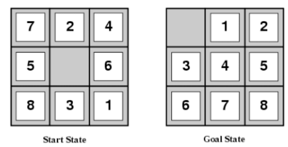
Local Search

Search so far..

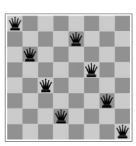
- A*, BFS, DFS etc
  – Given set of states, get to goal state



  – Need to know the *path* as well

Example: *n*-queens

- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal



- How would you *represent* the state space of this problem?
- How is the problem different from the 8-puzzle?

Local search algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution
- State space = set of "complete" configurations
- Find configuration satisfying constraints, e.g., n-queens
- In such cases, we can use local search algorithms
- Keep a single "current" state, try to improve it
  – Assume access to a function, Eval(x) that tells you how good X is

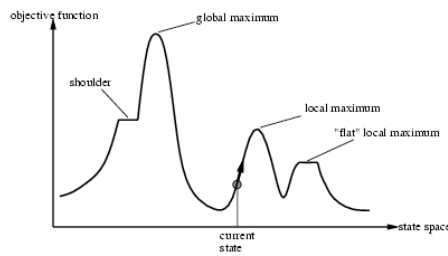
Hill-climbing search

- "Like climbing Everest in thick fog with amnesia"

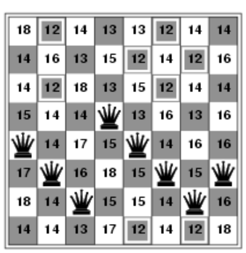
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node
   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
       neighbor ← a highest-valued successor of current
       if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
       current ← neighbor
```

Hill-climbing search

- Problem: depending on initial state, can get stuck in local maxima

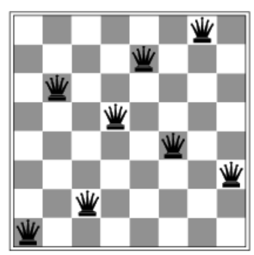


Hill-climbing search: 8-queens problem



- *h* = number of pairs of queens that are attacking each other, either directly or indirectly
- *h = 17* for the above state

Hill-climbing search: 8-queens problem



- A local minimum with *h = 1*

Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency

```
function SIMULATED-ANNEALING( problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
      T ← schedule[t]
      if T = 0 then return current
      next ← a randomly selected successor of current
      ΔE ← VALUE[next] – VALUE[current]
      if ΔE > 0 then current ← next
      else current ← next only with probability e^(Δ E/T)
```

Properties of simulated annealing search

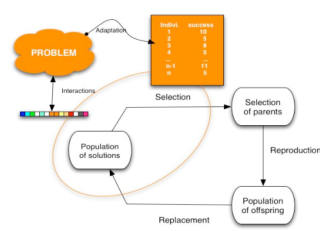
- One can prove: If T decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1
- Widely used in VLSI layout, airline scheduling, etc

Local beam search

- Keep track of k states rather than just one
- Start with k randomly generated states
- At each iteration, all the successors of all k states are generated
- If any one is a goal state, stop; else select the k best successors from the complete list and repeat.

Genetic algorithms

- Variant of local beam search with *sexual recombination.*



Genetic Algorithms

- View optimization by analogy with evolutionary theory → Simulation of natural selection
- View configurations as *individuals* in a *population*
- View *Eval* as a measure of *fitness*
- Let the least-fit individuals die off without reproducing
- Allow individuals to *reproduce* with the best-fit ones selected more often
- Each *generation* should be overall better fit (higher value of Eval) than the previous one
- If we wait long enough the population should evolve so toward individuals with high fitness (i.e., maximum of *Eval*)

Genetic Algorithms: Implementation

- Configurations represented by strings:

X = | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |

- Analogy:
  - The string is the chromosome representing the individual
  - String made up of genes
  - Configuration of genes are passed on to offsprings
  - Configurations of genes that contribute to high fitness tend to survive in the population
- Start with a random population of *P* configurations and apply two operations
  - *Reproduction*: Choose 2 "parents" and produce 2 "offsprings"
  - *Mutation*: Choose a random entry in one (randomly selected) configuration and change it

Genetic Algorithms: Reproduction

Parents:

| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |

| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |

Genetic Algorithms: Reproduction

Parents:

| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |

| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |

Select random crossover point:

| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |

| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |

Genetic Algorithms: Reproduction

Parents:

1 0 0 1 1 0 0 1

1 0 1 1 0 0 0 1

Select random crossover point:

1 0 0 | 1 1 0 0 1

1 0 1 | 1 0 0 0 1

Offspring: 1 0 0 1 0 0 0 1   1 0 1 1 1 0 0 1

- Each offspring receives part of the genes from each of the parents
- Implemented by crossover operation

Genetic Algorithms: Mutation

- Random change of one element in one configuration
  - →Implements random deviations from inherited traits
  - →Corresponds loosely to “random walk”: Introduce random moves to avoid small local extrema

1 0 0 1 1 0 0 1

1 0 0 1 0 0 0 1   1 1 1 1 1 0 0 1   1 1 0 1 1 0 0 1

1 1 1 1 1 0 0 1

Select a random individual   Select a random entry   Change that entry

Basic GA Outline

- Create initial population $X = \{X_1,..,X_P\}$
- Iterate:
  1. Select K random pairs of pa
  2. For each pair of parents
     1.1 Generate offsprings (Y_1
  r each offspring Y_i:
  Replace randomly selected element of the population by Y_i
  With probability μ:
  Apply a random mutation to Y_i
- Return the best individual in the population

Stopping condition is not obvious?

Possible strategy: Select the best rP individuals ($r < 1$) for reproduction and discard the rest → Implements selection of the fittest

Variation: Generate only one offspring

Genetic Algorithms: Selection

- Discard the least-fit individuals through threshold on *Eval* or fixed percentage of population
- Select best-fit (larger *Eval*) parents in priority
- Example: Random selection of individual based on the probability distribution

$$\Pr(\text{individual } X \text{ selected}) = \frac{Eval(X)}{\sum_{Y \in population} Eval(Y)}$$

- Example (*tournament*): Select a random small subset of the population and select the best-fit individual as a parent
- Implements “survival of the fittest”
- Corresponds loosely to the greedy part of hill–climbing (we try to move uphill)

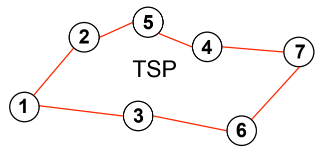
GA and Hill Climbing

- Create initial population $X = \{X_1,..,X_P\}$
- Iterate:
  1. Select K random
  2. For each pair of parents (X, X'):
     1.1 Generate offsprings (Y_1, Y_2) using crossover operation
     offspring Y_i:
     randomly selected element of the
     by Y_i
     With probability μ:
     Apply a random mutation to Y_i
- Return the best individual in the population

Hill-climbing component: Try to move uphill as much as possible

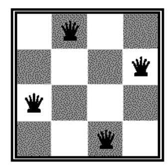
Random walk component: Move randomly to escape shallow local maxima

How would you set up these problems to use GA search?

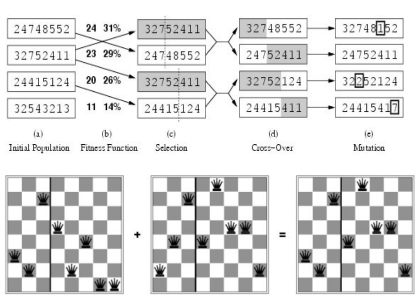


SAT
$A \lor \neg B \lor C$
$\neg A \lor C \lor D$
$B \lor D \lor \neg E$
$\neg C \lor \neg D \lor \neg E$
$\neg A \lor \neg C \lor E$
………

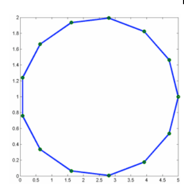
N-Queens



GA for N-queens



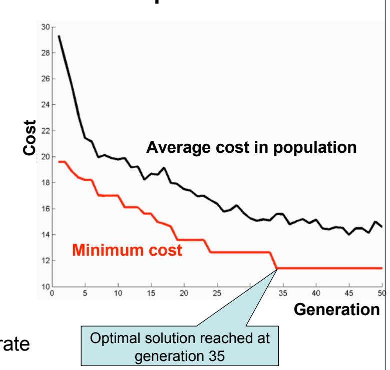
TSP Example

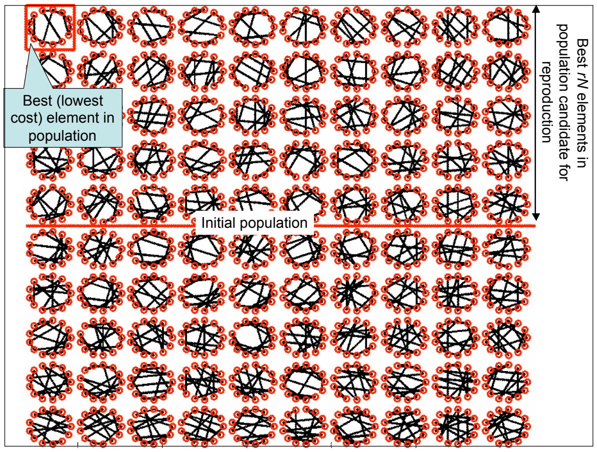


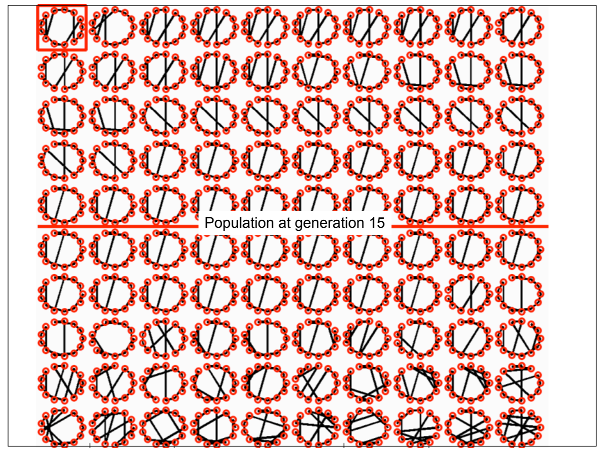
$N = 13$

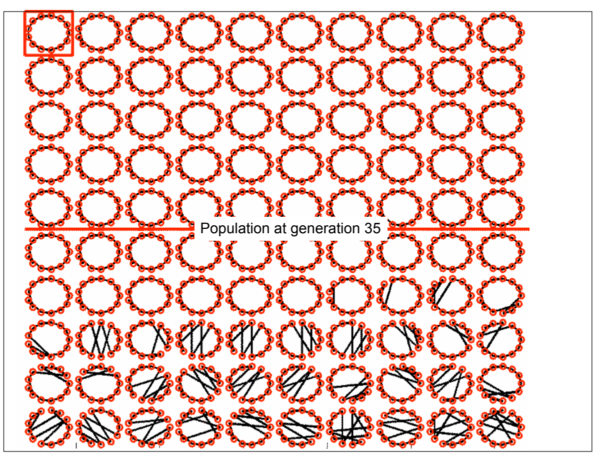
P = 100 elements in population

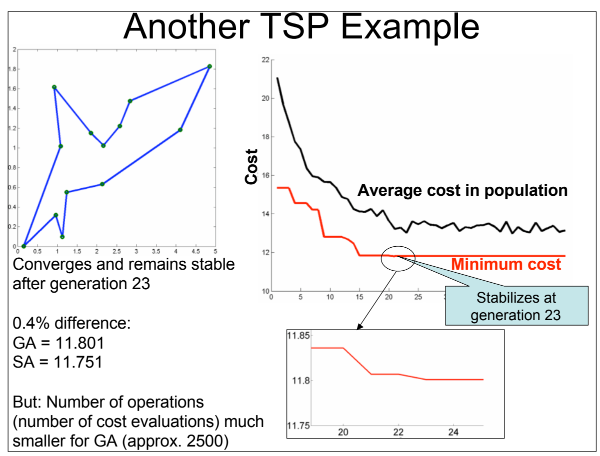
μ = 4% mutation rate
r = 50% reproduction rate
(K = rP)




Best (lowest cost) element in population
Initial population
Best r/N elements in population candidate for reproduction


Population at generation 15


Population at generation 35


Another TSP Example
Cost
Average cost in population
Minimum cost
Stabilizes at generation 23
Converges and remains stable after generation 23
0.4% difference:
GA = 11.801
SA = 11.751
But: Number of operations (number of cost evaluations) much smaller for GA (approx. 2500)

GA Discussion

- Many parameters to tweak: μ, P, r
- Many variations on basic scheme. Examples:
  - Multiple-point crossover
  - Dynamic encoding
  - Selection based on rank or relative fitness to least fit individual
  - Multiple fitness functions
  - Combine with a local optimizer (for example, local hill-climbing) → Deviates from "pure" evolutionary view
- In many problems, assuming correct choice of parameters, can be surprisingly effective

GA Discussion

- Why does it work at all?
- Limited theoretical results (informally!):
  - Suppose that there exists a partial assignment of genes *s* such that:

$$\underset{X \text{ contains } s}{\text{Average of }} Eval(X) \geq \underset{Y \in \text{Population}}{\text{Average of }} Eval(Y)$$

  - Then the number of individuals containing *s* will increase in the next generation
- Key consequence: The design of the representation (the chromosomes) is critical to the performance the GA. It is probably more important than the choice of parameters of selection strategy, etc.