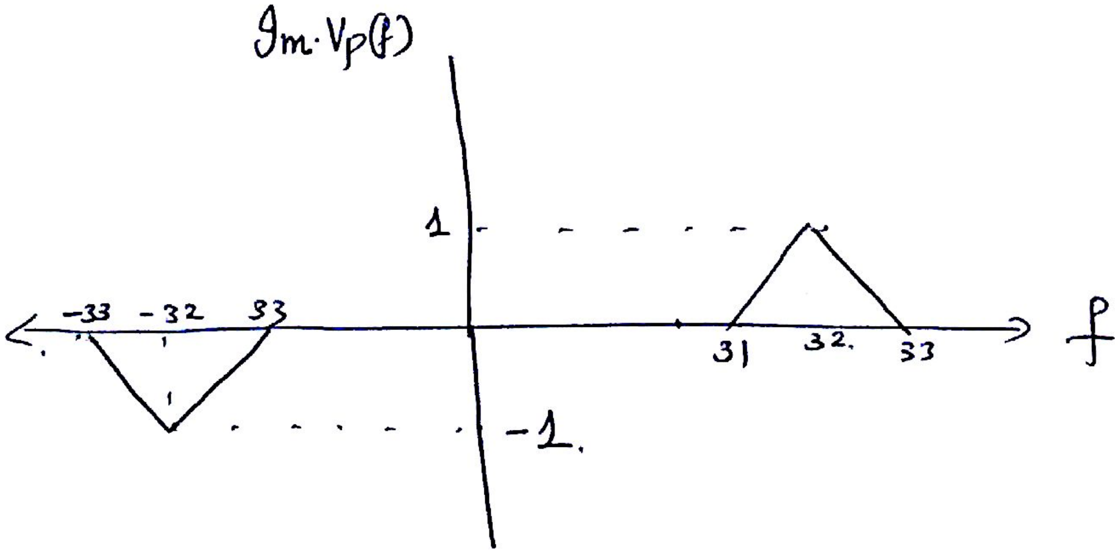
4 (a)

Since $V_p(t)$ is real valued s/g

$$V_p(f) = V_p^*(-f)$$

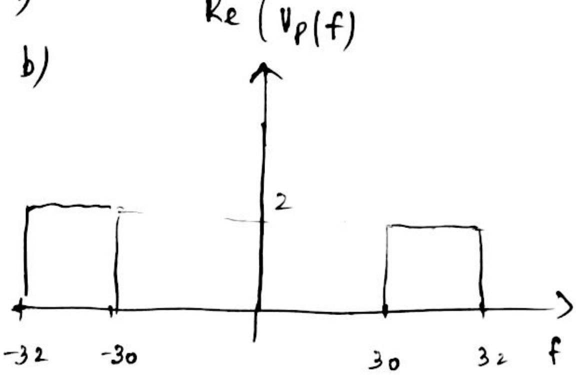
$\Rightarrow$

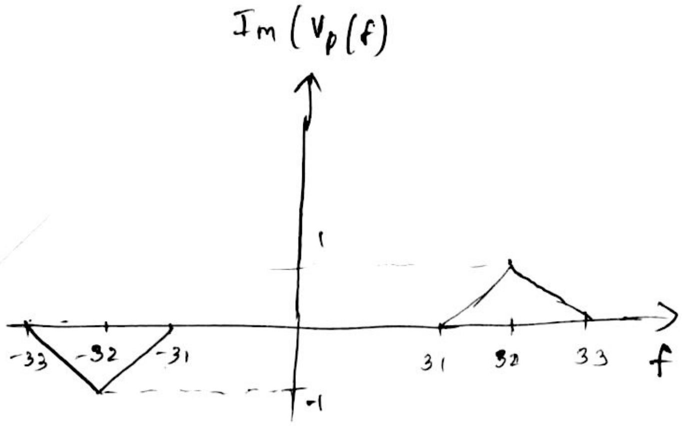




4)

b)

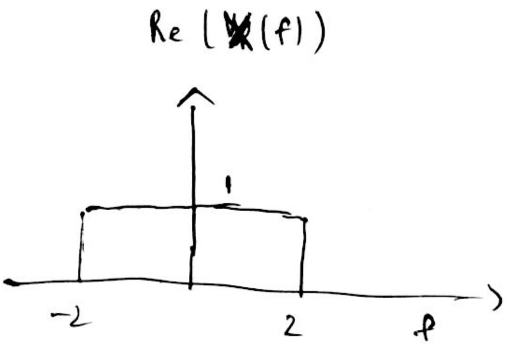


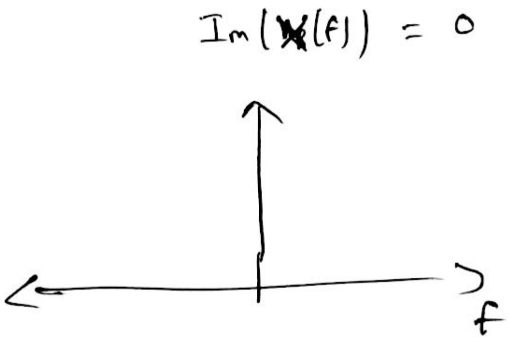


$$v_p(t)\cos(2\pi\, 30\, t) \longleftrightarrow \frac{V_p(f+30) + V_p(f-30)}{2}$$

After passing though LPF,

we get, $X(f)$ as,





$\Rightarrow$ $X(f) = \text{rect}(-2, 2)$

$v_p(t)\cos(60\pi t) \to \boxed{LPF} \to x(t)$

$\Rightarrow$ $x(t) = 4\,\text{sinc}(4t)$