Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

In [13]:
```
import pandas as pd
import numpy as np
import scipy
from scipy import stats
```

In [14]:
```
z=(50-45)/8
print('Probability that the Service Manager will not meet his commitment',np.round(1-stats.norm.cdf(z),5))
```

```
Probability that the Service Manager will not meet his commitment 0.26599
```

Hence, correct option is B.

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

In [19]:
```
#We'll find probability of employees having age greater than 44
z_score1=(44-38)/6
p1=1-stats.norm.cdf(z_score1)
#We'll now find probability of employees having age between 38 - 44
z_score2=(38-38)/6
p2=1-(p1+(1-stats.norm.cdf(z_score2)))
if p1>p2:
    print('Given statement is true')
else:
    print('Given statement is false')
```

```
Given statement is false
```

In [47]:
```
#We'll find probability of employees having age under 30
z_score3=(30-38)/6
p3=1-stats.norm.cdf(np.abs(z_score3))
#To find number of employees under age of 30
N=np.round(((400*p3)/100)*100)
if N==36:
    print('Given statement is true')
else:
    print('Given statement is false')
```

```
Given statement is true
```

Hence,

A) False
B) True

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Answer:

We know that,

if X1, X2, . . . , Xn are independent, and Xi ~ Normal(μi , σi^2) for i = 1, . . . , n,

then a1X1+a2X2+. . .+anXn ~ Normal [(a1μ1+. . .+anμn), (a1^{2}σ1^2 +. . .+an^{2}σn^2)]

Also, X1+X2+. . .+Xn ~ Normal [(μ1+μ2+. . .+ μn),(σ1^2+ σ2^2+. . .+ σn^2)]

Hence,

2X1 = (2μ, 4σ^2)

X1+X2 = N(2μ, 2σ^2)

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

```
In [92]: #Given That
         μ=100
         σ=20
         #Z value at 0.1 percentile
         z1=stats.norm.ppf(0.005)
         #Z value at 0.99 percentile
         z2=stats.norm.ppf(0.995)
```

```
In [93]: #We know z=(x-μ)/σ, hence, x=z*σ+μ
         a=z1*20+100
         b=z2*20+100
         print('a = ',a)
         print('b = ',b)
```

```
a =  48.483413929021985
b =  151.516586070978
```

Hence, correct option is D.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit$_1$ ~ N(5, 3^2) and Profit$_2$ ~ N(7, 4^2) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5th percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

In [97]:
```
#Mean profit from different divisions of a company
Mean = 5+7
print('Mean profit in Rupees is ',Mean*45,' Millions')
#Variance of profits from different divisions of a company=SD**2=SD1**2+SD2**2
SD=np.sqrt((9)+(16))
print('Standard Deviation in Rupees is ',SD*45,' Millions')
```

```
Mean profit in Rupees is  540  Millions
Standard Deviation in Rupees is  225.0  Millions
```

In [98]:
```
# A. To specify a rupee range such that it contains 95% probability for annual profit of company
print('Range is ',stats.norm.interval(0.95,540,225),'Millions')
```

```
Range is  (99.00810347848784, 980.9918965215122) Millions
```

In [102]:
```
# B. To specify 5th percentile of profit, we'll use x = z*σ + μ
# Calculating z for 5th percentile
z5=stats.norm.ppf(0.05)
print('5th Percentile of profit in Rupees is ',np.round(z5*225+540),' Millions')
```

```
5th Percentile of profit in Rupees is  170.0  Millions
```

In [106]:
```
# C. Probability of making loss i.e. P(X<0)
#Probability of division 1 making loss
Div1=stats.norm.cdf(0,5,3)
#Probability of division 2 making loss
Div2=stats.norm.cdf(0,7,4)
if Div1>Div2:
    print('Division 1 have more probability of making loss than the division 2')
else:
     print('Division 2 have more probability of making loss than the division 1')
```

```
Division 1 have more probability of making loss than the division 2
```

Hence,

A) Range is (99.00810347848784, 980.9918965215122) Millions

B) 5th Percentile of profit in Rupees is 170.0 Millions

C) Division 1 have more probability of making loss than the division 2