Q11 Consider the following IP problem.

Maximize $\quad Z = 220x_1 + 80x_2,$

subject to

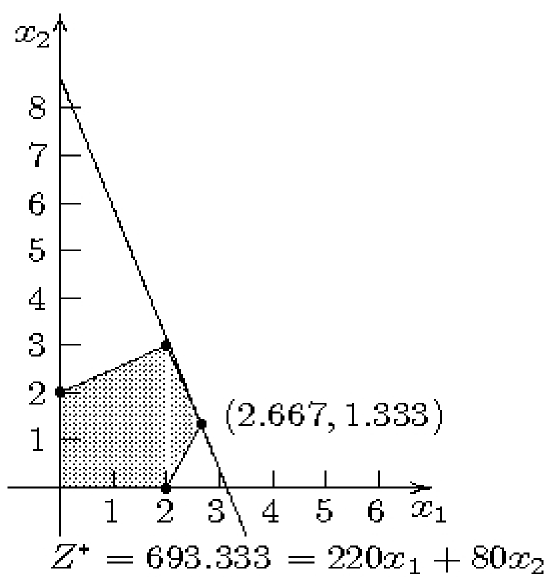
$$5x_1 + 2x_2 \leq 16$$
$$2x_1 - x_2 \leq 4$$
$$-x_1 + 2x_2 \leq 4$$

and

$$x_1 \geq 0, \quad x_2 \geq 0,$$
x_1, x_2 are integers.

Initialization:

Relaxing the integer constraints, the graph below reveals that the optimal solution of the LP relaxation of the whole problem is $(x_1, x_2) = (2.667, 1.333)$ with an objective function value of Z = 693.333.



This LP-relaxation of the whole problem possesses feasible solutions and its optimal solution has noninteger values for x_1 and x_2, so the whole problem is not fathomed and we are ready to move on to the first full iteration.

Iteration 1:

The only remaining (unfathomed) subproblem at this point is the whole problem, so we use it for branching and bounding. In the above optimal solution for its LP-relaxation, both integer-restricted variables (x_1 and x_2) are noninteger, so we select the first one (x_1) to be the branching variable. Since $x_1^* = 2.667$ in this optimal solution, we will create two new subproblems below by adding the respective constraints,

$$x_1 \leq [x_1^*] \qquad \text{and} \qquad x_1 \geq [x_1^*] + 1,$$

where $[x_1^*]$ is the greatest integer $\leq x_1^*$, so $[x_1^*] = 2$.

Subproblem 1:

The original problem plus the additional constraint,

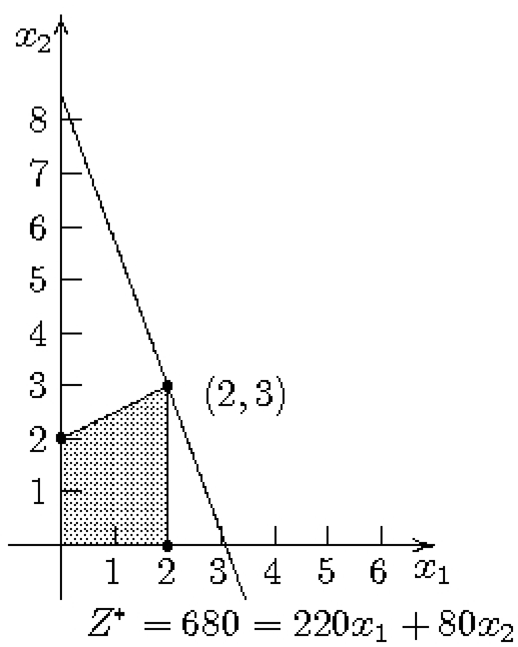
$$x_1 \leq 2.$$

Subproblem 2:

The original problem plus the additional constraint,

$$x_1 \geq 3.$$

For subproblem 1, the following graph shows that the optimal solution for its LP-relaxation is $(x_1, x_2) = (2, 3)$ with $Z = 680$.



Since the solution $(x_1, x_2) = (2, 3)$ is integer-valued, subproblem 1 is fathomed by fathoming test 3 and this solution becomes the first incumbent.

Incumbent = (2, 3) with $Z^* = 680$.

Now consider subproblem 2. Referring back to the graph of the LP-relaxation for the whole problem, it can be seen that the new constraint $x_1 \geq 3$ results in having no feasible solutions. Therefore, subproblem 2 is fathomed by fathoming test 2.

At this point, there are no remaining (unfathomed) subproblems, so the optimality test indicates that the current incumbent is optimal for the original whole problem, so no additional iterations are needed.

$$(x_1^*, x_2^*) = (2, 3) \qquad \text{with } Z^* = 680.$$

Q7

We first define decision variables as follows.

$$\text{Let } s_i = \begin{cases} 1 & \text{if skirt i is taken,} \\ 0 & \text{if skirt i is not taken,} \qquad \text{for i } = 1, 2, 3. \end{cases}$$

$$\text{Let } p_i = \begin{cases} 1 & \text{if slack i is taken,} \\ 0 & \text{if slack i is not taken,} \qquad \text{for i } = 1, 2, 3. \end{cases}$$

$$\text{Let } t_i = \begin{cases} 1 & \text{if top i is taken,} \\ 0 & \text{if top i is not taken,} \qquad \text{for i } = 1, 2, 3, 4. \end{cases}$$

Let $t_5 = 1$ indicate the use of the Icelandic sweater (“top #5”).

$$\text{Let } d_i = \begin{cases} 1 & \text{if dress i is taken,} \\ 0 & \text{if dress i is not taken,} \qquad \text{for i } = 1, 2, 3. \end{cases}$$

$$\text{Let } x_{ij} = \begin{cases} 1 & \text{if both skirt i and top j are taken,} \\ 0 & \text{otherwise,} \qquad \text{for relevant combinations of i and j.} \end{cases}$$

$$\text{Let } y_{ij} = \begin{cases} 1 & \text{if both slack i and top j are taken,} \\ 0 & \text{otherwise,} \qquad \text{for relevant combinations of i and j.} \end{cases}$$

The formulation of this problem is

$$\begin{aligned} \text{Maximize } Z = \; & x_{11} + x_{12} + x_{15} + x_{21} + x_{24} + x_{32} + x_{33} + x_{34} + x_{35} \\ & + y_{11} + y_{13} + y_{21} + y_{22} + y_{24} + y_{25} + y_{33} + y_{34} + y_{35} \\ & + d_1 + d_2 + d_3, \end{aligned}$$

subject to

$$\begin{aligned} & 600\, s_1 + 450\, s_2 + 700\, s_3 + 600\, p_1 + 550\, p_2 + 500\, p_3 + 350\, t_1 \\ & + 300\, t_2 + 300\, t_3 + 450\, t_4 + 600\, d_1 + 700\, d_2 + 800\, d_3 \leq 4{,}000 \end{aligned}$$

$$5{,}000\, s_1 + 3{,}500\, s_2 + 3{,}000\, s_3 + 3{,}500\, p_1 + 6{,}000\, p_2 + 4{,}000\, p_3$$
$$+ 4{,}000\, t_1 + 3{,}500\, t_2 + 3{,}000\, t_3 + 5{,}000\, t_4 + 6{,}000\, d_1 + 5{,}000\, d_2 + 4{,}000\, d_3 \leq 32{,}000$$

$x_{ij} \leq 1/2\,(s_i + t_j)$ for $i = 1, 2, 3$, $j = 1, 2, 3, 4, 5$

$y_{ij} \leq 1/2\,(p_i + t_j)$ for $i = 1, 2, 3$, $j = 1, 2, 3, 4, 5$

s_i, p_i, d_i binary for $i = 1, 2, 3$

t_i binary for $i = 1, 2, 3, 4$

x_{ij}, y_{ij} binary for relevant combinations of i and j.