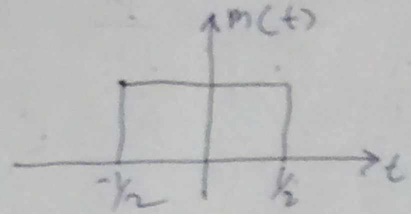
1) a) $m(t) = \sum_{-\infty}^{\infty} (-1)^n \, p(t-n)$

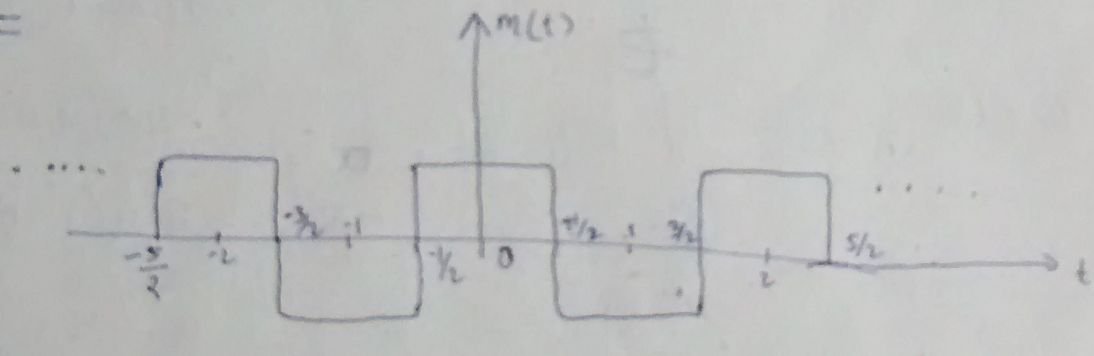
$p(t) = I_{[-1/2, 1/2]}(t)$ $\equiv$



in $m(t)$ if n is even $p(t-n)$ is ~~positive~~ added.
and if n is odd $p(t-n)$ is substracted.

and as the duration of $p(t)$ is 1, so there will not be any overlap between $p(t-n)$ for different values of n

$m(t) =$



$$\phi(t) = 20\pi \int_{-\infty}^{t} m(\tau)\, d\tau + a$$

it is given $\phi(0) = 0 \Rightarrow 20\pi \int_{-\infty}^{0} m(\tau)\, d\tau + a = 0$

$$20\pi \int_{-\infty}^{-1/2} m(\tau)\, d\tau + 20\pi \int_{-1/2}^{0} m(\tau)\, d\tau + a = 0$$

$$0 + 20\pi \cdot \frac{1}{2} + a = 0 \Rightarrow a = -10\pi$$

$\phi(0) = 0$

for $0 < t < 0.5$ $\quad \phi(t) = 20\pi \int_{-\infty}^{t} m(\tau) d\tau + a$

$$= 20\pi \int_{-\infty}^{0} m(\tau) d\tau + a + 20\pi \int_{0}^{t} m(\tau) d\tau$$

$$= 0 + 20\pi \int_{0}^{t} m(\tau) d\tau$$

$$= 20\pi t$$

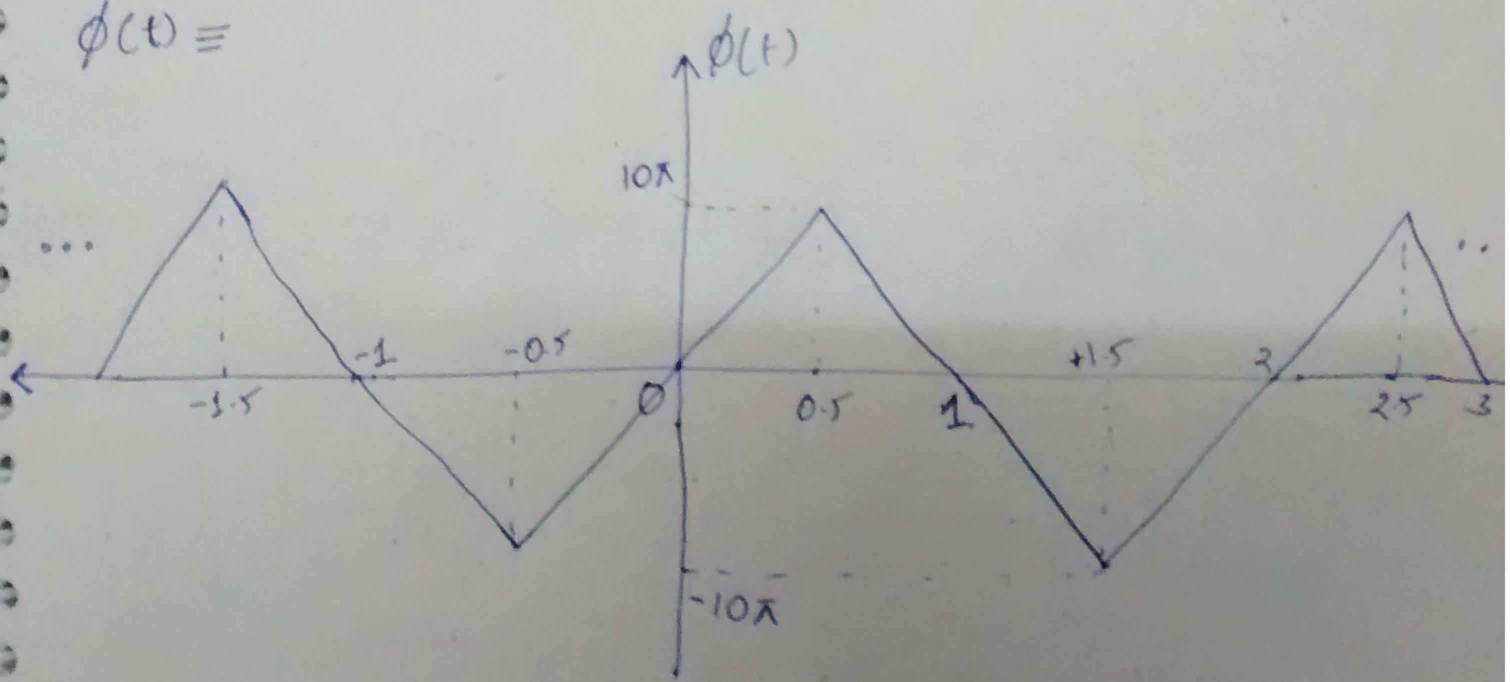
$\|^{ly}$ $0.5 < t < 1.5$ $\quad \phi(t) = 20\pi \int_{0}^{t} m(\tau)$

$$= 20\pi \int_{0}^{0.5} m(\tau) d\tau + 20\pi \int_{0.5}^{t} m(\tau) d\tau$$

$$= 20\pi \cdot (0.5) - 20\pi (t - 0.5)$$

$$= 20\pi - 20\pi t$$

$\phi(t) \equiv$



Tutorial 7: Q1.B

Q1.B Given that the FM signal $u(t) = 20\cos(2\pi f_c t + \phi(t))$ and $\phi(t) = 20\pi \int_{-\infty}^{t} m(\tau)d\tau + a$. Hence, the instantaneous frequency can be written as $f(t) = f_c + 10m(t)$. Since, $m(t)$ has maximum value of 1, the maximum frequency deviation $f_\Delta = 10$. It is also given that the Bandwidth W of message $m(t)$ is $W = 2$. Hence, by Carlson's rule the transmitted bandwidth is $2f_\Delta + 2W = 24$Hz.

Tutorial 7: Q1.C

Q1.C: Suppose that a very narrow ideal BPF (with bandwidth less than 0.1) is placed at $f_c + \alpha$. For which (if any) of the following choices of will you get non zero power at the output of the BPF:

(i) $\alpha = .5$, (ii) $\alpha = .75$, (iii) $\alpha = 1$.

Ans): It is clear from Q1.a and Q1.b that $\phi(t)$ is a periodic signal with period 2. The FM output is given by $u(t) = 20\cos(2\pi f_c t + \phi(t)) = Re(20\tilde{u}(t)e^{j2\pi f_c t})$, where $\tilde{u}(t) = e^{j\phi(t)}$. Since, $\phi(t)$ is periodic with period $T_0 = 2$, $e^{j\phi(t)}$ is also periodic with period T_0. Hence, $e^{j\phi(t)}$ can be expanded in Fourier series as

$$e^{j\phi(t)} = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f_0 t}, \text{where} f_0 = 1/T_0 = 0.5. \tag{1}$$

This implies that

$$u(t) = \sum_{k=-\infty}^{\infty} 20|c_k| \cos(2\pi(f_c + kf_0)t + \arg(c_k)) \tag{2}$$

Thus the FM signal $u(t)$ has components at f_c, $f_c \pm 0.5$, $f_c \pm 1$ etc. Hence, the BPF with $\alpha = 0.5$ and $\alpha = 1$ will have non zero output.

Tutorial - 7

2 (a)

$$\theta(t) = 2\pi K_f \int_0^t m(\tau)\, d\tau + \theta(0)$$

$$m(t) = \frac{1}{2\pi K_f} \frac{d\theta(t)}{dt}$$

$$0 = 2\pi K_f \int_0^1 K\, d\tau + \theta(0)$$

($K \rightarrow$ Amplitude of square wave)

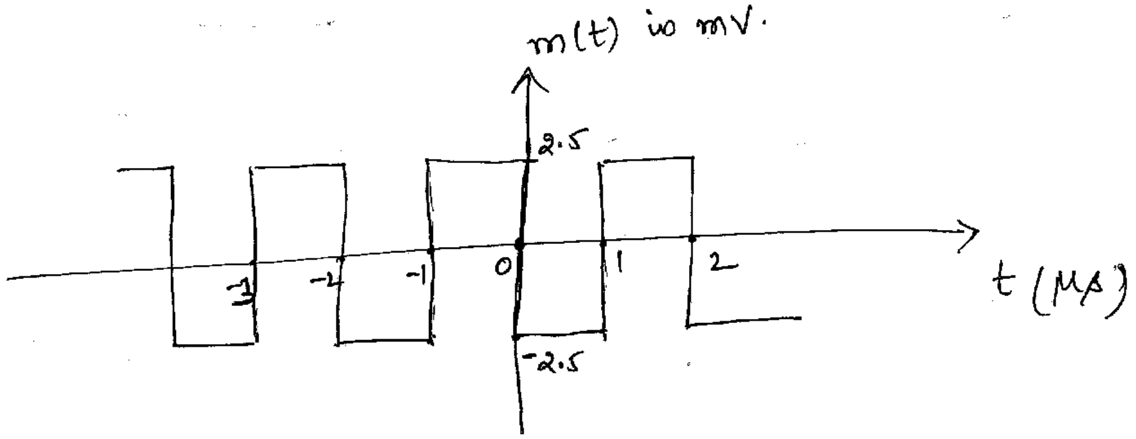
$\downarrow$

$\because$ Diff. of Triangle is square.

$$0 = 2\pi \times \underset{\downarrow\ 2\,MHz/mV}{K_f} \overset{mV \times 1\mu s}{(K)} + 10\pi$$

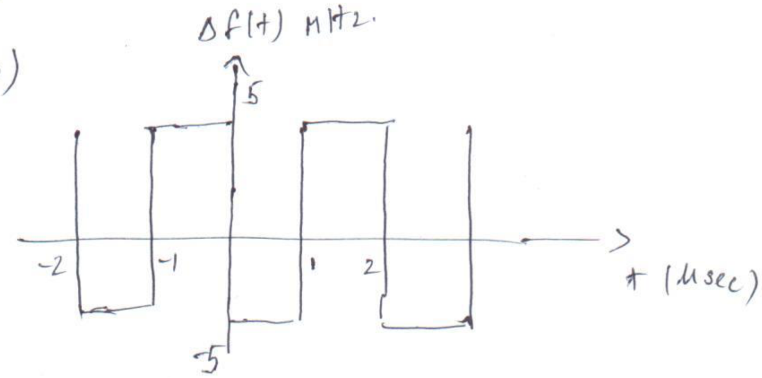
$$= 2\pi \times 2\, \frac{MHz}{\cancel{mV}} \times K\, \frac{\cancel{mV}}{1\,\cancel{MHz}} = 4\pi K \mp 10\pi$$

$$\Rightarrow K = (-2.5)\ mV$$



2)

b)



$$(\Delta f)_{max} = 5 \text{ MHz.}$$

Bandwidth of the message $\cong 0.5 \times 10^6$ (Considering only 1st harmonic)

$$B = 0.5 \text{ MHz.}$$

By Carson's rule

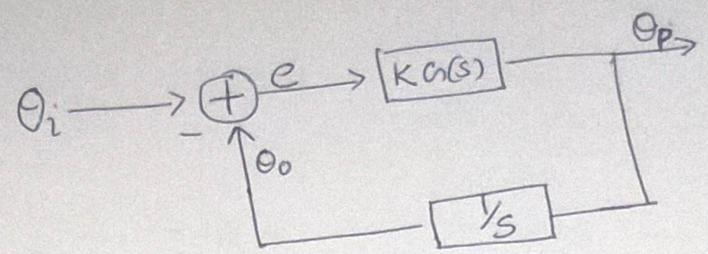
$$B_{FM} = 2B + 2(\Delta f)_{max}.$$

$$= 2 \times 0.5 + 2 \times 5$$

$$B_{FM} = 11 \text{ MHz.}$$

Q3

(a)



$$H_m(s) = \frac{\theta_p(s)}{\theta_i(s)}$$

$$= \frac{KG(s)}{1 + \frac{KG(s)}{s}}$$

$$= \frac{sKG(s)}{s + KG(s)}$$

$$H(s) = \frac{\theta_o(s)}{\theta_i(s)}$$

$$\theta_o(s) = \frac{\theta_p(s)}{s} = \frac{H_m(s)\theta_i(s)}{s}$$

$$= \frac{KG(s)}{s + KG(s)} \times \theta_i(s)$$

$$\Rightarrow \frac{\theta_o(s)}{\theta_i(s)} = \frac{KG(s)}{s + KG(s)}$$

$$H_e(s) = \frac{e(s)}{\theta_i(s)} = \frac{\theta_i(s) - \theta_o(s)}{\theta_i(s)}$$

$$= 1 - \frac{\theta_o(s)}{\theta_i(s)}$$

$$= 1 - \frac{KG(s)}{s + KG(s)}$$

$$= \frac{s}{s + KG(s)}.$$

Communication Systems; Tutorial 7

solution

March 14, 2016

3.b

$H_m(s)$. Output is taken from the input of VCO

5.b

$H(s)$, Out put is taken from output of VCO

8) d

$$H_M(s) = \frac{s k G(s)}{s + k G(s)}$$

$$H(s) = \frac{k G(s)}{s + k G(s)}$$

$$H_e(s) = \frac{s}{s + k G(s)}$$

So $$H_M(s) = \frac{2 \times (s+8)}{s + \frac{2(s+8)}{s}}$$

$$= \frac{2s(s+8)}{s^2 + 2s + 16}$$

$$H(s) = \frac{2s+16}{s^2+2s+16} \qquad H_e(s) = \frac{s^2}{s^2+2s+16}$$

The denominator is $s^2 + 2s + 16$ " of the form $s^2 + 2\xi\omega_n s + \omega_n^2$

$$\Rightarrow \omega_n = 4 \text{ rad/s}$$

$$\xi = \frac{1}{4}$$