# Analog communication techniques

The "message" signal in an analog communication system is a signal $m(t)$. Typically, $m(t)$ is a bandlimited (or approximately bandlimited) signal. Examples are speech, audio, etc.

Examples of analog communication systems are:

Radio – AM/FM
Television – Broadcast TV.

The channels in these systems are passband channels.

$\Rightarrow$ We need to transmit passband signals even though the message signal is typically a baseband signal. We have already studied about how to do this.

A passband signal $u_p(t)$ can be expressed as

$$
\begin{aligned}
u_p(t) &= u_c(t)\cos 2\pi f_c t - u_s(t)\sin 2\pi f_c t \\
&= e(t)\cos(2\pi f_c t + \theta(t)) \\
&= \mathrm{Re}\Big(\underbrace{u(t)}_{u_c(t)+ju_s(t)} e^{j2\pi f_c t}\Big).
\end{aligned}
$$

Modulation is the process of encoding the information signal into the transmitted signal.

$$m(t) \longrightarrow \boxed{\text{Modulation}} \longrightarrow u_p(t)$$

Two broad classes of modulation

**Amplitude modulation**

$m(t)$ appears in $u_c(t)$, $u_s(t)$, or both

**Angle modulation**

$m(t)$ appears in $\theta(t)$.

At the receiver, we need to "demodulate".

$u_p(t)$ or a noisy version of $u_p(t)$ $\longrightarrow$ $\boxed{\text{Demodulation}}$ $\longrightarrow$ $\hat{m}(t)$. Estimate of $m(t)$.

Initially, we will assume $u_p(t)$ is received. Later, we will discuss the noisy channel.

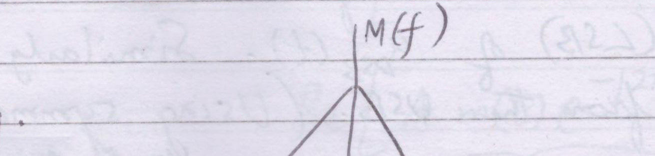
## $\boxed{\text{Amplitude Modulation (AM)}}$

There are a number of variants. We will discuss a few of them.

### 1) Double Sideband Suppressed-carrier (DSB-SC)

Let $m(t)$ be a bandlimited signal.

e.g.




$$u_{DSB}(t) = A\, m(t) \cos 2\pi f_c t$$

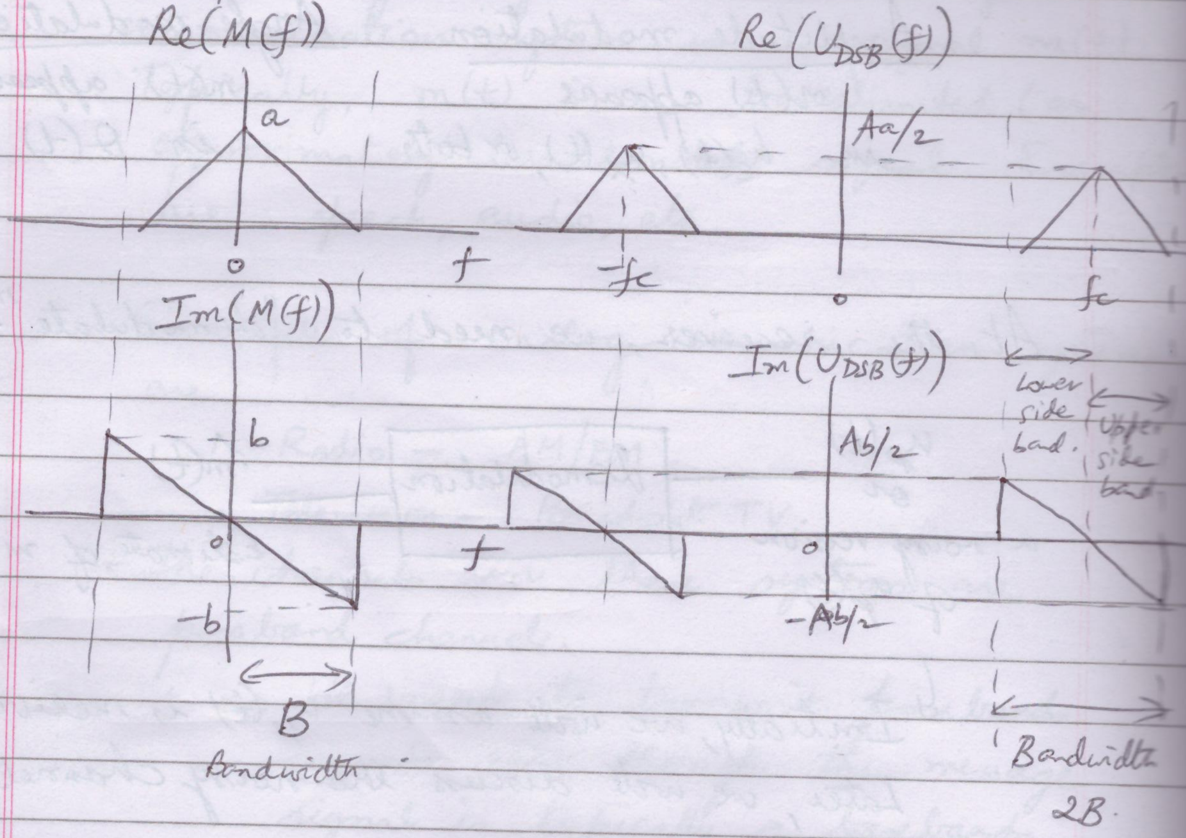
i.e. $m(t)$ directly determines $u_c(t)$.

& $u_s(t) = 0$

Spectrum of DSB-SC AM signal

$$U_{DSB}(f) = \frac{A}{2}\left[M(f-f_c) + M(f+f_c)\right].$$

Example:




Note: (1) For a baseband signal $m(t)$ of bandwidth $\underline{B\ Hz}$, we get a passband signal $u_{DSB}(t)$ of bandwidth $\underline{2B\ Hz}$.

(2) If $m(t)$ is real, the upper side band (USB) of $u_{DSB}(t)$ can be determined from the lower side band (LSB) of $u_{DSB}(t)$. Similarly, the LSB from the USB. (Using symmetry properties)

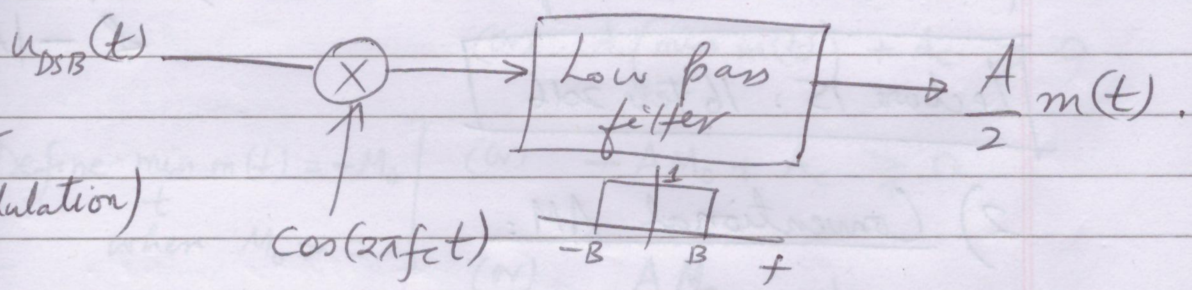
Since both the LSB & USB are transmitted, this scheme is a DSB (Double sideband) modulation scheme.

The reason for calling it a suppressed carrier scheme will become clear after we discuss the next scheme.

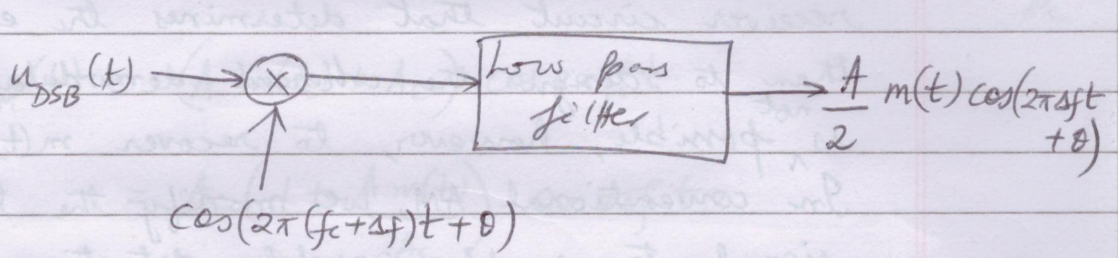
## Demodulation of DSB-SC:

→ Need to recover the I component ($u_c(t)$) of $u_{DSB}(t)$. We know how to do this provided we have a cosine at the receiver that is perfectly synchronized with the cosine at the transmitter.

(Called coherent demodulation)




→ Suppose we have $\cos(2\pi f_c t + \underbrace{2\pi\Delta f t + \theta})$ at the receiver.



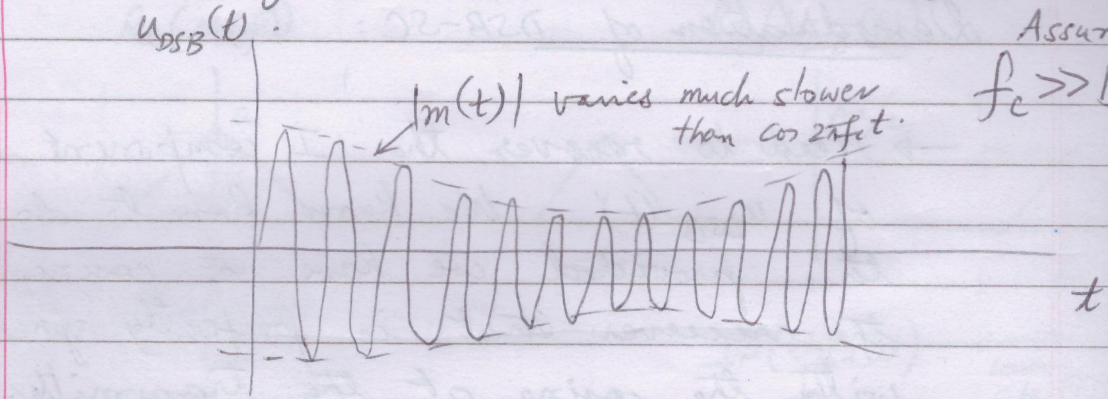

$$u_{DSB}(t) = \mathrm{Re}\left[A\, m(t)\, e^{j2\pi f_c t}\right]$$

$$= \mathrm{Re}\left[\underbrace{A\, m(t)\, e^{-j2\pi\Delta f t}\, e^{-j\theta}}\, e^{j(2\pi(f_c+\Delta f)t+\theta)}\right]$$

Need to drive $\Delta f$ and $\theta$ to zero to get a good estimate of $m(t)$. Need to synchronize the local oscillator at the rx to the received carrier.

One possible approach for synchronization is to use phase-locked loops (PLLs).

Visualizing the DSB-SC signal $u_{DSB}(t)$:




Assume. $f_c >> B$

The envelope (dashed line) is $|A\, m(t)|$.
& $-|A m(t)|$

## Lecture 15: 16 Feb 2016

## 2) Conventional AM:

Observe that the envelope of the DSB-SC signal $u_{DSB}(t)$ was equal to $A|m(t)|$. It turns out that it is easier to design a receiver circuit that determines the envelope than to design the coherent demodulator. It is not possible, however, to recover $m(t)$ from $|m(t)|$. In conventional AM, we modify the transmit signal to enable envelope detection to obtain $m(t)$.

$$u_{AM}(t) = \underbrace{A\, m(t) \cos 2\pi f_c t}_{u_{DSB}(t)} + \underbrace{A_c \cos(2\pi f_c t)}_{\text{Additional term.}}$$

$$= (A m(t) + A_c) \cos(2\pi f_c t).$$

Note that $U_{AM}(f) = U_{DSB}(f) + \frac{A_c}{2}\left(\delta(f-f_c) + \delta(f+f_c)\right)$

The envelope of $u_{AM}(t)$ is

$$e(t) = \left| A\, m(t) + A_c \right|.$$

If we can ensure that $A\, m(t) + A_c > 0$ for all $t$, then $e(t) = A\, m(t) + A_c$ and we can obtain $m(t)$ from $e(t) \Rightarrow$ Envelope detector will work.

* How should we choose $A_c$ to make this happen?

Need $A_c$ such that $A\, m(t) + A_c \geq 0$ for all $t$.

(or) $A\left(\min_t m(t)\right) + A_c \geq 0$.

[Define $\min_t m(t) = -M_0$ where $M_0 > 0$]

(or) $-AM_0 + A_c \geq 0$

(or) $\dfrac{AM_0}{A_c} \leq 1$.

(or) $a_{mod} \leq 1$ where $a_{mod} \triangleq \dfrac{AM_0}{A_c}$.

$$u_{AM}(t) = \left(A\, m(t) + A_c\right) \cos 2\pi f_c t$$

$$= A_c\left(1 + \frac{A\, m(t)}{A_c}\right) \cos 2\pi f_c t$$

$$= A_c\left(1 + a_{mod} \frac{m(t)}{M_0}\right) \cos 2\pi f_c t$$

$$= A_c\left(1 + a_{mod}\, m_n(t)\right) \cos 2\pi f_c t$$

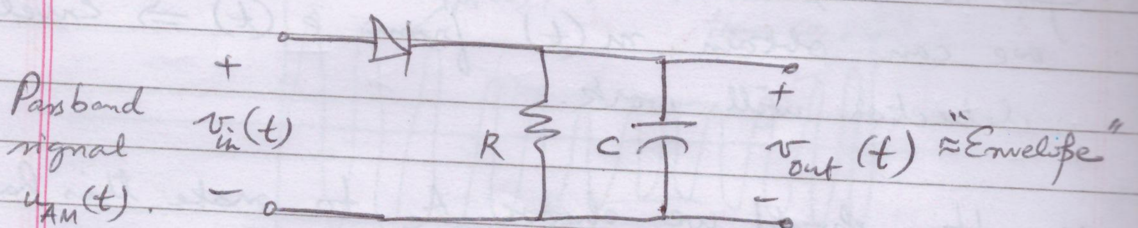
where $m_n(t) \triangleq \dfrac{m(t)}{M_0}$, a normalized version of $m(t)$ such that

$$\min_t m_n(t) = -1.$$

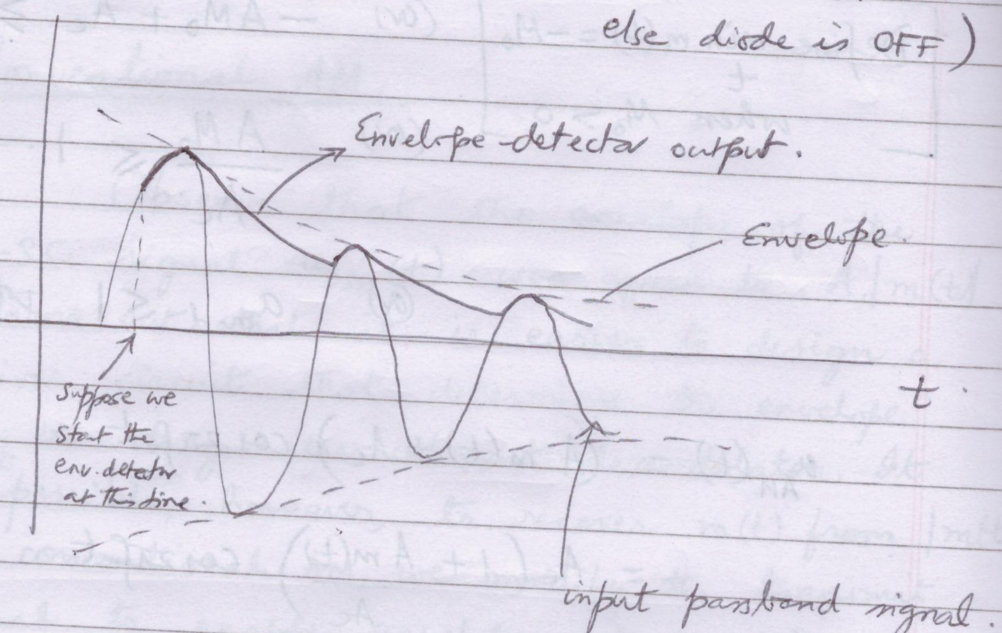
$a_{mod}$ is called the "modulation index".

* How can we do envelope detection?

Consider the following circuit.




Let the diode be ideal. (i.e. if $v_{in}(t) > v_{out}(t)$ then diode is ON & $v_{out}(t) = v_{in}(t)$. else diode is OFF)




For the envelope-detector output to be "close" to the true envelope, we need

$$\frac{1}{B} \gg \frac{1}{RC} \gg \frac{1}{f_c}$$

RC: Time constant

$\frac{1}{B}$: to be able to track variations in $m(t)$

$\frac{1}{f_c}$: to be able to track the envelope & not the carrier.

eg: $B = 5\,kHz$ $f_c = 500\,kHz$

Need $2\mu s \ll RC \ll 200\mu s$

example R & C : $R = 50\Omega$, $C = 400nF$.

* Envelope detector is simple to implement. However, significant fraction of the transmit power is wasted in transmitting $A_c \cos 2\pi f_c t$ which does not contain any information about the message $m(t)$.

  This may be OK in an application like broadcast radio since there is only one transmitter & a large no. of receivers.

$\boxed{\text{Lecture 16 : 18 Feb 2016}}$

Power efficiency (of conventional AM):

$$\text{Power efficiency} \triangleq \frac{\text{Power of } A_c a_{mod} m_n(t) \cos 2\pi f_c t}{\text{Power of } u_{AM}(t)}$$

$$= \frac{\overline{A_c^2 a_{mod}^2 m_n^2(t) \cos^2 2\pi f_c t}}{\overline{A_c^2 (1 + a_{mod} m_n(t))^2 \cos^2 2\pi f_c t}}$$

$$= \frac{\frac{A_c^2}{2} a_{mod}^2 \overline{m_n^2(t)}}{\frac{A_c^2}{2}\left(1 + a_{mod}^2 \overline{m_n^2(t)}\right)}$$ assuming $\overline{m_n(t)} = 0$.

$$= \frac{a_{mod}^2 \overline{m_n^2}}{1 + a_{mod}^2 \overline{m_n^2}}$$

Note that $a_{mod} \leq 1$ for envelope detection to work

Also $\min_t m_n(t) = -1$.

If $\overline{m_n^2} < 1$, then power efficiency $< \frac{1}{2}$.

Example: If $m_n(t) = \sin(2\pi f_m t)$, $(a_{mod} = 1)$

$$\overline{m_n^2} = \frac{1}{2}$$

$$\Rightarrow \text{Power efficiency} = \frac{1/2}{1 + 1/2} = \frac{1}{3}.$$

See example 3.2.1 in the book:

$m(t) = 2\sin(2000\pi t) - 3\cos(4000\pi t)$

$a_{mod} = 0.7$

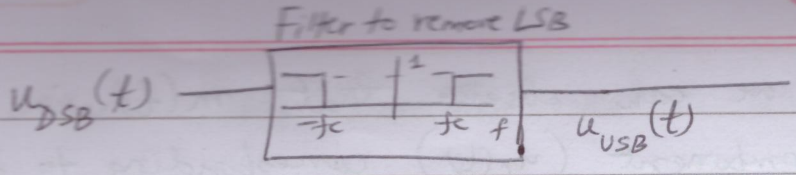
$f_c = 580\,kHz$

Power efficiency $= \underline{0.24}$.

## 3) Single-sideband modulation (SSB)

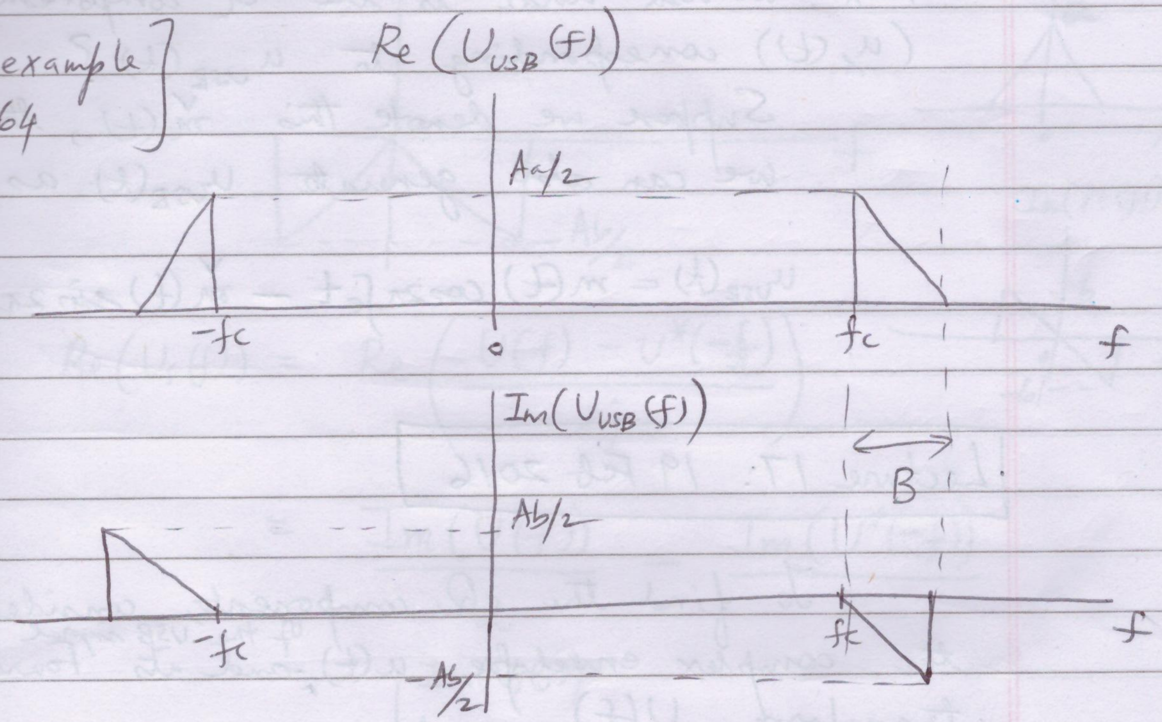
While discussing DSB-SC modulation, we observed that (see fig. in p64) either the upper sideband (USB) or the lowerside band (LSB) of the DSB-SC signal is sufficient to recover $m(t)$.

* Consider the USB signal. How can we recover $m(t)$ from this?

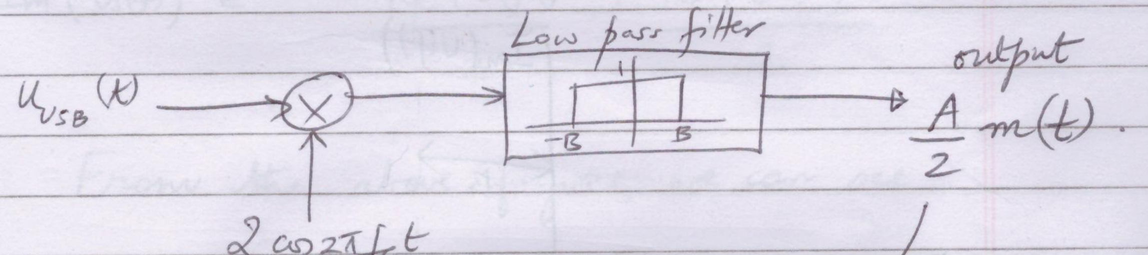


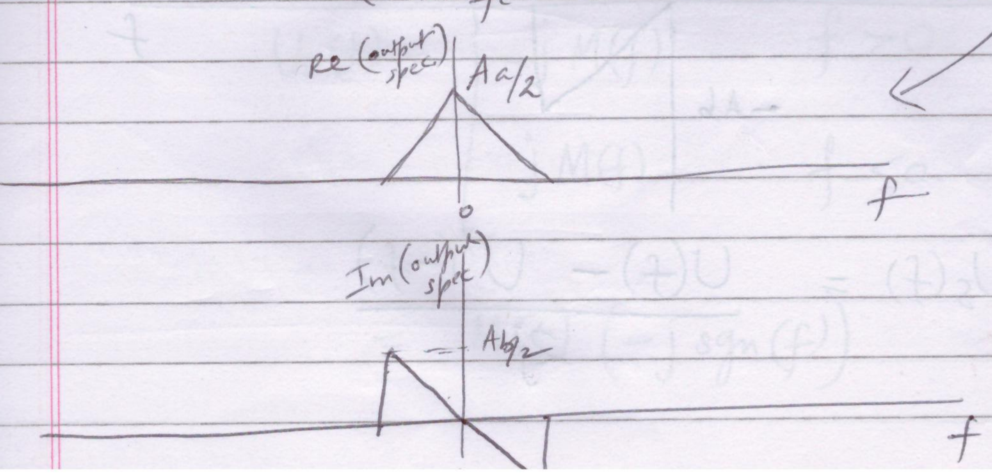

The spectrum of the USB signal is shown below.

[For the example in p64]




Note that we can get $M(f)$ by shifting left by $f_c$ & adding with another shift right by $f_c$. Thus, the same demodulation method used for DSB-SC actually works for USB as well!



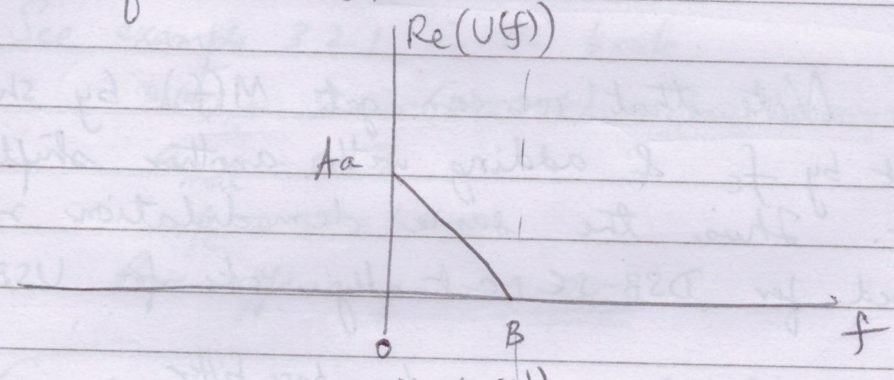
Thus, we can observe that $m(t)$ is the I component ($u_c(t)$) corresponding to the real passband USB signal $u_{USB}(t)$.

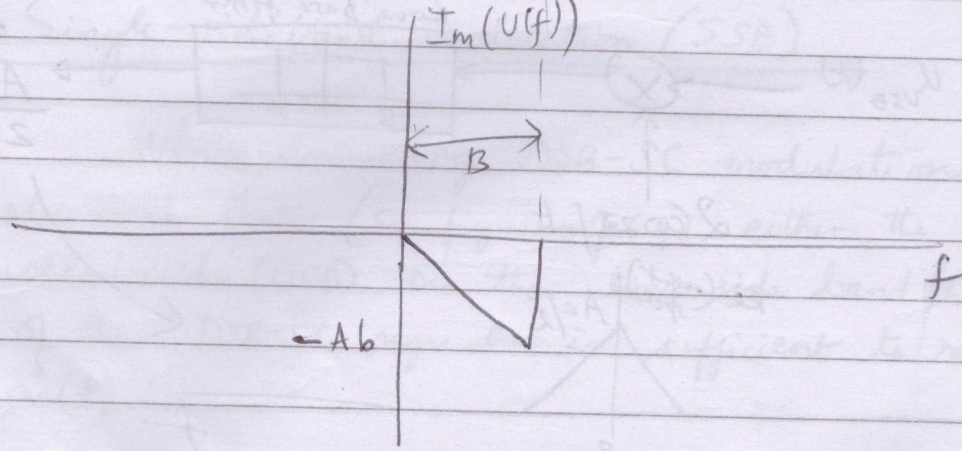
* Next, we ask what is the Q component ($u_s(t)$) corresponding to $u_{USB}(t)$?
Suppose we denote this $\check{m}(t)$, then we can also generate $u_{USB}(t)$ as

$$u_{USB}(t) = m(t)\cos 2\pi f_c t - \check{m}(t)\sin 2\pi f_c t.$$

## Lecture 17: 19 Feb 2016

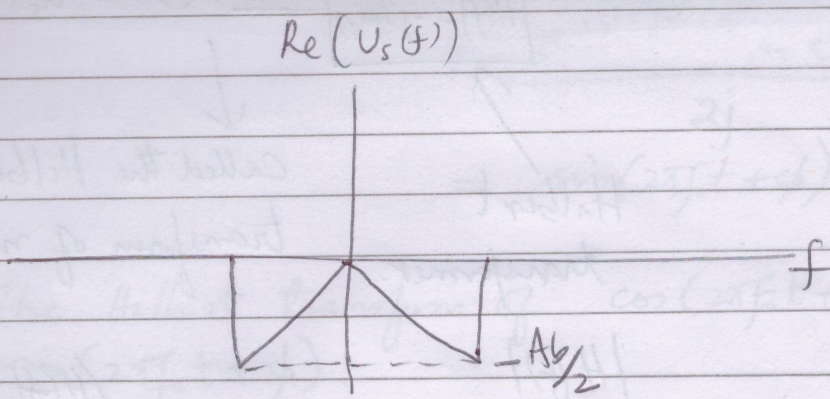
To find the Q component, consider the complex envelope $u(t)$ of the USB signal and its Fourier transform $U(f)$.

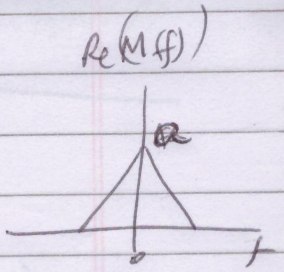


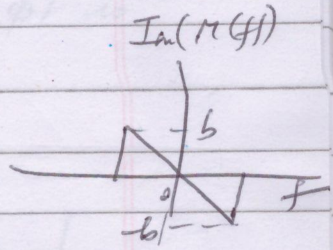




$$U_s(f) = \frac{U(f) - U^*(-f)}{2j}$$

since $u_s(t) = \mathrm{Im}(u(t)) = \dfrac{u(t) - u^*(t)}{2j}$

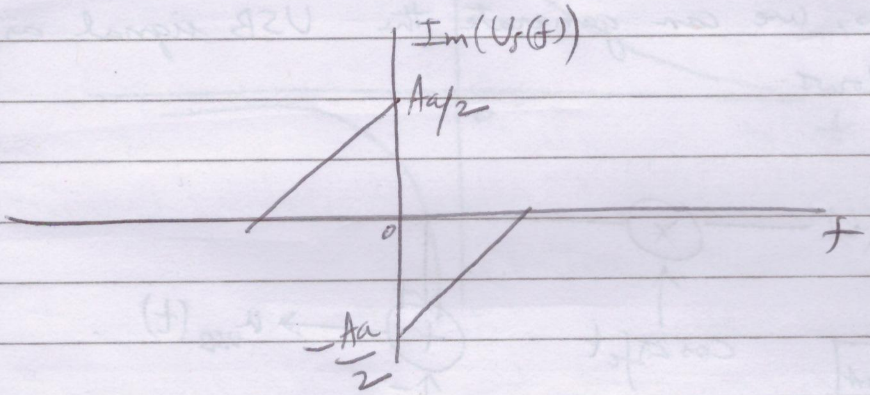



$$\mathrm{Re}(U_s(f)) = \mathrm{Re}\left(\frac{U(f) - U^*(-f)}{2j}\right)$$

$$= \frac{\mathrm{Im}(U(f))}{2} - \frac{\mathrm{Im}(U^*(-f))}{2}$$




$$\mathrm{Im}(U_s(f)) = -\frac{\mathrm{Re}(U(f))}{2} + \frac{\mathrm{Re}(U^*(-f))}{2}$$

From the above figures, we can see

$$U_s(f) = \begin{cases} -j\,M(f) & f > 0 \\ j\,M(f) & f < 0 \end{cases}$$

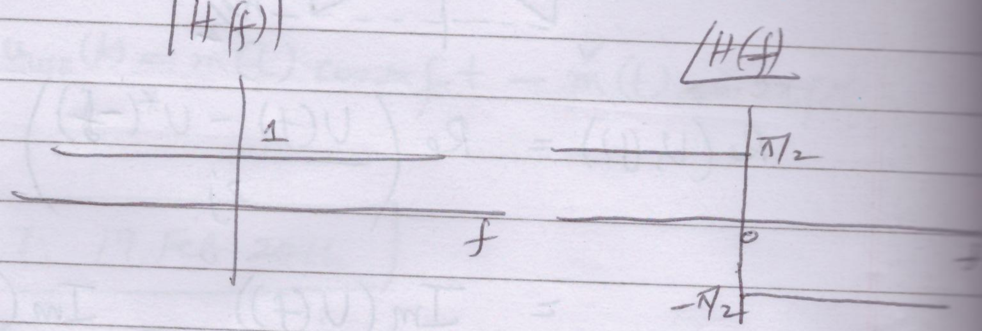
$$= M(f)\left(-j\,\mathrm{sgn}(f)\right).$$

Thus, we have

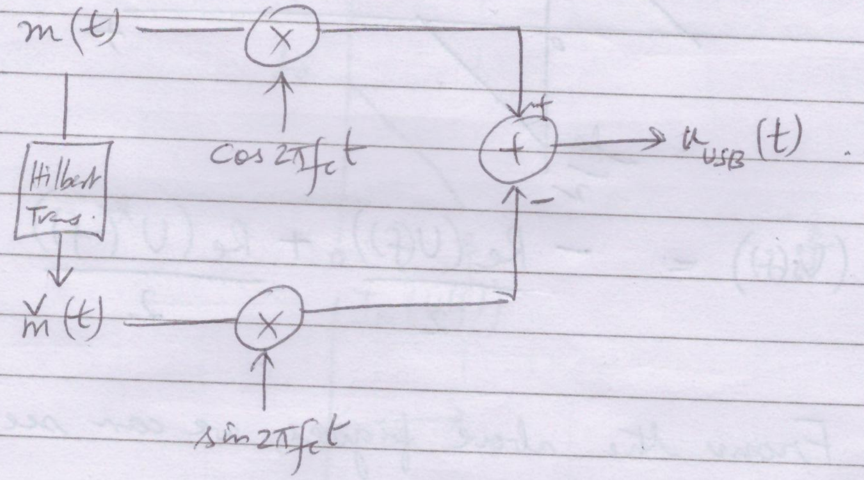
$m(t)$ — $H(f) = -j\,\mathrm{sgn}(f)$ — $\check{m}(t) = u_s(t)$.

Hilbert transformer.

Called the Hilbert transform of $m(t)$.




* Thus, we can generate the USB signal as follows.




* Hilbert transform of a sinusoid:

Suppose $m(t) = \cos(2\pi f_0 t + \phi)$

$$= \frac{e^{j(2\pi f_0 t + \phi)} + e^{-j(2\pi f_0 t + \phi)}}{2}$$

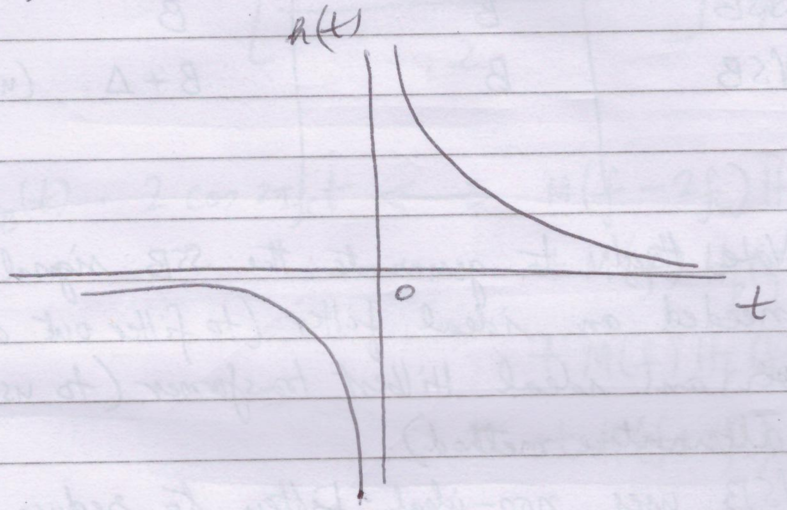
$$m(t) \longrightarrow \boxed{H(f) = -j\,\mathrm{sgn}(f)} \longrightarrow \frac{-j e^{j(2\pi f_0 t+\phi)} + j e^{-j(2\pi f_0 t+\phi)}}{2}$$

$$= \frac{e^{j(2\pi f_0 t+\phi)} - e^{-j(2\pi f_0 t+\phi)}}{2j}$$

$$= \sin(2\pi f_0 t+\phi).$$

The Hilbert transform of $\cos(2\pi f_0 t+\phi)$ is $\sin(2\pi f_0 t+\phi)$.

* $H(f) = -j\,\mathrm{sgn}(f) \longleftrightarrow h(t) = \dfrac{1}{\pi t}$.




* How can be obtain the LSB signal?

$$u_{LSB}(t) = m(t)\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t$$

(Can be shown as in the case of the USB signal by obtaining the I & Q components of $u_{LSB}(t)$).

# Lecture 18: (23 Feb 2016)

## Vestigial-sideband (VSB) modulation:

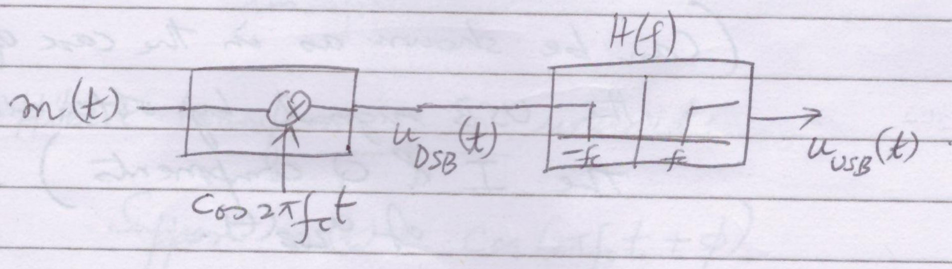
(Used in broadcast television systems).

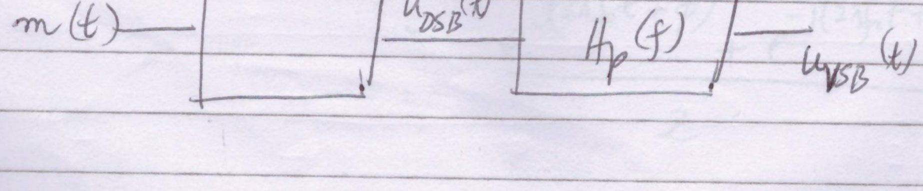
* So far:

| | Message BW | Transmit signal BW |
|---|---|---|
| DSB-SC | B | 2B |
| Conventional AM | B | 2B |
| SSB | B | B |
| VSB | B | $B+\Delta$ (usually $\Delta$ is small compared to B) |

Note that to generate the SSB signal, we needed an ideal filter (to filter out a side band) or an ideal Hilbert transformer (to uses the alternative method).
VSB uses non-ideal filters to reduce the BW requirement to $B+\Delta$ (a little more than B).
What kind of non-ideal filters will work?

* Consider the following problem.



For what conditions on $H_p(f)$, can be recover $m(t)$ as the I component of $u_{VSB}(t)$?

Need: $u_{VSB}(t) \longrightarrow \otimes \longrightarrow \boxed{\text{LPF}} \longrightarrow m(t)$, with $2\cos 2\pi f_c t$ into the multiplier.

$\updownarrow$

$$U_{VSB}(f) = U_{DSB}(f)\, H_p(f)$$

$$= \left[\frac{M(f-f_c) + M(f+f_c)}{2}\right] H_p(f)$$

$$u_{VSB}(t) \cdot 2\cos 2\pi f_c t \longleftrightarrow M(f-2f_c)\,H_p(f-f_c) + M(f)\,H_p(f-f_c) + M(f)\,H_p(f+f_c) + M(f+2f_c)\,H_p(f+f_c)$$

$\downarrow$ LPF $\downarrow$

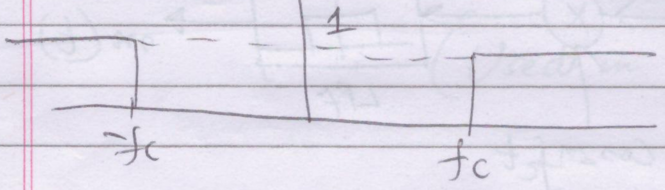
$$M(f)\left[H_p(f-f_c) + H_p(f+f_c)\right].$$

$\Rightarrow$ We need $H_p(f-f_c) + H_p(f+f_c)$ to be constant for $-B \leq f \leq B$.

$$H_p(f-f_c) + H_p(f+f_c) = \text{constant for } |f| \leq B.$$

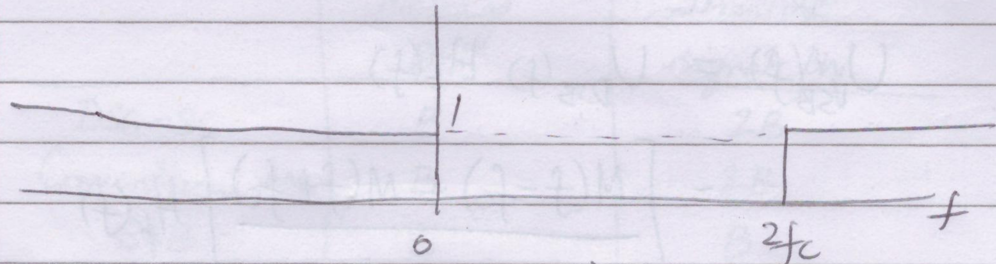
Examples :

(1) $H_p(f)$

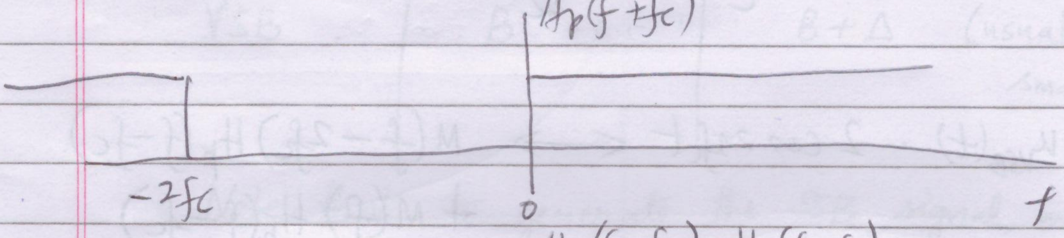



(We know this gives the USB signal)

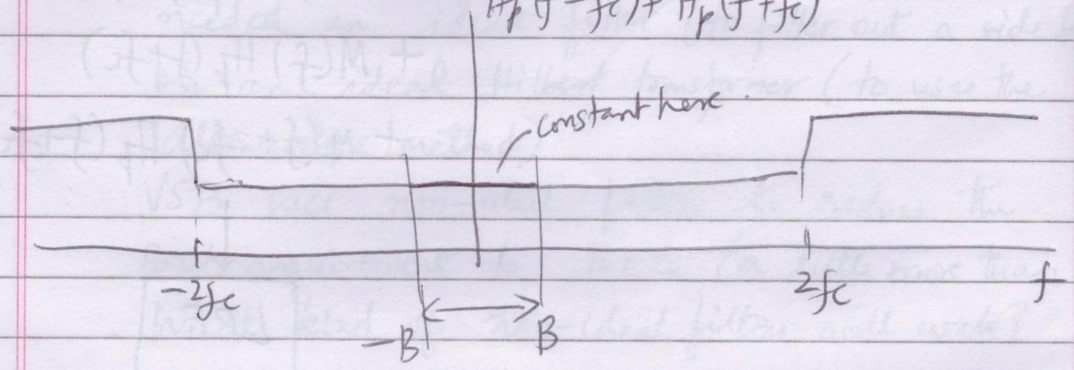
$H_p(f - f_c)$



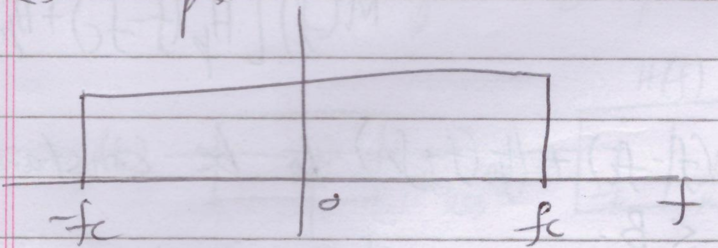

$H_p(f + f_c)$




$H_p(f - f_c) + H_p(f + f_c)$

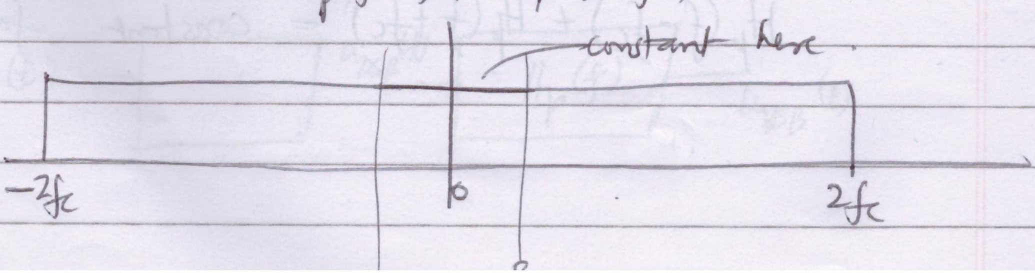



(2) $H_p(f)$

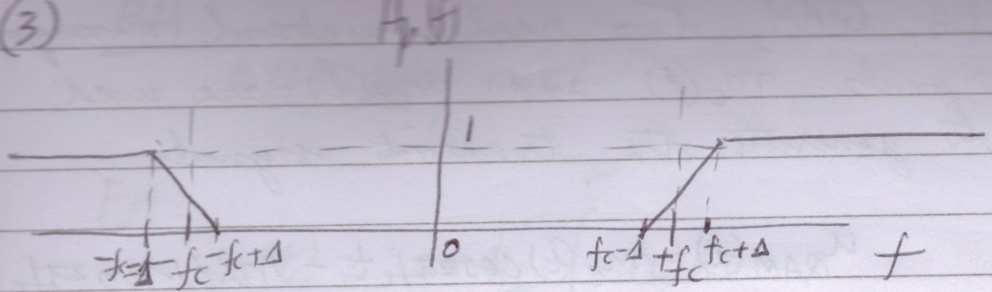


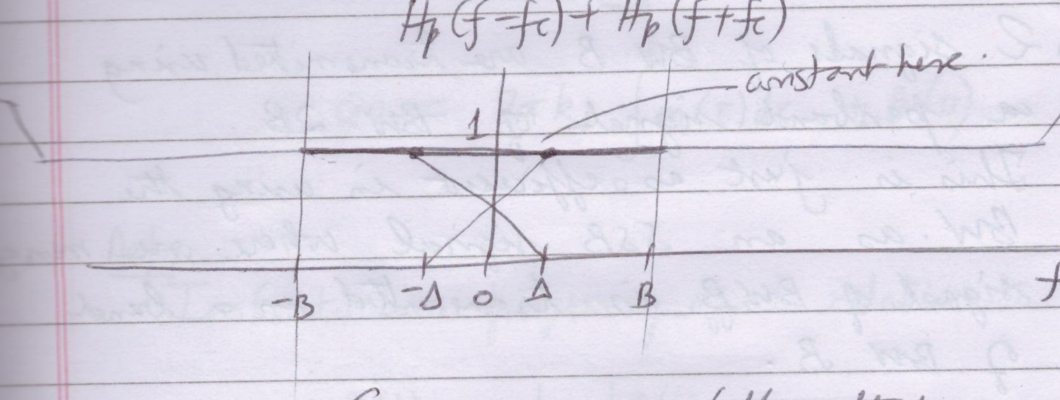

(We know this gives the LSB signal)

$H_p(f - f_c) + H_p(f + f_c)$



(3)




Example of a filter that can be used for VSB modulation.

$BW = B+\Delta$. (of the transmit signal).

## Quadrature Amplitude Modulation (QAM):

So far

One real baseband message signal → Modulator → Real passband signal.

DSB-SC $\quad m(t) \quad u_{DSB}(t) = \underbrace{m(t)}_{I\ comp} \cos 2\pi f_c t$

Q comp = 0

| | I | Q |
|---|---|---|
| DSB-SC | $m(t)$ | 0 |
| Conv. AM | $(A_c m(t)+A_c)$ | 0 |
| SSB | $m(t)$ | $\check{m}(t)$ |
| QAM | $m_c(t)$ | $m_s(t)$ |

In QAM, two real baseband message signals $m_c(t)$ and $m_s(t)$ are used to generate the transmit signal

$$u_{QAM}(t) = m_c(t)\cos 2\pi f_c t - m_s(t)\sin 2\pi f_c t.$$

$\Rightarrow$ 2 signals of BW B are transmitted using a passband signal of BW 2B.
This is just as efficient in using the BW as an SSB signal where one message signal of BW B is transmitted in a band of BW B.

$\boxed{\text{Lecture 19: (25 Feb 2016)}}$

## Angle Modulation:

We know that we can express any real passband signal $u_p(t)$ as

$$u_p(t) = \underbrace{e(t)}_{\text{envelope}} \cos(2\pi f_c t + \theta(t)).$$

In angle modulation schemes, $\theta(t)$ is a function of the message signal $m(t)$ and $e(t)$ is usually a constant (say $A_c$) independent of $m(t)$.

Two main types of angle modulation

1) Phase modulation (PM)
2) Frequency modulation (FM).

PM:

$$\theta(t) = k_p\, m(t)$$

FM:

$$\theta(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau$$

$$\text{(or)} = 2\pi k_f \int_{0}^{t} m(\tau)\, d\tau + \theta(0)$$

Defn

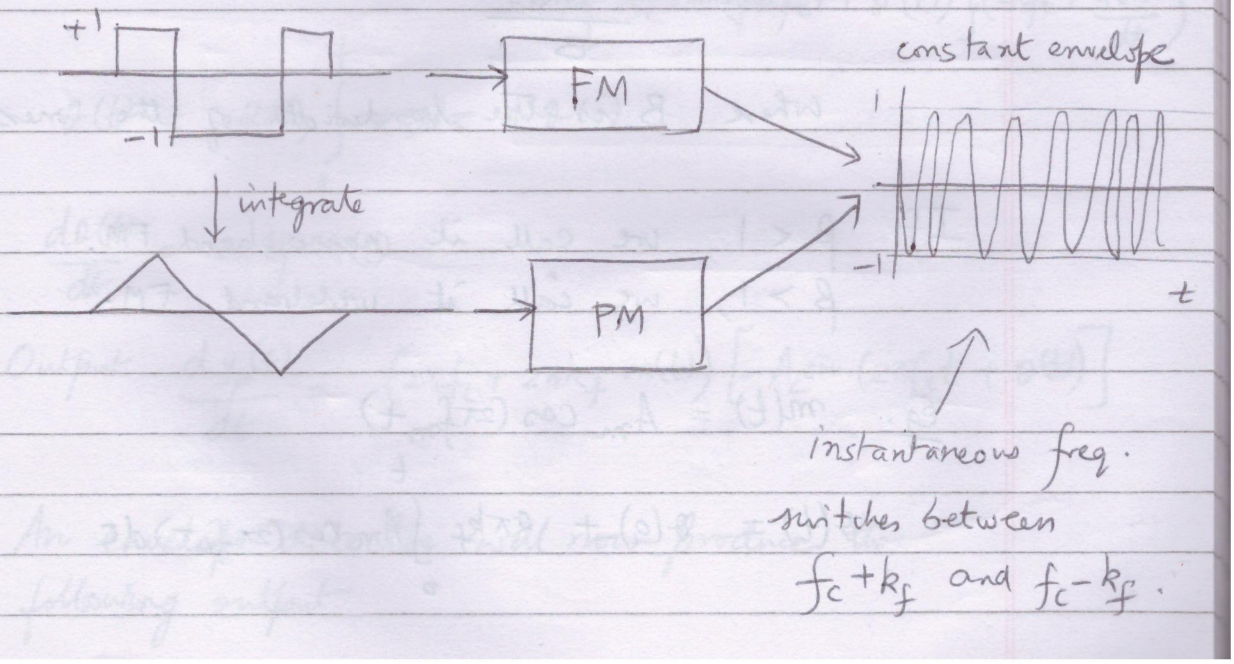
Instantaneous frequency offset relative to the carrier

$$= f(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

In FM, $f(t) = k_f\, m(t)$.

* Observe that FM is equivalent to PM with the integral of the message.
Similarly, for differentiable messages, PM can be interpreted as FM with the input being the derivative of the message.

Example (See Fig 3.21 in book)



* Because of the equivalence above, we will restrict our attention to FM.

[ There are some differences as well. When the message has discontinuities as in the example above, the PM signal will have abrupt changes in phase while the FM signal has continuous phase.

$\Rightarrow$ We can expect lesser bandwidth for FM compared to PM for the same level of variations in the message. ]

Defn:

Frequency deviation: The maximum deviation in instantaneous frequency due to the message $m(t)$ is given by

$$\Delta f_{max} = k_f \max_t |m(t)|.$$

The modulation index $\beta$ is defined as

$$\beta = \frac{\Delta f_{max}}{B}$$

where $B$ is the bandwidth of the message.

If $\beta < 1$, we call it narrowband FM.
$\beta > 1$, we call it wideband FM.

Eg. $m(t) = A_m \cos(2\pi f_m t)$

$$\theta(t) = \theta(0) + 2\pi k_f \int_0^t A_m \cos(2\pi f_m \tau)\, d\tau$$

$$= \theta(0) + \frac{A_m k_f}{f_m} \sin(2\pi f_m t)$$

$$\Delta f_{max} = A_m k_f$$

$$B = f_m$$

$$\Rightarrow \beta = \frac{A_m k_f}{f_m}$$

$$\Rightarrow \theta(t) = \underbrace{\beta \sin(2\pi f_m t)}_{\text{phase deviation}} + \theta(0)$$

FM modulation: Using a voltage-controlled oscillator (VCO)
(Direct FM modulation)

FM demodulation:

$\longrightarrow$ An Ideal discriminator produces $d\theta(t)/dt$

Consider $y_p(t) = A_c \cos(2\pi f_c t + \theta(t))$ $\longrightarrow$ $\boxed{\frac{d}{dt}(\;)}$ $\longrightarrow$

$$\frac{d}{dt}\left(A_c \cos(2\pi f_c t + \theta(t))\right)$$

$$= \left[-A_c \sin(2\pi f_c t + \theta(t))\right] \cdot \left(2\pi f_c + \frac{d\theta(t)}{dt}\right)$$

$$\theta(t) = 2\pi k_f \int_0^t m(\tau) d\tau + \theta(0)$$

$$\frac{d\theta(t)}{dt} = 2\pi k_f \, m(t)$$

Output $\frac{dy_p(t)}{dt} = (2\pi f_c + 2\pi k_f m(t)) \left[-A_c \sin(2\pi f_c t + \theta(t))\right]$

An envelope detector will now produce the following output

$$\left|A_c\left(2\pi f_c + 2\pi k_f m(t)\right)\right| = 2\pi A_c \left|f_c + k_f m(t)\right| .$$

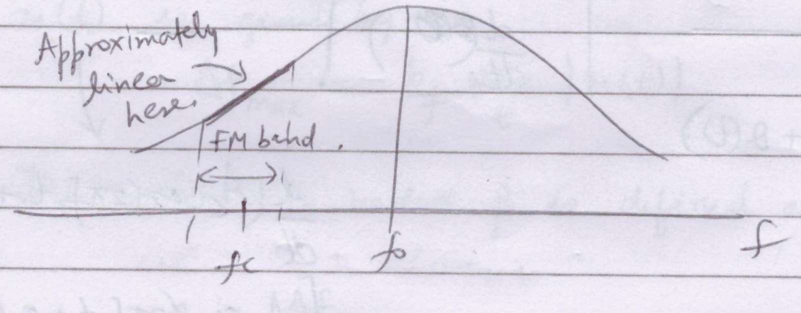
For a properly designed FM system $f_c + \underbrace{k_f m(t)}_{\text{instantaneous freq. deviation}} >$

$$A_c \cos(2\pi f_c t + \theta(t)) \longrightarrow \boxed{\frac{d}{dt}(\ )} \longrightarrow \boxed{\text{Envelope detector}} \longrightarrow \left[f_c + k_f m(t)\right]$$

Remove DC to get a scaled version of $m(t)$

$\boxed{\frac{d}{dt}(\ )}$ has $H(f) = j\,2\pi f$.

In practice, we need this only over the band occupied by the FM signal, e.g.,




$\boxed{\text{Lecture 20: (26 Feb 2016)}}$

## FM spectrum:

$$u_{FM}(t) = A_c \cos\left(2\pi f_c t + \theta(t)\right)$$

where $\theta(t) = \theta(0) + 2\pi k_f \int_0^t m(\tau)\, d\tau$.

* For a general $m(t)$, it is not possible to analytically evaluate the spectrum of $u_{FM}(t)$.

(In AM, we could easily obtain the spectrum & bandwidth of the modulated signal in terms of the bandwidth of the message signal).

We will show /discuss the following.

* Under the strict definition of bandwidth, (as in defn (1) in p38) the BW of $u_{FM}(t)$ is infinite.

  $\Rightarrow$ We need the other approximate BW defns here. e.g. energy-containment BW.

* For $m(t) = A_m \cos(2\pi f_m t)$, we can obtain the spectrum of $u_{FM}(t)$.

* For general $m(t)$ of BW B, we will provide an approximate formula (Carson's rule) for the BW of $u_{FM}(t)$.

* Some discussion on narrowband & wideband FM

<u>FM spectrum for a sinusoidal message:</u>

$$m(t) = A_m \cos(2\pi f_m t).$$

$$\Rightarrow \theta(t) = \beta \sin(2\pi f_m t).$$

$$u_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)).$$

Let us consider the complex envelope of $u_{FM}(t)$.

$$u(t) = A_c e^{j\theta(t)} = A_c e^{j\beta \sin(2\pi f_m t)}.$$

$$\underbrace{e^{j\beta \sin(2\pi f_m t)}}_{\downarrow} \longleftrightarrow ?$$

Periodic

Fundamental period $= \frac{1}{f_m}$.

$\Rightarrow$ we can write

$$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} u[n]\, e^{j 2\pi n f_m t}.$$

where $u[n]$ are the Fourier coefficients given by

$$u[n] = \frac{1}{(1/f_m)} \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j\beta \sin(2\pi f_m t)}\, e^{-j 2\pi n f_m t}\, dt.$$

Let $x = 2\pi f_m t$. $\left(\frac{dx}{2\pi f_m} = dt\right)$

$$t = -\frac{1}{2f_m} \Rightarrow x = -\pi$$

$$t = \frac{1}{2f_m} \Rightarrow x = \pi$$

$$\cancel{\frac{1}{2f_m}} = \cancel{\frac{2\pi t}{x}}$$

$$u[n] = \frac{f_m}{\cancel{2\pi}} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \frac{dx}{2\pi f_m}$$

$$\Rightarrow \boxed{u[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)}\, dx.}$$

This integral cannot be evaluated in closed-form. However, it can be evaluated numerically and is called the Bessel function of $n^{th}$ order of the first kind.

$$u[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)}\, dx \triangleq \underline{J_n(\beta)}.$$

Therefore, we have

$$\boxed{e^{j\beta \sin(2\pi f_m t)} \longleftrightarrow \sum_{n=-\infty}^{\infty} J_n(\beta)\, \delta(f - n f_m)}.$$

<u>Some facts</u>:

(1) For integer $n$, $J_n(\beta) = (-1)^n J_{-n}(\beta) = (-1)^n J_n(-\beta)$

(2) $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$ (From Parseval's theorem)

(3) $J_n(\beta) \to 0$ as $n \to \infty$

We can approximate $e^{j\beta \sin(2\pi f_m t)}$ using a finite number of terms from the summation

$$e^{j\beta \sin(2\pi f_m t)} \approx \sum_{n=-K}^{K} J_n(\beta)\, e^{j2\pi n f_m t},$$

where $K$ is chosen for good approximation.

One approximation is to choose $K$ such that

$$J_0^2(\beta) + 2\sum_{n=1}^{K} J_n^2(\beta) \geq \alpha \quad (\text{say } 0.99)$$

Power containment BW.

$\boxed{\text{Lecture 21 : (1 Mar 2016)}}$ (Q1 papers returned, + some clarification)

<u>Carson's Rule:</u>

This rule gives an estimate of the bandwidth of the FM transmit signal.

$$B_{FM} \approx 2B + 2\,\Delta f_{max} = 2B(\beta+1),$$

where $B$ is the bandwidth of the message signal $m(t)$ and $\Delta f_{max}$ is the maximum deviation in instantaneous frequency. (Further note that $\beta = \frac{\Delta f_{max}}{B}$).

We have already derived the BW of the FM signal when $m(t) = A_m \cos(2\pi f_m t)$. In this case, the 99% power-containment bandwidth is $\approx 2B(\beta+1)$ for integer values of $\beta = 1, 2, \ldots, 10$. Thus, for these $\beta$, the power-containment bandwidth matches with Carson's rule.

## Narrowband and wideband FM.

Narrowband : $\beta \ll 1$
Wideband : $\beta > 1$.

By Carson's rule
$$B_{FM} \approx 2B \text{ for narrowband FM.}$$

* Another interpretation of narrowband FM is given below.

$$u_{FM}(t) = A_c \cos(2\pi f_c t + \theta(t))$$
$$= A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau + \theta(0)\right).$$

$$u_{FM}(t) = A_c \cos\theta(t) \cos 2\pi f_c t - A_c \sin\theta(t) \sin 2\pi f_c t$$

For narrowband FM, let $|\theta(t)| \ll 1$.

Then, we can approximate $\cos\theta(t) \approx 1$
$\sin\theta(t) \approx \theta(t)$.

$$\Rightarrow u_{FM}(t) \approx \underbrace{A_c \cos 2\pi f_c t}_{\text{Carrier.}} - \underbrace{A_c \theta(t) \sin 2\pi f_c t}_{\text{Similar to DSB-SC AM signal}}.$$

$$\Rightarrow \text{BW of } u_{FM}(t) \approx 2B.$$

* For wideband FM, BW is more a fn. of $\Delta f_{max}$ than B.

# Lecture 22: (3 Mar 2016)

So far, we have discussed demodulation under the assumption that the received signal is exactly the modulated signal. In practice, there are more issues

(1) Other signals occupy neighbouring frequency bands (e.g. other radio stations)
$\Rightarrow$ We need selective filtering.

(2) We need to able to demodulate multiple signals (e.g. different radio signals (stations) at different times)
$\Rightarrow$ need tuning capability

(3) Amplification is needed since the received signal is attenuated by the channel.

Designing such filters, tuners and amplifiers poses some implementation problems.

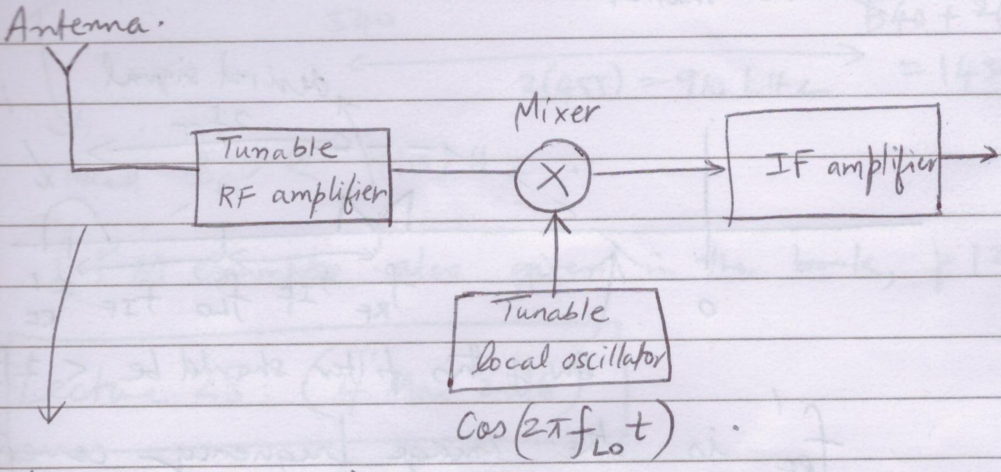
* For example, it may be difficult to design a tunable filter of the same quality over a wide range of $f_c$. (large variation in $B/f_c$)
* Or, it may be difficult to design a narrow filter of BW $B$ around $f_c \gg B$. (small $\frac{B}{f_c}$)
* may be difficult to amplify at very high $f_c$.

These issues motivated the following design for the demodulator called the <u>superheterodyne receiver</u>.

The usual receiver where we directly demodulate the signal from $f_c$ to baseband will be termed <u>direct-conversion receivers</u>.

Direct-conversion is becoming more popular for cellular receivers.

Superheterodyne receivers are popular for AM/FM radio.




$A\cos(2\pi f_{RF} t + \theta)$

$$A\cos(2\pi f_{RF} t + \theta) \cdot \cos(2\pi f_{LO} t) = \frac{A}{2}\cos(2\pi(f_{RF} - f_{LO})t + \theta) + \frac{A}{2}\cos(2\pi(f_{RF} + f_{LO})t + \theta).$$

We can either choose $f_{LO} = f_{RF} - f_{IF}$.

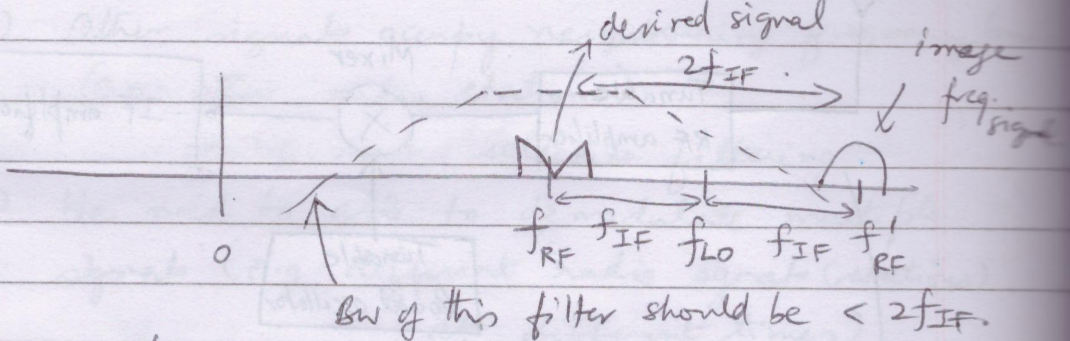
(or) $f_{LO} = f_{RF} + f_{IF}$.

In both cases, we get the desired signal at $f_{IF}$ after the mixer.

* Image frequency:
  We can also get undesired signals at $f_{IF}$ after the mixer. $\Rightarrow$ We need to filter these signals out before the mixer (in the RF amplifier). See the following example.

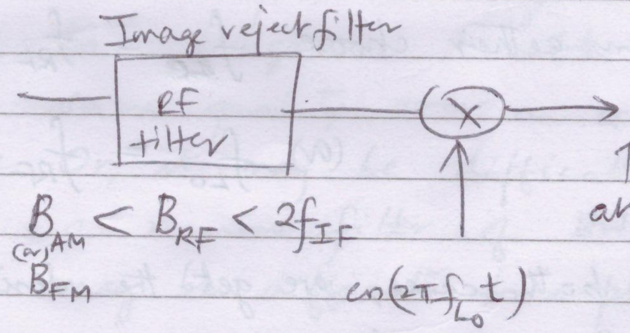
Fix $f_{IF}$, $f_{LO}$.

Then, we can get both the signal at $f_{LO} - f_{IF}$ & $f_{LO} + f_{IF}$ at $f_{IF}$ after the mixer.




$f'_{RF}$ is the image frequency corresponding to $f_{RF}$ w.r.t $f_{LO}$.

In order to avoid the signal at the image frequency to interfere with the desired signal after shifting to IF, we need an RF filter with bandwidth $< 2f_{IF}$.




$B_{AM}$ (or) $B_{FM} < B_{RF} < 2f_{IF}$

<u>Example</u>:

AM: $f_{IF} = 455$ kHz

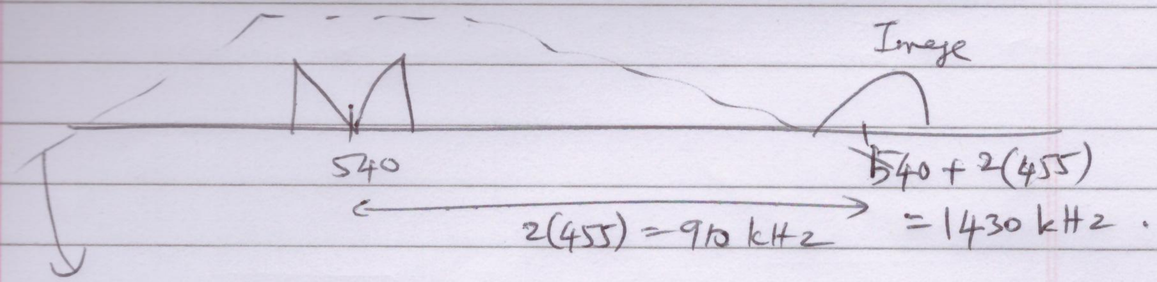
AM Band 540 kHz to 1600 kHz

$$f_{LO} = f_{RF} + f_{IF}$$

$\underbrace{995 \text{ kHz to } 2140 \text{ kHz}}$.

Oscillator needs to be tunable in this range.

Suppose we want to tune to 540 kHz.




Need $B_{RF} < 910$ kHz.

(FM example also given in the book, p 120.)

$\boxed{\text{Lecture 23: (4 Mar 2016)}}$