Assignment 1 : Linear systems
(EE 6417:Allied Topics Control)

1. Consider a simple pendulum of length L and let m be the mass attached at the end of the pendulum. The equation of motion is given by:

$$\ddot{\theta} + \frac{g}{L} sin\theta + \frac{k}{M}\dot{\theta} = 0$$

   Let k be the coeffiecient of friction at pivot point and g be the gravity.
   1) Linearize the equations of the motion about the equillibrium points.
   2) Obtain the state space representation of the system. The output of the system is the angle θ.

2. Consider the following differential equation of a system
   $\dddot{x}(t) + 3\ddot{x}(t) + 3\dot{x}(t) + x(t) = \dddot{u}(t) + 2\ddot{u}(t) + 4\dot{u}(t) + u(t)$.
   Find the state state space representation of the system and also obtain a block diagram representation of the system.

3. Obtain the state variable representation and the block diagram for the following transfer function:

$$T(s) = \frac{Y(s)}{U(s)} = \frac{s^2 + 2s + 10}{s^3 + 4s^2 + 6s + 10}$$

4. Compute the Jordan normal form for the following matrix

$$\begin{bmatrix} 5 & 4 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 3 & 0 \\ 1 & 1 & -1 & 2 \end{bmatrix} \tag{1}$$

5. Consider the following state and output equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -6 & 1 & 0 \\ -11 & 0 & 1 \\ -6 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix} u \text{ and } y = [1\ 0\ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{2}$$

   Show that the state equation and the output equation can be transformed into the following form by a suitable transformation matrix.

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

   Then obtain y in terms of z_1, z_2 and z_3.

6. Prove the following properties of matrix exponentials:
   1) If $A = diag\{a_{11}, a_{22}, \ldots, a_{nn}\}$ then $e^A = diag\{e^{a_{11}}, e^{a_{22}}, \ldots, e^{a_{nn}}\}$
   Compute e^{At}, given

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

   2) If B is any $n \times n$ matrix and Q be any non-singular matrix then

$$e^{Q^{-1}BQ} = Q^{-1}e^B Q$$

   3) $e^A e^B = e^{(A+B)}$, iff $AB = BA$.

7. Given the system equations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \tag{3}$$

   Find the solution in terms of initial conditions $x_1(0)$, $x_2(0)$ and $x_3(0)$.