1

Time Avg: of a function $g(t)$:

$$\bar{g} = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} g(t)\, dt.$$

Average over a period $\bar{g}_p = \frac{1}{T_p} \int_0^{T_p} g(t)\, dt.$
($T_p \to$ period of $g(t)$)

Consider $y(T) = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} g(t)\, dt$ for some T.

The interval $[-\frac{T}{2}, \frac{T}{2}]$ can be split into some 'K' intervals of periods plus some residual interval of length say 'l'. ($< T_p$). , i.e $T = KT_p + l$. for some integer K, . $0 < l < T_p$.

Assuming $\left| \int_0^{l} g(t) \right| < A$, $\forall\, 0 < l < T_p$, we can bound $y(T)$ as, $\frac{1}{T}\left(K \int_{-\frac{T}{2}}^{\frac{T}{2}-l} g(t) + \int_{\frac{T}{2}-l}^{\frac{T}{2}} g(t)\right)$

$$\frac{K(\bar{g}_p T_p) - A}{KT_p + T_p} < y(T) < \frac{K(\bar{g}_p T_p) + A}{KT_p + 0}$$

Taking $T \to \infty$ ($\Rightarrow k \to \infty$) we get,

$$\bar{g}_p \leq \lim y(T) \leq \bar{g}_p \Rightarrow \bar{g} = \bar{g}_p$$ //

Tutorial 1 : Question 2.A

Given $y(t) = \int_{-\infty}^{t} e^{u-t} x(u)\ du = \int_{-\infty}^{\infty} e^{u-t}\mathcal{U}(t-u)x(u)\ du$, where $\mathcal{U}$ is the unit step function.
a) Let the transfer function be $h(t)$. $h(t) * x(t) = \int_{-\infty}^{\infty} h(t-u)x(u)\ du$. Comparing, we have $h(t-u) = e^{-(t-u)}\mathcal{U}(t-u)$. Hence, $h(t) = e^{-t}\mathcal{U}(t)$. Now since $y(t) = h(t) \star x(t)$, the system is LTI.

Tutorial-1

Q.(2.b): From 2.a, $h(t) = e^{-t}U(t)$.

$$\therefore H(f) = \int_{-\infty}^{\infty} h(t) \cdot e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-t}U(t) \cdot e^{-j2\pi ft}\, dt$$

$$= \int_{0}^{\infty} e^{-(1+j2\pi f)t} \cdot dt$$

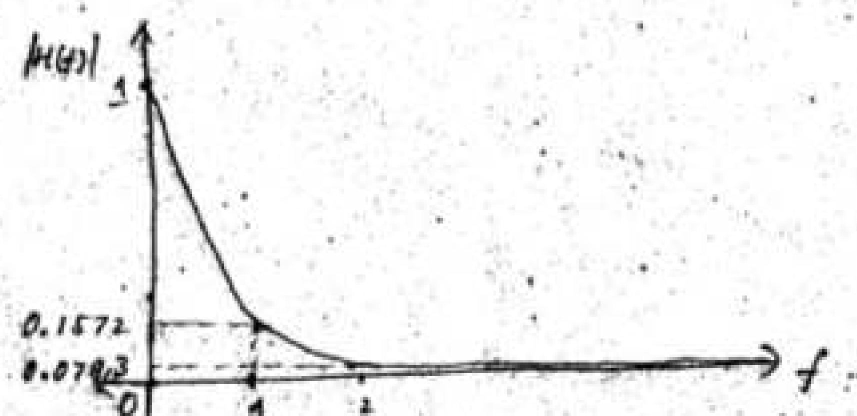
$$= \left[\frac{-e^{-(1+j2\pi f)t}}{1+j2\pi f} \right]_0^{\infty}$$

$$= \frac{1}{1+j2\pi f}$$

$$|H(f)| = \left| \frac{1}{1+j2\pi f} \right| = \left| \frac{1-j2\pi f}{(1+j2\pi f)(1-j2\pi f)} \right|$$

$$= \left| \frac{1-j2\pi f}{1+4\pi^2 f^2} \right| = \frac{1}{1+4\pi^2 f^2}\sqrt{1+4\pi^2 f^2}$$

$$= \frac{1}{\sqrt{1+4\pi^2 f^2}}$$

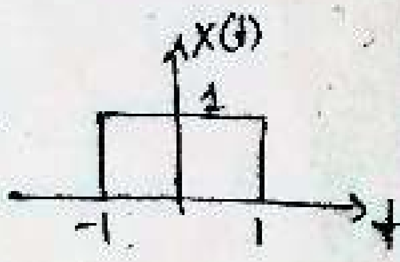


2c) $h(t) = e^{-t} u(t)$

$\Rightarrow H(f) = \frac{1}{1+j\omega}$

$Y(f) = H(f) X(f)$

but $x(t) = 2\frac{\sin 2t\pi}{2\pi t} \Rightarrow X(f) =$



energy of output $y(t) = \int_{-\infty}^{\infty} |y(t)|^2 dt$

we can use parseval's theorem here,

$$E_{y(t)} = \int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(\omega)|^2 d\omega.$$

$$E_{y(t)} = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} \frac{1}{1+\omega^2} d\omega$$

$$= \frac{1}{2\pi} \tan^{-1}\omega \Big|_{-2\pi}^{2\pi}$$

$$= \frac{1}{2\pi} 2\tan^{-1} 2\pi = \frac{\tan^{-1} 2\pi}{\pi}.$$

Problem: 3) a) $x_1(t) = e^{-t} I_{[0,\infty]}(t)$

$$x_2(t) = x_1(-t) = e^{t} I_{[0,\infty]}(-t)$$

$$y(t) = x_1 * x_2$$

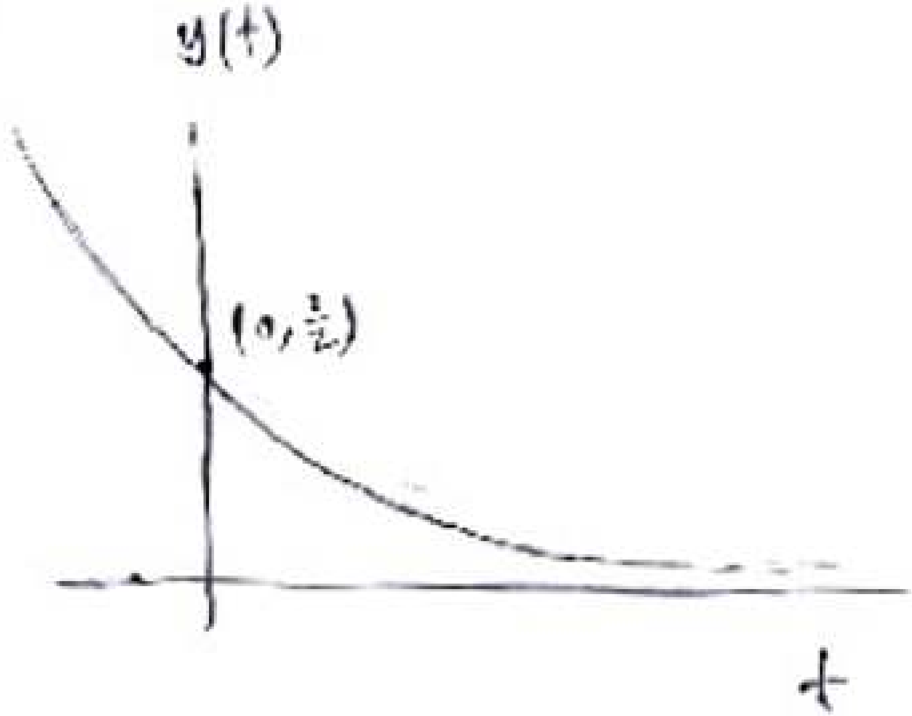
$$= \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau} I_{[0,\infty]}(\tau)\, e^{t-\tau} I_{[0,\infty]}(\tau - t)\, d\tau$$

$$= e^{t} \int_{t}^{\infty} e^{-2\tau}\, d\tau$$

$$= e^{t} \left. \frac{e^{-2\tau}}{-2} \right|_{t}^{\infty}$$

$$= \frac{e^{-t}}{2}$$



3(b)

Given $x_1(t) = I_{[0,2]}(t) - 3I_{[1,4]}(t)$

$$x_2(t) = I_{[0,1]}(t)$$

$$x_1(t) * x_2(t) = \underbrace{\left(I_{[0,2]} * I_{[0,1]}\right)(t)}_{\mathrm{I}} - 3\underbrace{\left(I_{[0,1]} * I_{[1,4]}\right)(t)}_{\mathrm{II}}$$

I

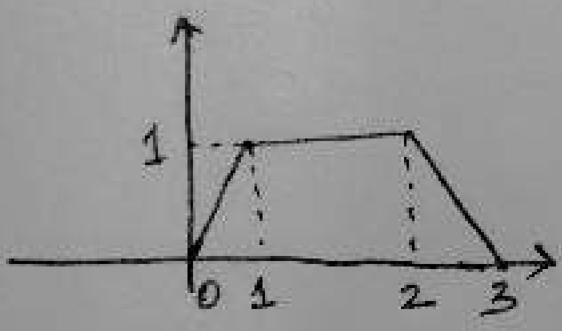
$\left(I_{[0,2]} * I_{[0,1]}\right)(t)$

By using LTI property

$= \underbrace{\left(I_{[-1,1]} * I_{[-1/2, 1/2]}\right)}(t - 3/2)$

↳ Refer Example in textbook.

=



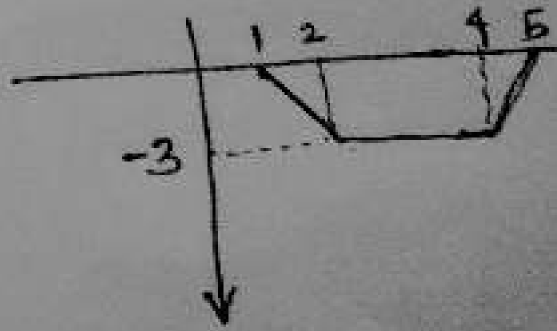
∵ Convolution of rectangular pulses results in trapezoidal pulse.

II

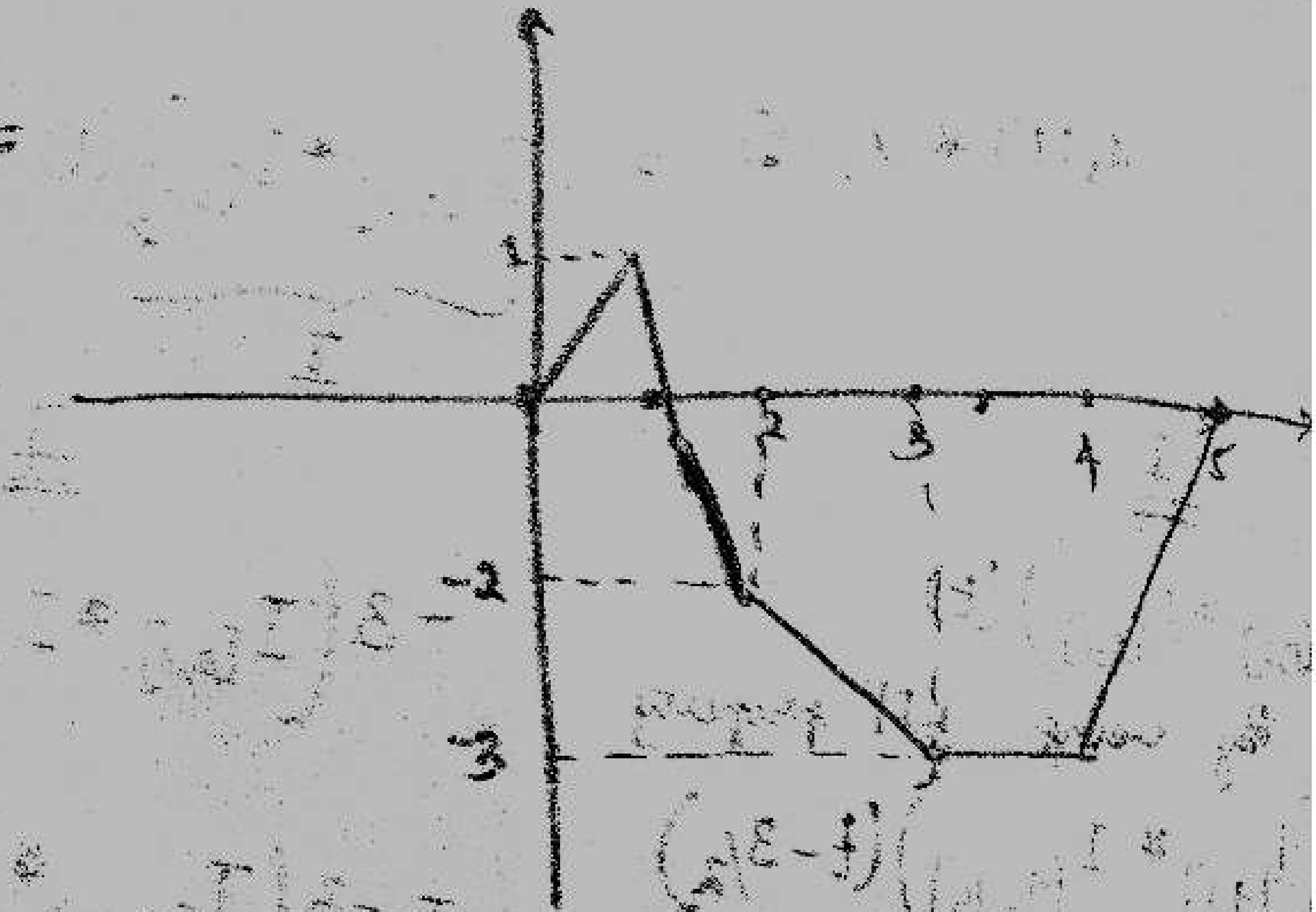
$-3\left(I_{[0,1]} * I_{[1,4]}\right)(t)$

$= -3\left(I_{[-1/2, 1/2]} * I_{[-2,2]}\right)(t - 7/2)$

=



I + II



4a) $U_t = \begin{cases} 1 ; 0 \le t \le 0.1 \\ 0 ; 0.1 \le t \le 0.5 \end{cases}$ $\quad \omega_0 = \frac{2\pi}{0.5}$

$U'_t = \delta(t) - \delta(t-0.1)$

a) Let Fourier series of U'_t C'_n

$C'_0 = 0$

$C'_n = \frac{1}{0.5} \int_0^{0.5} U'_t \, e^{-j\omega_0 n t} \, dt$

$= 2 \int_0^{0.5} (\delta(t) - \delta(t-0.1)) \, e^{-j\omega_0 n t} \, dt$

$\Rightarrow C'_n = 2\left(1 - \left(\exp\left(-j\frac{2\pi}{5}\right)\right)^n\right) ; n \neq 0$

b) Let C_n be the fourier series of $u(t)$

$C_n = \frac{C'_n}{jn\omega_0}$ for $n \neq 0$

$C_0 =$ DC value of U_t $= 0.2$

$C_n = \frac{2}{jn\omega_0} \left[1 - \left(e^{-j\frac{2\pi}{5}}\right)^n\right]$

$= \frac{1}{2\pi j n} \left[1 - \left(e^{-j\frac{2\pi}{5}}\right)^n\right] ; n \neq 0$

4) (c) $u(t)$ is a signal of period $T = .5\ \mu sec$.

$T = .5 \times 10^{-6}$ sec

$\Rightarrow f_0 = 2000$ kHz is the fundamental frequency

So $u(t)$ is a signal composed of a DC Term + components at Integer multiples of the fundamental frequency.

$u(t) \longrightarrow$ [LPF] $\longrightarrow$ DC Term only.

100 kHz

($\because$ Every other component has a frq. > 100 kHz)

So output $\underline{y(t) = .2 \quad \forall t}$

1) (d) Time period of $x(t)$, $T = 0.5\,\mu sec$

$\Rightarrow$ fundamental frequency, $f_0 = \frac{1}{T} =$

$= 2000\,KHz$

The signal is contains frequencies $0, nf_0$ where n is an integer.

frequencies present are 0, $2000\,KHz$, $4000\,KHz$, ... etc

Since the cut off frequency of L.P.F = $300\,KHz$.

$\therefore$ the output only contains ~~0 frequencies~~ dC component (ie component corresponding to '0 frequency)

D.C component of the signal $= C_0$

$$C_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$$= \frac{1}{0.5}\int_0^{0.5} x(t)\,dt$$

$$= \frac{1}{0.5}\int_0^{0.1} 1\,dt$$

$$= \underline{\underline{0.2}}$$