① ⓐ The VCO output is an FM signal with

$$\Delta f_{max} = k_f \max_t m(t) = 25 \text{ kHz/mV} \times 2 \text{ mV} = 50 \text{ kHz}$$

Message is periodic with period 100 micro seconds, hence its fundemental freq. is 10 kHz. Approximating its bandwidth by first harmonic, $B \approx 10$ kHz.

By Carson's formula

$$B_{FM} \approx 2\Delta f_{max} + 2B \approx 120 \text{ kHz}.$$

ⓑ Complex envelope of VCO output is given by $e^{j\theta(t)}$, where

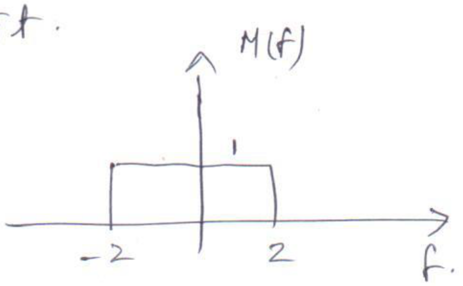
$$\theta(t) = 2\pi k_f \int m(\tau)\,d\tau$$

For periodic messages with zero DC value (as is the case for m(t) here), $\theta(t)$, and hence, $e^{j\theta(t)}$ has same period as the message. We can therefore express the convex envelope as a Fourier series with complex exponentials at frequencies nf_m, where $f_m = 10$ kHz is the fundemental freq. of msg, and where 'n' takes integer values. Thus, FM signal has discrete components at $f_c + nf_m$, where $f_c = 5$ MHz in this problem. A bandpass filter at 5.005 MHz with 5 kHz doesn't capture any of these components (∵ it spans only [5.0025, 5.0075] MHz, where as fourier components are at 5 MHz, 5.01 MHz. Thus power at output is zero.

2)

a) $u_{DSB-SC} = 10\, m(t) \cos 300\pi t.$

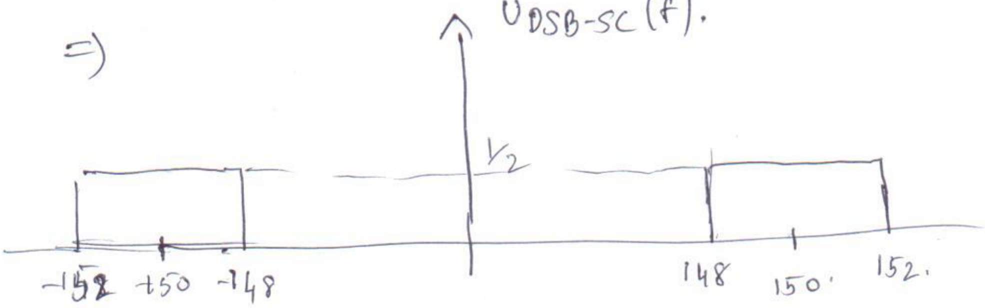
$M(f) = I_{[-2,2]}(f)$



$$U_{DSB-SC}(f) = \frac{I_{[-2,2]}(f-f_c) + I_{[-2,2]}(f+f_c)}{2}.$$

$f_c = 150$

$\Rightarrow$

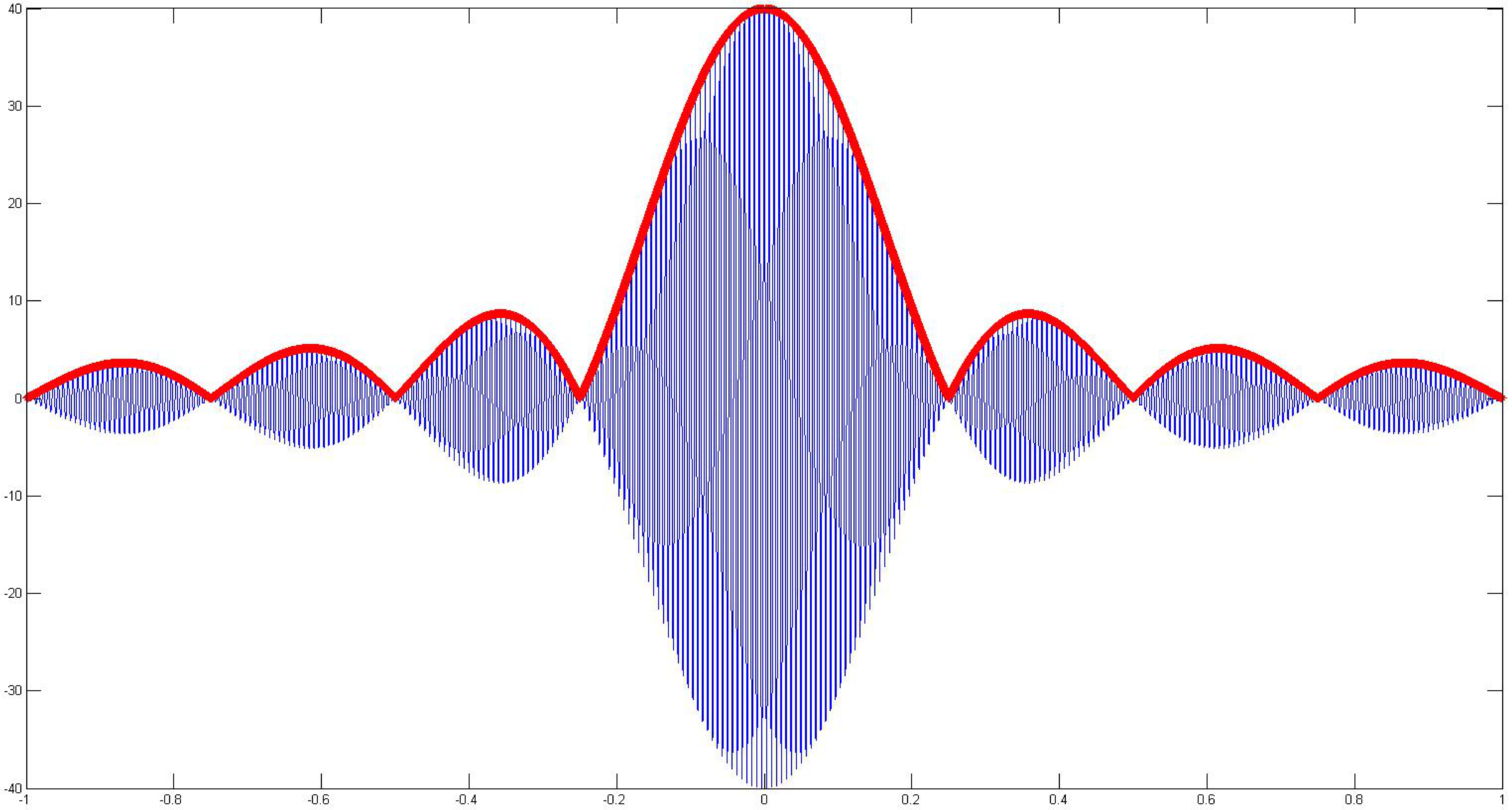


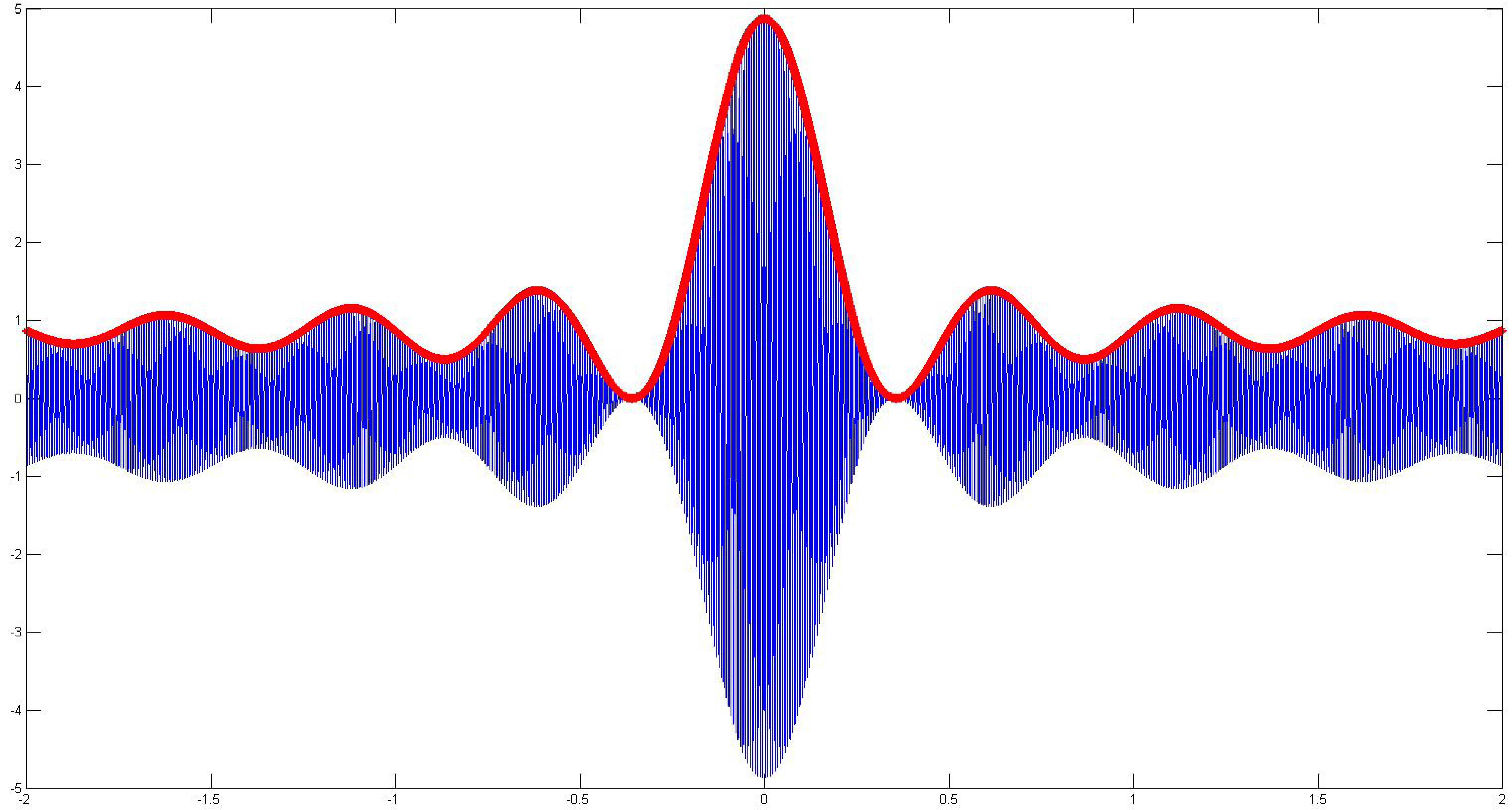
Power $= \frac{1}{4} \times 4 + \frac{1}{4} \times 4 = 2.$

Bandwidth $= 4$

2b) See 2b.jpg. This will be |(m(t))|.

2b)See 2c.jpg. The min value of A, A_{min}=min(m(t))=min(4sinc(4t))


40
30
20
10
0
-10
-20
-30
-40
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

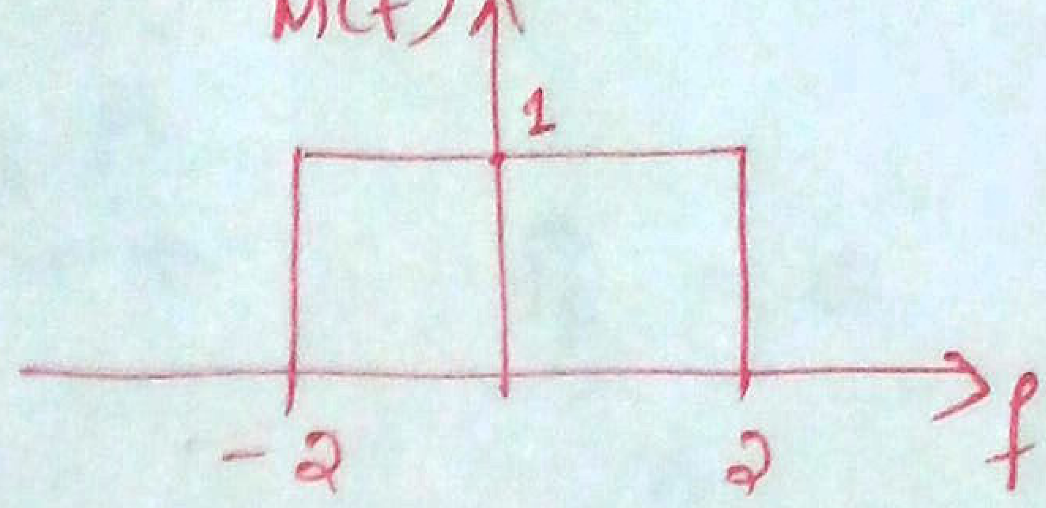


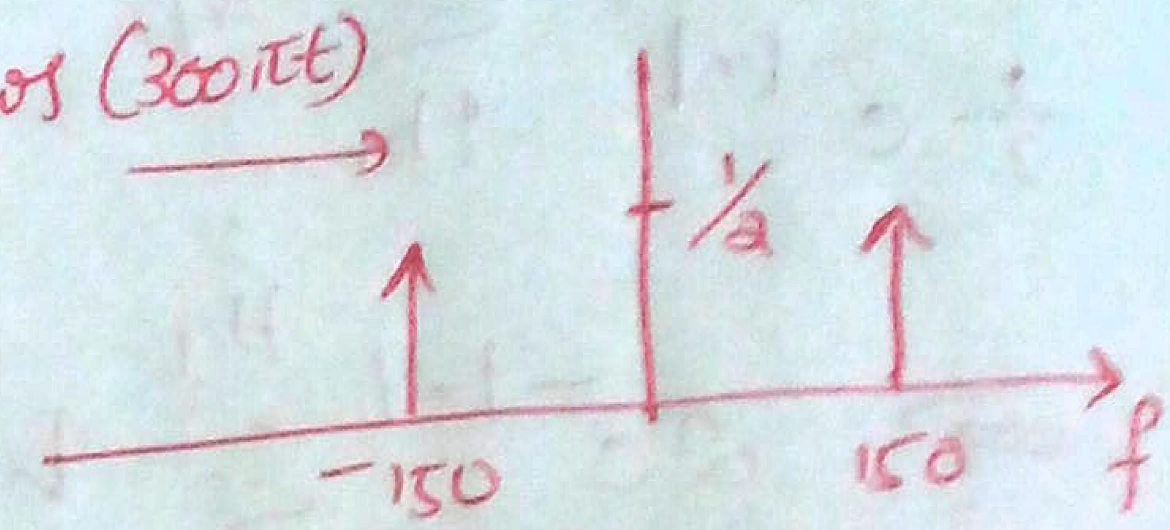
5
4
3
2
1
0
-1
-2
-3
-4
-5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

(2) . d. $M(f) = I_{[-2,2]}(f)$.

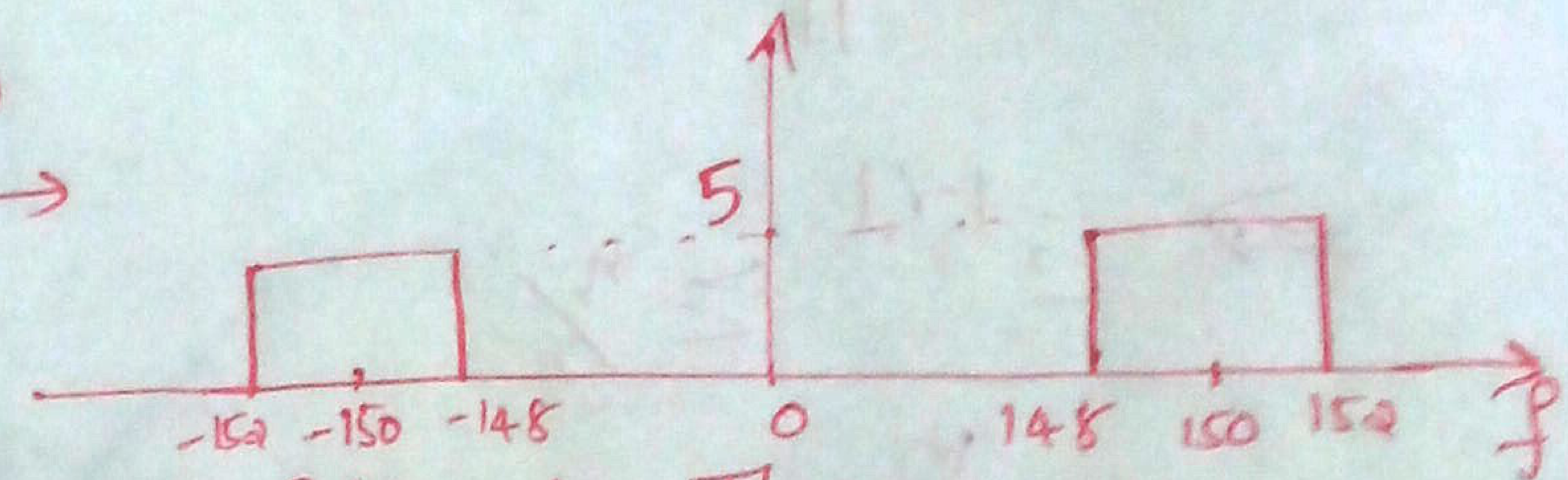
$U_{DSB-SC} = 10\, m(t) \cos(300\pi t)$

After HPF, $U_p(t) = U_c(t) \cos(300\pi t) - U_s(t) \sin(300\pi t)$.



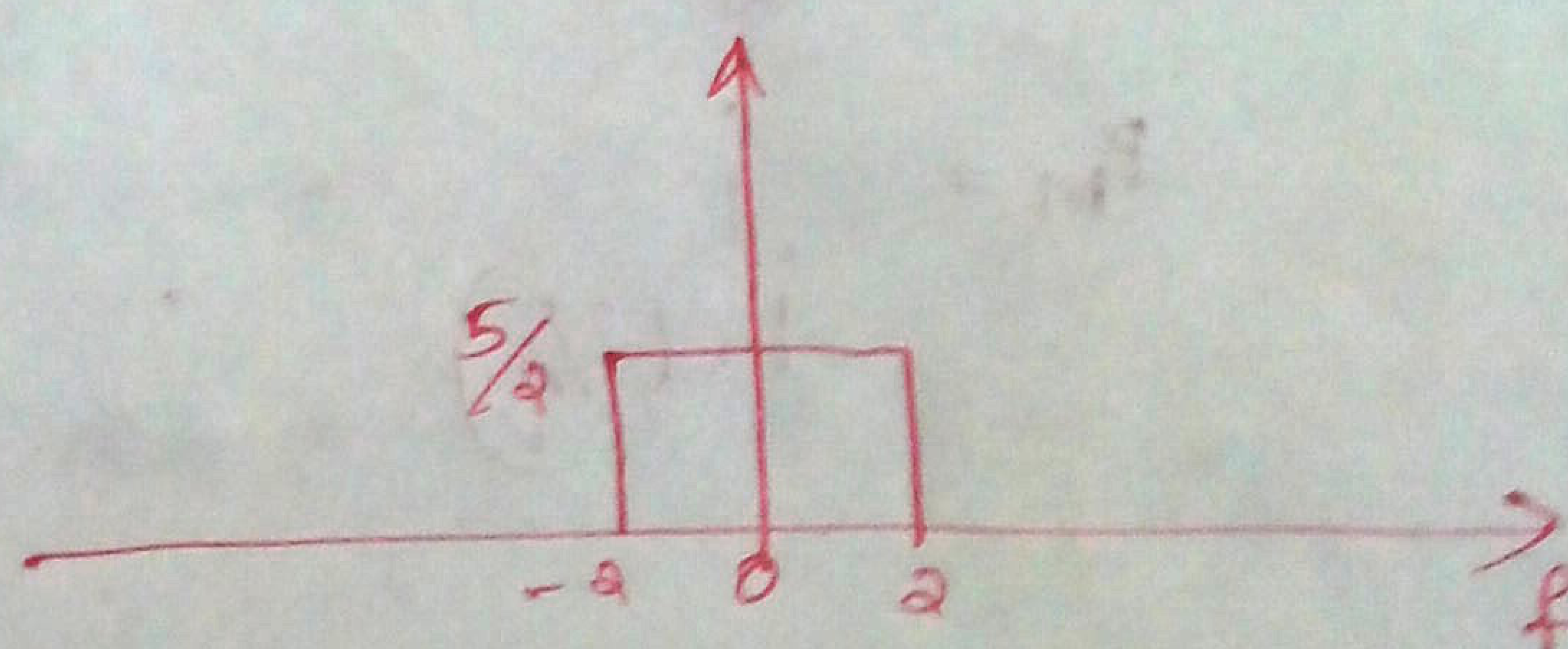


$10 \cos(300\pi t) \cdot m(t) \xrightarrow{FT}$



U_{DSB-SC} is highpass filtered. Then $U_c(t)$ can be obtained by multiplying $U_p(t)$ with $\cos(300\pi t)$ and lowpass filtering (This step gives $\frac{1}{2} U_c(t)$.)

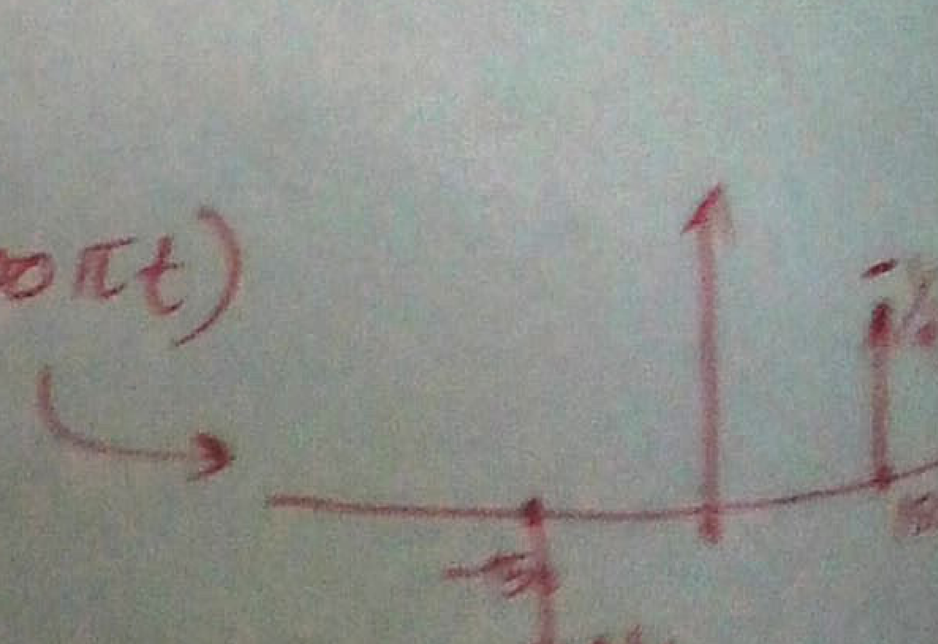
$\frac{1}{2} U_c(t) \xrightarrow{FT}$



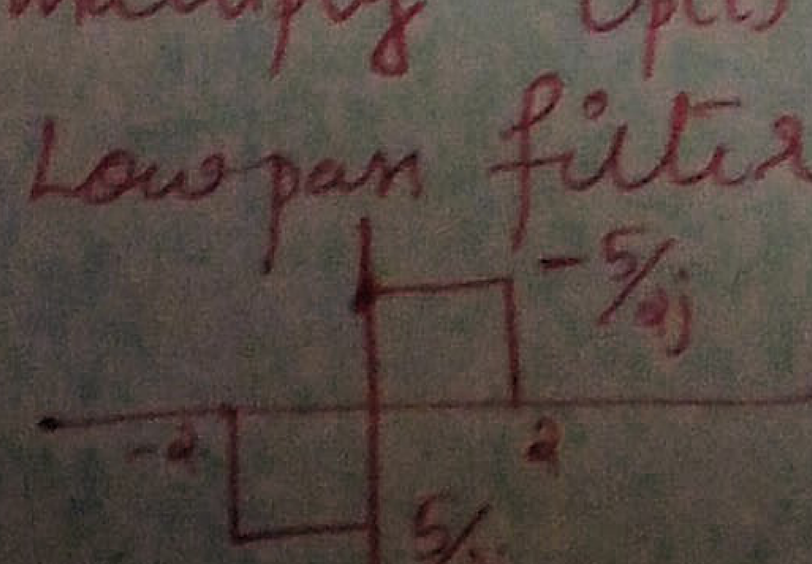
$U_c(t) = 20 \operatorname{sinc}(4t)$

Similarly, $U_p(t)$ can be obtained by using the following steps

- multiply $U_p(t)$ by $-\sin(300\pi t)$
- Lowpass filter



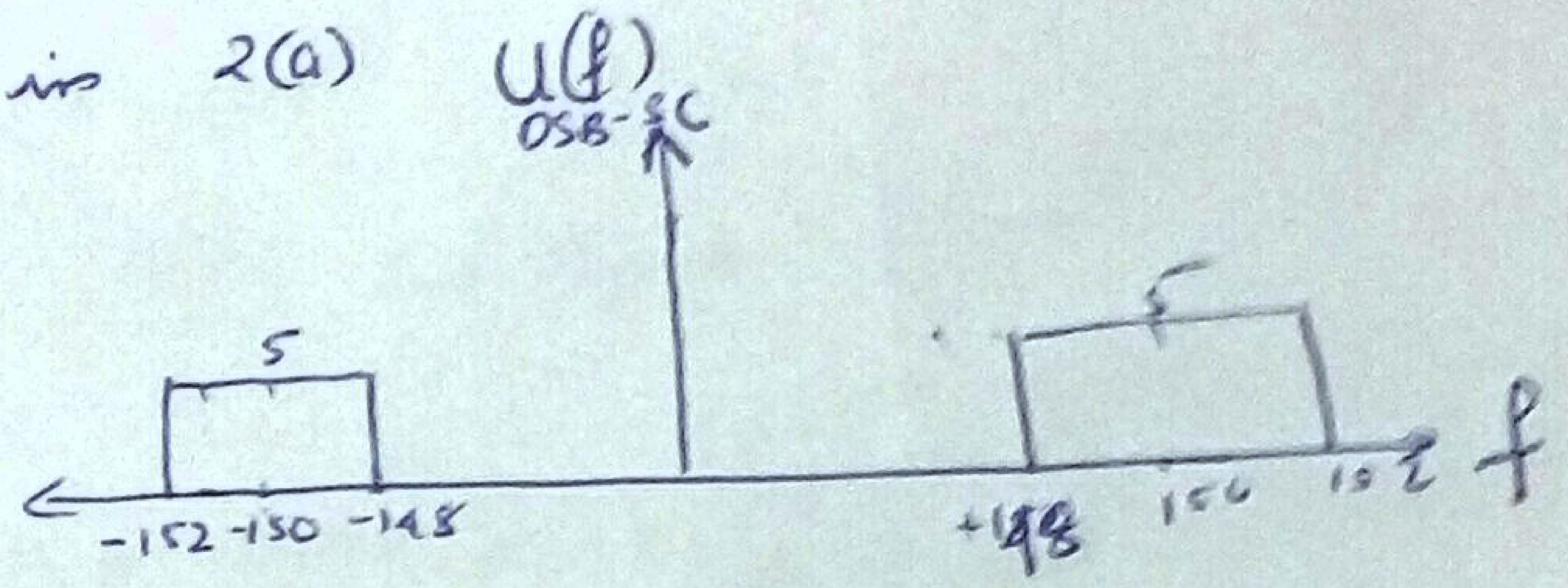
$\frac{1}{2} U_s(t) \xrightarrow{FT}$

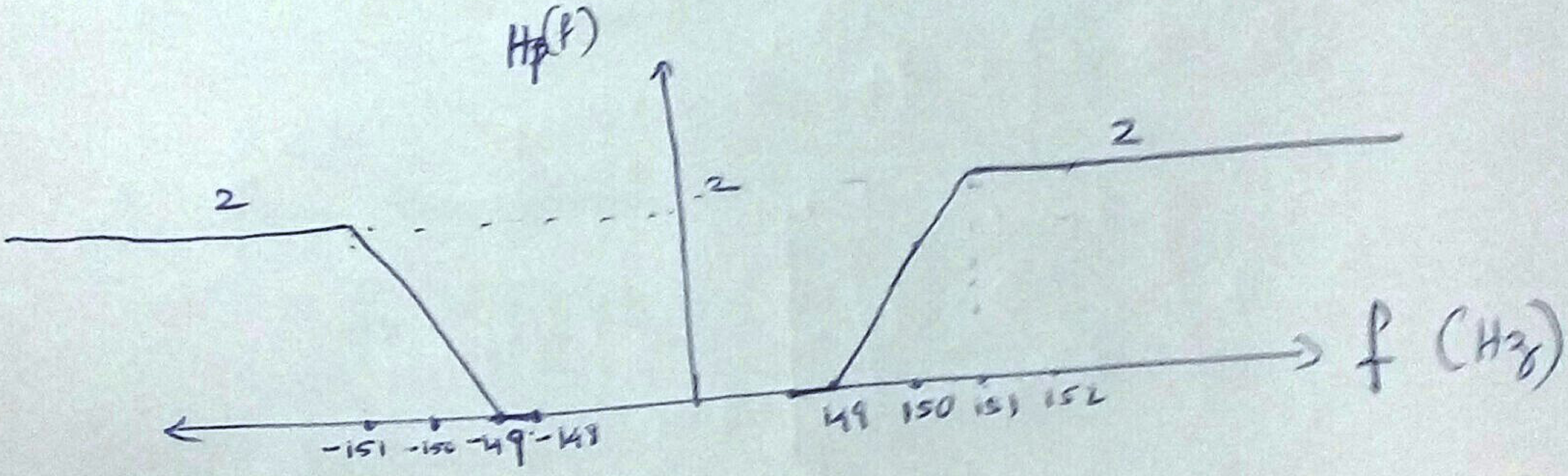


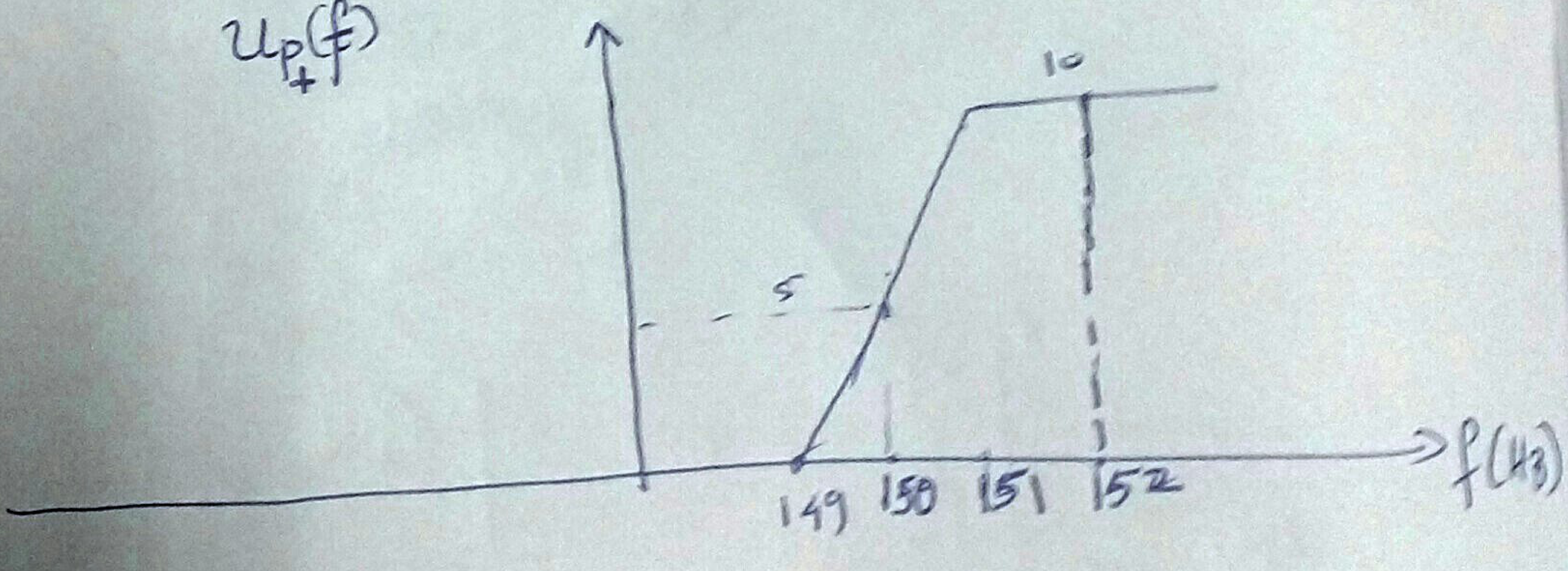
$U_s(t) = -20 \operatorname{sinc}(2t) \sin(2\pi t)$

2 e)

Signal in 2(a)

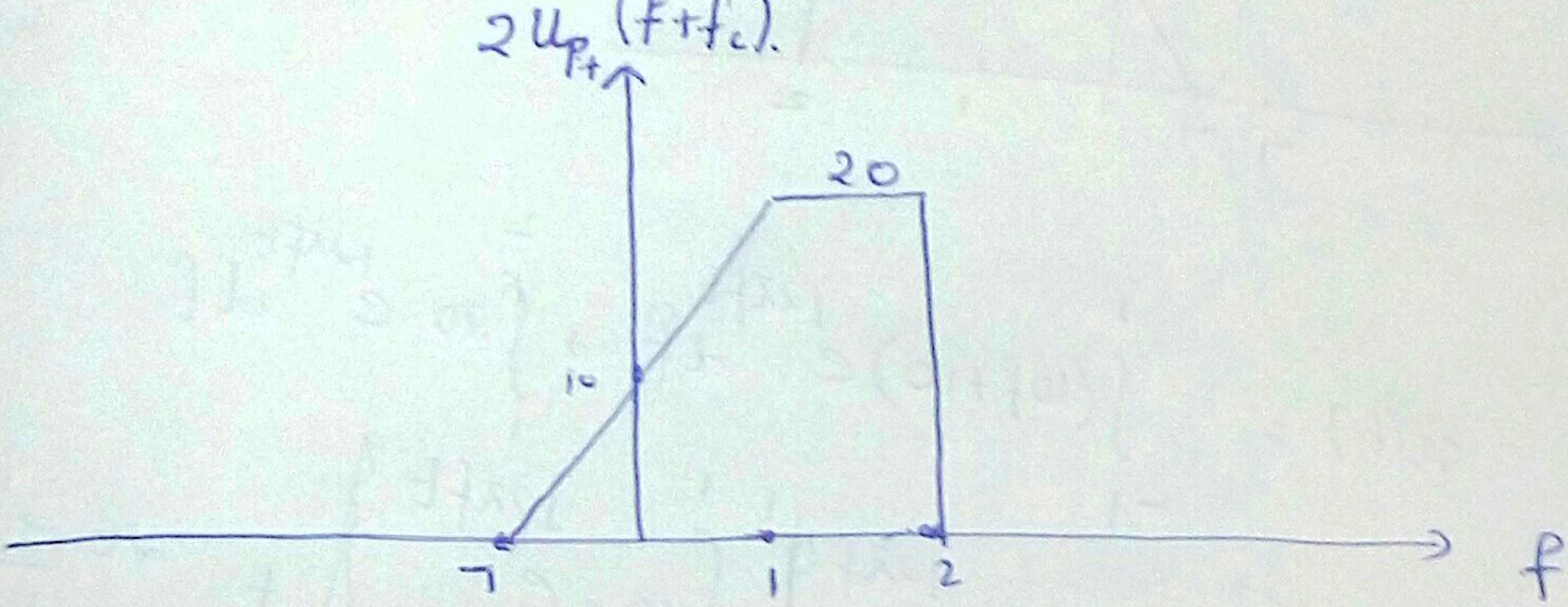






To find $u_c(t)$ & $u_s(t)$

Consider the signal



Complex base band signal

$$u(t) = u_c(t) + j\,u_s(t)$$

$$u(t) = \int_{-\infty}^{\infty} 2\,U_{P+}(f+f_c)\, e^{j2\pi f t}$$

$$= \frac{20}{\pi t}\sin 4\pi t + j\left[\frac{20}{(2\pi t)^2}\sin 2\pi t - \frac{20}{2\pi t}\cos 4\pi t\right]$$

$$\therefore \quad u_c(t) = \frac{20}{\pi t}\sin 4\pi t$$

$$u_s(t) = \frac{20}{(2\pi t)^2}\sin 2\pi t - \frac{20}{2\pi t}\cos 4\pi t$$

3) (a) from the given graph

$\theta(t) = 2.77 \sin 2\pi f_m t$

$= \frac{A_m K_f}{f_m} \sin 2\pi f_m t$

$\therefore \beta = \cancel{2.77} \frac{A_m k_f}{f_m} = 2.77.$

(b). from the graph

$f_m = 5 KHz$

(c) $B.W = 2 f_m (\beta + 1)$

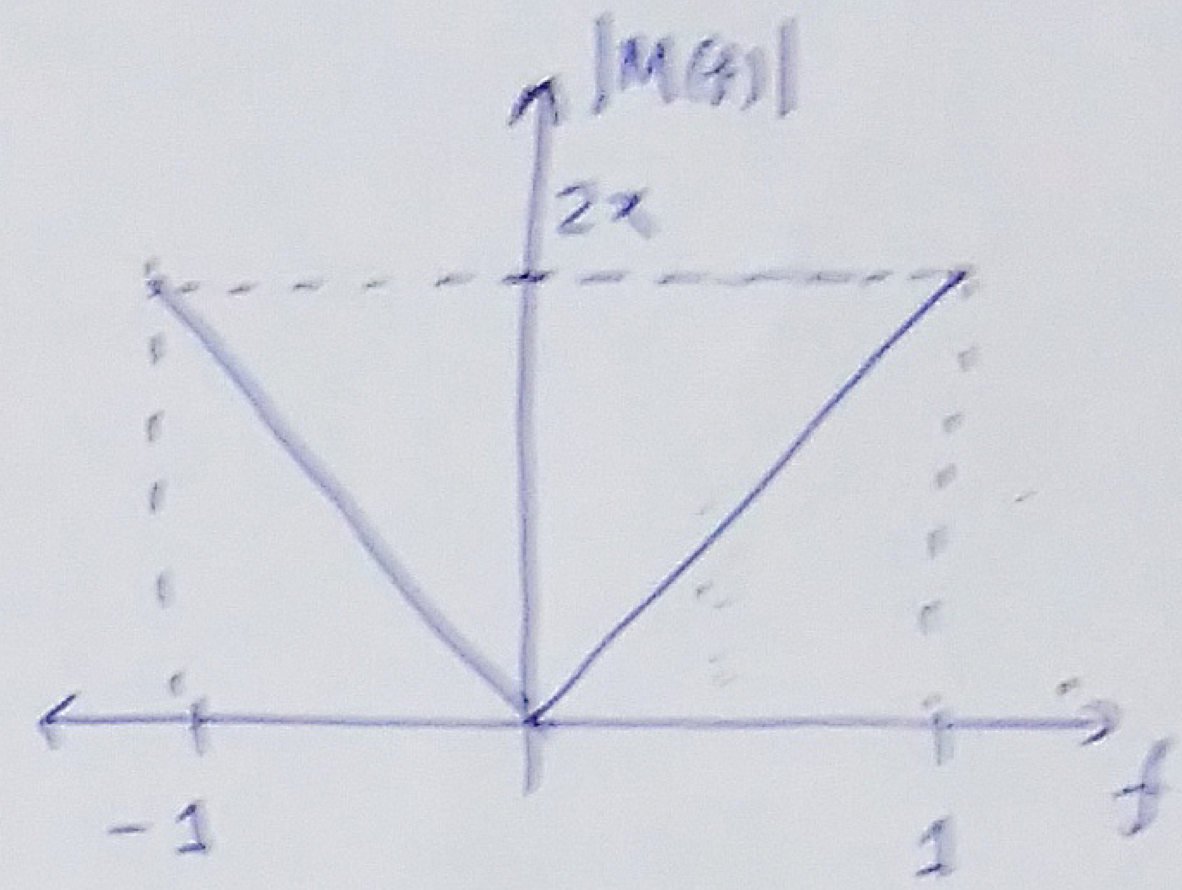
$= 2 \times 5 \times 3.77$

$= 37 KHz.$

$\boxed{4a}$

$$M(f) = \begin{cases} j2\pi f & |f| < 1 \\ 0 & \text{else.} \end{cases}$$

$$u(t) = A\cos\left(2\pi f_c t + \phi(t)\right)$$



$$\frac{1}{2\pi}\frac{d}{dt}\phi(t) = k_f\, m(t).$$

$$= m(t).$$

$$\phi(t) = \phi(0) + 2\pi k_f \int_0^t m(\tau).d\tau.$$

$$= 2\pi \int_0^t m(\tau).d\tau. \qquad (\phi(0) = 0\ ;\ k_f = 1;)$$

$$m(t) = \int_{-\infty}^{\infty} M(f).e^{j2\pi ft}.df = \int_{-1}^{1} j2\pi f.e^{j2\pi tf}.df = j2\pi\int_{-1}^{1} f.e^{j2\pi tf}.df$$

$$= j2\pi\left[\left[\frac{f}{j2\pi t} - \frac{1}{(j2\pi t)^2}\right]e^{j2\pi tf}\right]_{-1}^{1} = j2\pi\left[\left(\frac{+1}{j2\pi t} - \frac{1}{(j2\pi t)^2}\right)e^{j2\pi t} - \left(\frac{-1}{j2\pi t} - \frac{1}{(j2\pi t)^2}\right)e^{-j2\pi t}\right]$$

$$= j2\pi\left[\frac{1}{j2\pi t}\left(e^{j2\pi t} + e^{-j2\pi t}\right) - \left\{\frac{1}{4\pi^2 t^2}\left(e^{j2\pi t} - e^{-j2\pi t}\right)\right]\right.$$

$$= \frac{1}{t}\left(e^{j2\pi t} + e^{-j2\pi t}\right) + \frac{1}{j2\pi t^2}\left(e^{j2\pi t} - e^{-j2\pi t}\right)$$

$$= 2\frac{\cos 2\pi t}{t} + \frac{1}{\pi t^2}\sin 2\pi t$$

$$\phi(t) = 2\pi\int_0^t m(\tau).d\tau = 2\pi\int_0^t\left(\frac{2}{\tau}\cos 2\pi\tau + \frac{1}{\pi\tau^2}\sin 2\pi\tau\right).d\tau.$$

$$m(t) = \frac{2\cos 2\pi t}{t} + \frac{\sin 2\pi t}{\pi t^2}$$

$$t = \frac{1}{4}$$

$$m\left(\tfrac{1}{4}\right) = 0 + \frac{16}{\pi}$$

since, $f = f_c + K_f\, m(t)$ and $K_f = 1$

frequency deviation $= K_f\, m(t)$

$$\text{freq dev}\Big|_{t=\frac{1}{4}} = \frac{16}{\pi}\ Hz$$

4) c) max frequency deviation $\approx \frac{16}{\pi}\ Hz$

$$BW = 2 \cdot \frac{16}{\pi} = \frac{32}{\pi}\ Hz.$$