EE3005: Communication Systems

Problem Set 7: Angle Modulation

1. Let $p(t) = I_{[-\frac{1}{2},\frac{1}{2}]}(t)$ denote a rectangular pulse of unit duration. Construct the signal

$$m(t) = \sum_{n=-\infty}^{\infty} (-1)^n p(t-n)$$

The signal $m(t)$ is input to an FM modulator, whose output is given by

$$u(t) = 20\cos(2\pi f_c t + \phi(t))$$

where

$$\phi(t) = 20\pi \int_{-\infty}^{t} m(\tau)d\tau + a$$

and a is chosen such that $\phi(0) = 0$.

(a) Carefully sketch both $m(t)$ and $\phi(t)$ as a function of time.

(b) Approximating the bandwidth of $m(t)$ as $W \approx 2$, estimate the bandwidth of $u(t)$ using Carson's formula.

(c) Suppose that a very narrow ideal BPF (with bandwidth less than 0.1) is placed at $f_c + \alpha$. For which (if any) of the following choices of will you get nonzero power at the output of the BPF: (i) $\alpha = .5$, (ii) $\alpha = .75$, (iii) $\alpha = 1$.

2. A VCO with a quiescent frequency of 1 GHz, with a frequency sweep of 2 MHz/mV produces an angle modulated signal whose phase deviation $\theta(t)$ from a carrier frequency f_c of 1 GHz is shown in Figure 7.1.

(a) Sketch the input $m(t)$ to the VCO, carefully labeling both the voltage and time axes.

(b) Estimate the bandwidth of the angle modulated signal at the VCO output. You may approximate the bandwidth of a periodic signal by that of its first harmonic.

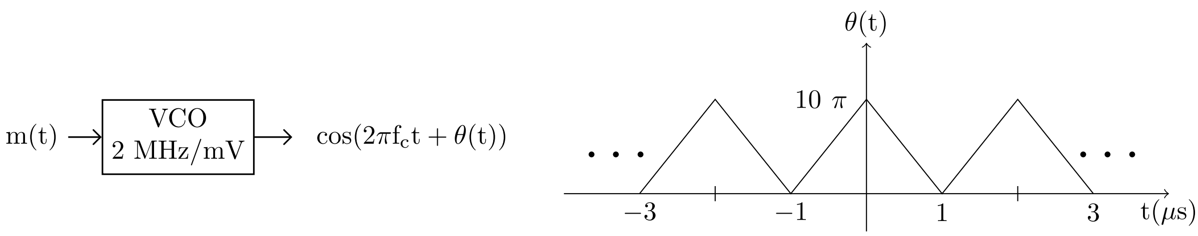

Figure 7.1: Set-up for Problem 2.

3. Consider the linearized model in Figure 7.2, with the following notation: loop filter $G(s)$, loop gain K, and VCO modeled as $1/s$. Recall from your background on signals and systems that a second order system of the form $\frac{1}{s^2+2\zeta\omega_n s+\omega_n^2}$ is said to have natural frequency ω_n (in radians/second) and damping

factor ζ.

Let $H(s)$ denote the gain from the PLL input to the output of the VCO. Let$H_e(s)$ denote the gain from the PLL input to the input to the loop filter. Let $H_m(s)$ denote the gain from the PLL input to the VCO input.

(a) Write down the formulas for $H(s)$, $H_e(s)$, $H_m(s)$, in terms of K and $G(s)$.

(b) Which is the relevant transfer function if the PLL is being used for FM demodulation?

(c) Which is the relevant transfer function if the PLL is being used for carrier phase tracking?

(d) For $G(s) = \frac{s+8}{s}$ and K=2, write down expressions for $H(s)$, $H_e(s)$ and $H_m(s)$. What is the natural frequency and the damping factor?

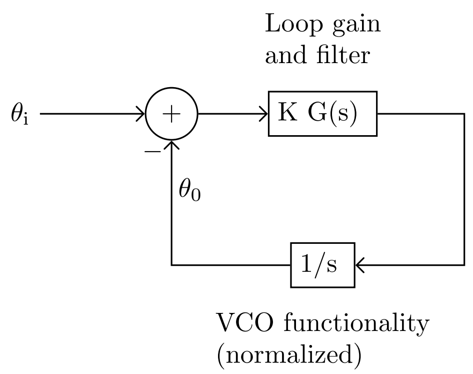

Figure 7.2: Linearized PLL model.