Q1 a)

Let
$x_1 =$ 1 if invest in project 1; 0 if not
$x_2 =$ 1 if invest in project 2; 0 if not
$x_3 =$ 1 if invest in project 3; 0 if not
$x_4 =$ 1 if invest in project 4; 0 if not
$x_5 =$ 1 if invest in project 5; 0 if not

Maximize $NPV = x_1 + 1.8x_2 + 1.6x_3 + 0.8x_4 + 1.4x_5$

subject to $\quad 6x_1 + 12x_2 + 10x_3 + 4x_4 + 8x_5 \le 20$

and $\quad x_1,\ x_2,\ x_3,\ x_4,\ x_5$, are binary variables.

Q2 a) maximize $70x_1 - 50000y_1 + 60x_2 - 40000y_2 + 90x_3 - 70000y_3 + 80x_4 - 60000y_4$

subject to:

$$y_1 + y_2 + y_3 + y_4 \le 2$$

$$y_3 \le y_1 + y_2 \qquad y_4 \le y_1 + y_2$$

$$5x_1 + 3x_2 + 6x_3 + 4x_4 \le 6000 + My_5$$

$$4x_1 + 6x_2 + 3x_3 + 5x_4 \le 6000 + M(1 - y_5)$$

$x_i \le My_i \quad i = 1, \ldots, 4$

$x_i \ge 0 \quad i = 1, \ldots, 4$

y_i binary $\quad i = 1, \ldots, 5$

$M = 100$ Million.

Modified Original

Q3

a.

let x_i = # units to produce of product i, $i=1,2,3$.

$$y_i = \begin{cases} 1 & \text{if product } i \text{ is produced} \\ 0 & \text{otherwise} \end{cases}$$

$$\max \quad 2x_1 + 3x_2 + 0.8x_3 - 3y_1 - 2y_2$$

s.t.

$$0.2x_1 + 0.4x_2 + 0.2x_3 \le 1$$

$$x_1 \le M y_1$$

$$x_2 \le M y_2$$

$$x_1 \le 3, \quad x_2 \le 2, \quad x_3 \le 5$$

$$x_1, x_2, x_3 \ge 0 \text{ integer} \quad ; \ y_1, y_2 \text{ binary}$$

Q4

a) let $y_{ij} = \begin{cases} 1 & \text{if } x_i = j \\ 0 & \text{if not} \end{cases}$ (i.e. produce j units of i)

$i=1,2,3$, $j=1,2,(3,4,5)$

$$\max. \ -1y_{11} + 2y_{12} + 4y_{13} + 1y_{21} + 5y_{22} + 1y_{31} + 3y_{32} + 5y_{33} + 6y_{34} + 7y_{35}$$

s.t.

$$y_{11} + y_{12} + y_{13} \le 1$$

$$y_{21} + y_{22} \le 1$$

$$y_{31} + y_{32} + y_{33} + y_{34} + y_{35} \le 1$$

$$1y_{11} + 2y_{12} + 3y_{13} + 2y_{21} + 4y_{22} + 1y_{31} + 2y_{32} + 3y_{33} + 4y_{34} + 5y_{35} \le 5$$

y_{ij} binary

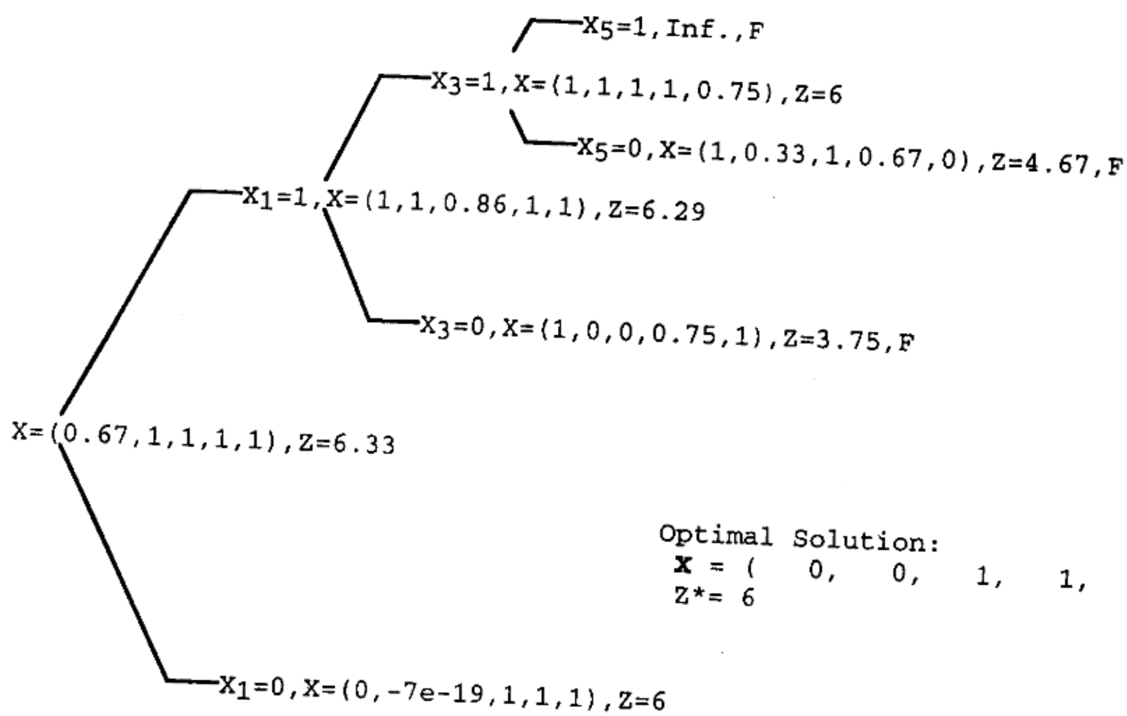
Q8 Optimal solution for the relaxed problem is
Z = 14.6, x1 = 2.6, x2 = 1.6

Rounding the optimal solution to the LP-relaxation
$(\bar{x}_1, \bar{x}_2) = (3, 2)$ is not feasible $4 \cdot 3 + 2 \not\leq 12$

Rounded Solutions	Constraints violated	Z
(3,2)	3rd	-
(3,1)	2nd and 3rd	-
(2,2)	none	12
(2,1)	none	11

Q9

Solution Tree:



Optimal Solution:
X = (0, 0, 1, 1, 1)
Z*= 6

Q10

(a) Initialization Step: Set $Z^*=+\infty$. Apply the bounding and fathoming steps and the optimality test as dexcribed below on the whole problem. If the whole problem is not fathomed, it becomes the initial subproblem for the first iteration below.

Iteration:

1. Branching: Choose the most recently created unfathomed subproblem (breaking ties by selecting the one with the smallest bound). Among the assignees not yet assigned for the current subproblem, choose the first one in the natural ordering to be the branching variable. Subproblems will correspond to each of the possible remaining assignments for the branching assignee.
   Form a subproblem for each remaining assignment by deleting the constraint that each of the unassigned assignees must perform exactly one assignment.

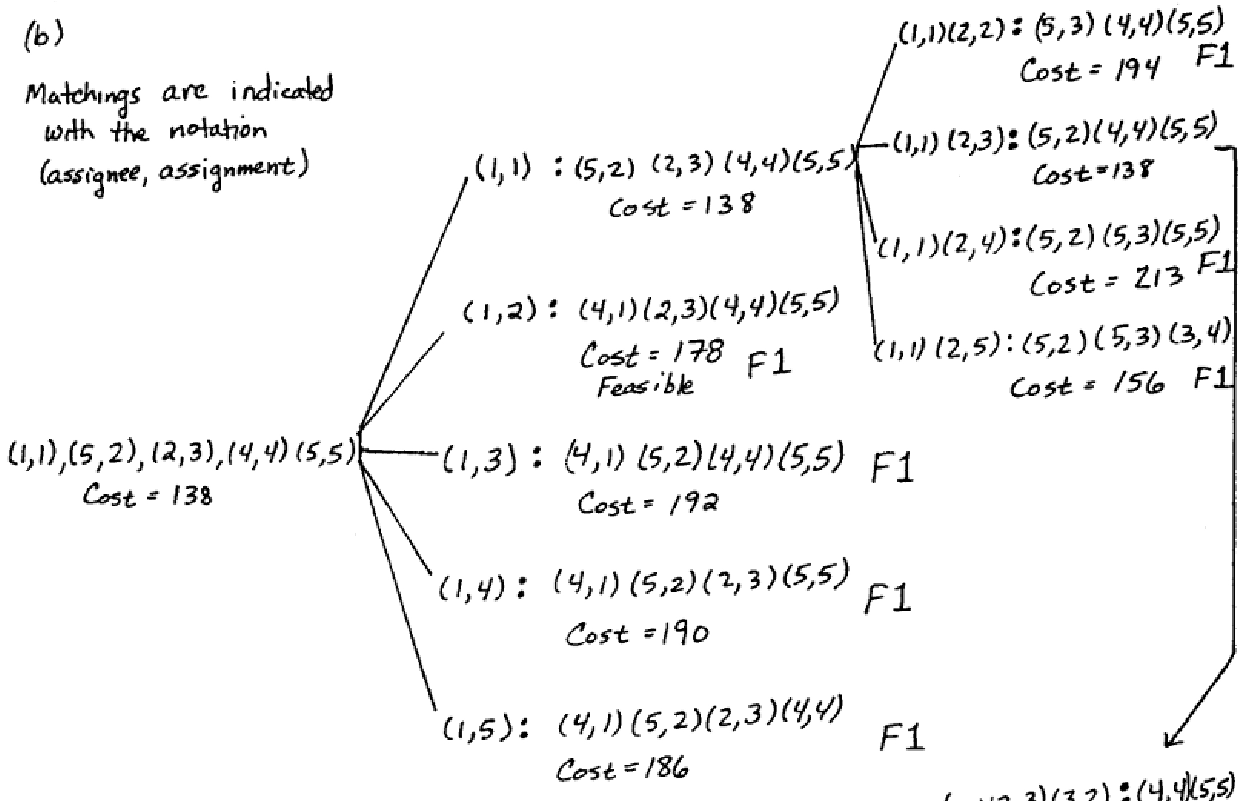
2. Bounding: For each new subproblem, obtain its bound by choosing the cheapest assignee for each remaining assignment and totaling the costs.

3. Fathoming: For each new subproblem, apply the two fathoming test below:
   Test 1. Its bound>= Z^*
   Test 2. The optimal solution for its relaxation is a feasible assignment (if this solution is better than the incumbent, it becomes the new incumbent and Test 1 is reapplied to all unfathomed subproblems with the new smaller Z^*).

Optimality Test: Identical to that in the text.

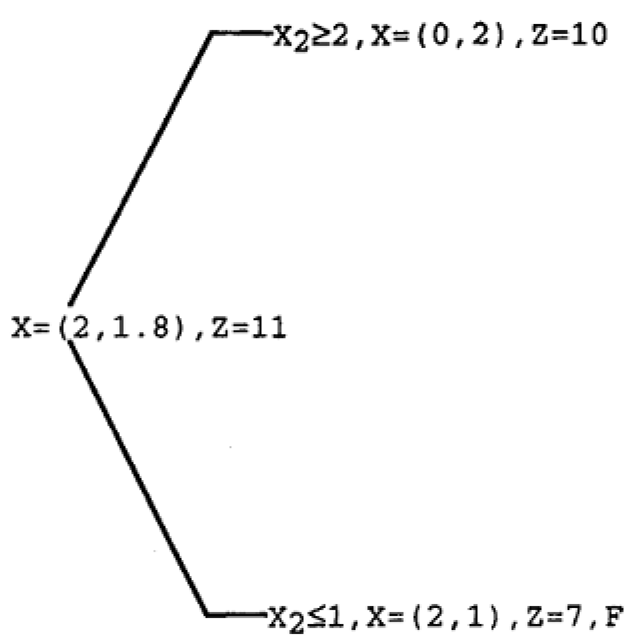
(b)

Matchings are indicated with the notation (assignee, assignment)



The optimal matching is
(1,1) (2,3) (3,2) (4,4) (5,5)
with Cost = 154.

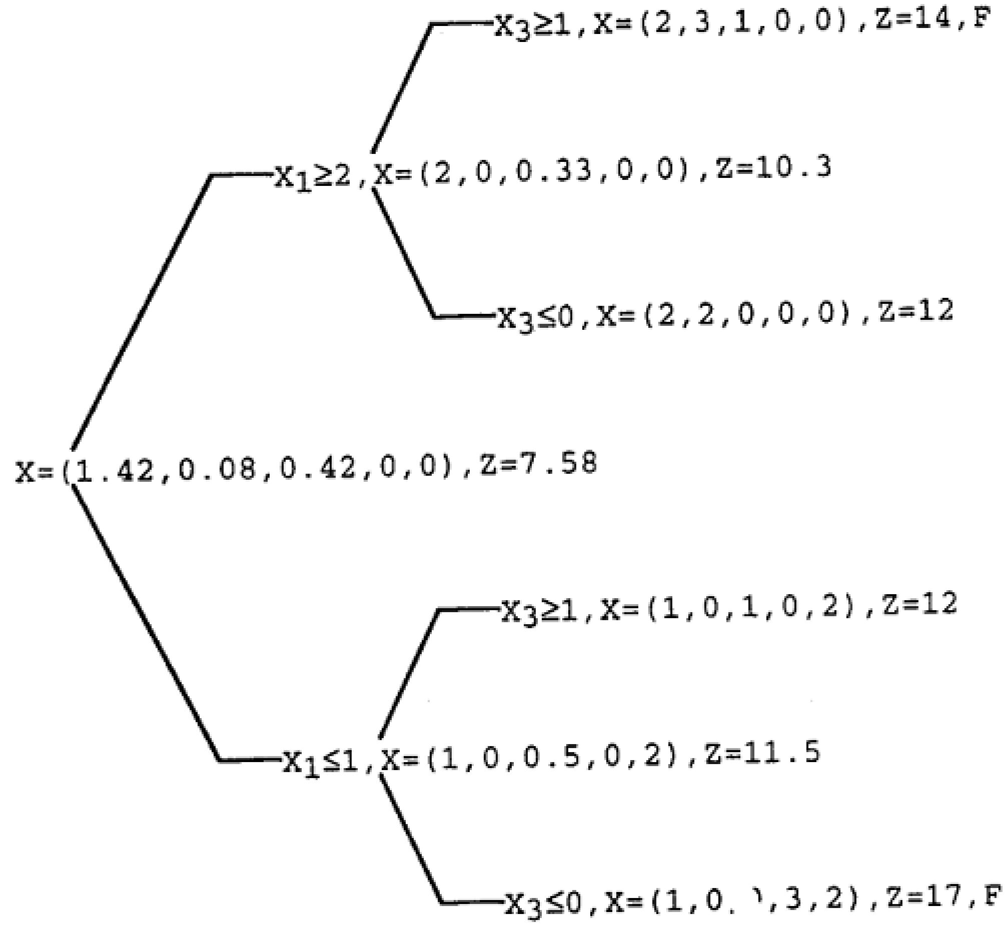
Q11



Optimal Solution:
X = (0, 2)
Z*= 10

Q12

Solution Tree:



Optimal Solution:
x = (1, 0, 1, 0, 2)
Z*= 12 <u>or</u> x=(2,2,0,0,0)

Q13

a) redundant, because even if all variables equal, $2+1+2 \leq 5$

b) not redundant, as (1,0,1) violates this ($8 \nleq 5$)

c) not redundant, as (0,0,0) (for example) violates

d) redundant, because $(0,1,1) \rightarrow 0-1-2 \geq -4$ still; this is the worst case, because we let variables with positive coefficients $=0$ and variables with negative coefficients $=1$ to try to violate the ≥ -4 condition, and we can't do it.

Q14

$$4x_1 - 3x_2 + x_3 + 2x_4 \leq 5$$

$b = 5 \quad S = 7 \quad$ and $S < b + a_1$

so $\tilde{a}_1 = S - b = 2$

$\bar{b} = S - a_1 = 3$

so we have:

$$2x_1 - 3x_2 + x_3 + 2x_4 \leq 3 \qquad \Delta$$

$b = 3 \quad S = 5 \quad$ and $\quad S < b + |a_2|$

so $a_2 = b - S = -2$

thus in Δ:

$$2x_1 - 2x_2 + x_3 + 2x_4 \leq 3$$

now $S = 5 \quad b = 3$

and $S \geq b + |a_j|$ for $j = 1, \dots, 4$.