DeFi Lending Protocols: Liquidation Risk and the Carry Trade

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Introduction and Motivation

Compound is a lending market that offers several different ERC-20 assets for borrowing and lending. All the tokens in a single market are pooled together so every lender earns the same variable rate and every borrower pays the same variable rate. In contrast to centralized lending markets, variable rates are determined by demand and supply and based on pre-defined algorithms. Interest rate models are based on utilization, which is a measure of the net demand for asset. Currencies with higher utilization, and consequently lower excess reserves, have higher interest rates to compensate lenders. Borrowers are also required to post excess collateral in an asset different from the one being borrowed. Borrowers face liquidation risk. For example, if a borrower's loan increases relative to the posted collateral, the protocol seizes their collateral to pay off the loan. The borrower faces a liquidation penalty: they are forced to sell their collateral to the protocol at a 5% discount.

In this paper, we aim to provide a model to explain equilibrium pricing of interest rates, and pin down the fundamental sources of risk that explain the cross-section of interest rates. We also examine whether classic no-arbitrage conditions in currency markets, like uncovered and covered interest rate parity, and if there are profitable carry trade strategies.

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Research Hypotheses

H1: How are the policy parameters of interest rate rules determined?

The supply and borrow interest rates are compounded every block (approximately 15 seconds on Ethereum producing approximately continuous compounding) and are determined by the utilization percentage in the market. Utilization is calculated as total borrow/total supply. The utilization rate u is used as an input parameter to a formula that determines the interest rates. In Figure 1, we plot the interest rate model for currencies on the Compound platform. This plots borrowing rates as a function of the utilization percentage in the market. The interest rate model for borrowing rates is given by the piece-wise equation 1. a_0 is the base rate, and is the rate corresponding to zero utilization. The slope parameter b_0 measures the sensitivity of interest rates to utilization. A feature of interest rate model is the kink, in which the slope parameter changes for utilization above a threshold rate $\bar{u}$, typically 80 per cent. The kink makes interest rates more sensitive to a higher utilization rate, $b_1 > b_0$. This corresponds to the literature on modeling excess reserve balances with a logistic function in Veyrune et al. (2018). The authors find that in money markets the interest rate schedule becomes steeper when excess reserves are smaller. Excess reserves are the inverse of the utilization rate, and is consistent with the behaviour of the kink in the Compound interest rate model.

Parameters like the base-rate and slope of the interest rate model are chosen by voters as part of the governance protocol. For example, the objective of the bank is to reduce the probability of excess reserves falling to zero and the bank being unable to meet withdrawals; a classic bank run. Policy parameters can be set such that the expected cost of withdrawals exceeding excess reserves is minimized, while at the same time accruing expected profits to the bank in terms of maximizing the spread between borrowing and lending rates. For example, Poole (1968) models the reserve management practice of a commercial bank that has a stochastic withdrawal rate. In line with reserve management, interest rate rules for more risky assets have a higher base-rate and slope parameters. Therefore risky assets are more sensitive to a decline in excess reserves due to the larger expected cost of withdrawals.

$$r = \begin{cases} a_0 + b_0 u, u \leq \bar{u} \\ a_0 + b_0 \bar{u} + b_1(u - \bar{u}), u > \bar{u} \end{cases} \tag{1}$$

H2: What are the determinants of the cross-section of interest rates?

In Figure 2, we plot the cross-section of borrowing rates and the utilization rates on the Compound platform. An interesting observation is that stablecoins (USDT, USDC, DAI) typically have high interest rates, and unstable cryptocurrencies (ETH, WBTC, ZRX) typically have low interest rates. We hypothesize that differences between high and low interest rate currencies reflect differences in liquidation risk of the collateral. For example, borrowing in more volatile currencies like ETH is risky because fluctuations in the ETH price can cause the position to liquidate, if their loan falls relative to the posted collateral. Therefore in equilibrium, the utilization rate is lower for risky currencies. The interest rate model in equation 1 sets a borrowing rate that is linear in the utilization rate. A lower utilization means lower borrowing rates as well, based on the algorithm.

H3: Are DeFi lending markets efficient?

Standard no-arbitrage conditions in currency markets include uncovered and covered interest rate parity. Lending protocols have no term structure; interest rates are compounded at an intra-day frequency. A related condition we can test is the carry trade: can investors short low interest rate currencies and go long in high interest rate currencies and make systematic profits. Excess returns in the carry trade in currency markets are a function of crash risk Brunnermeier et al. (2008).

Figure 1: Interest Rate Model

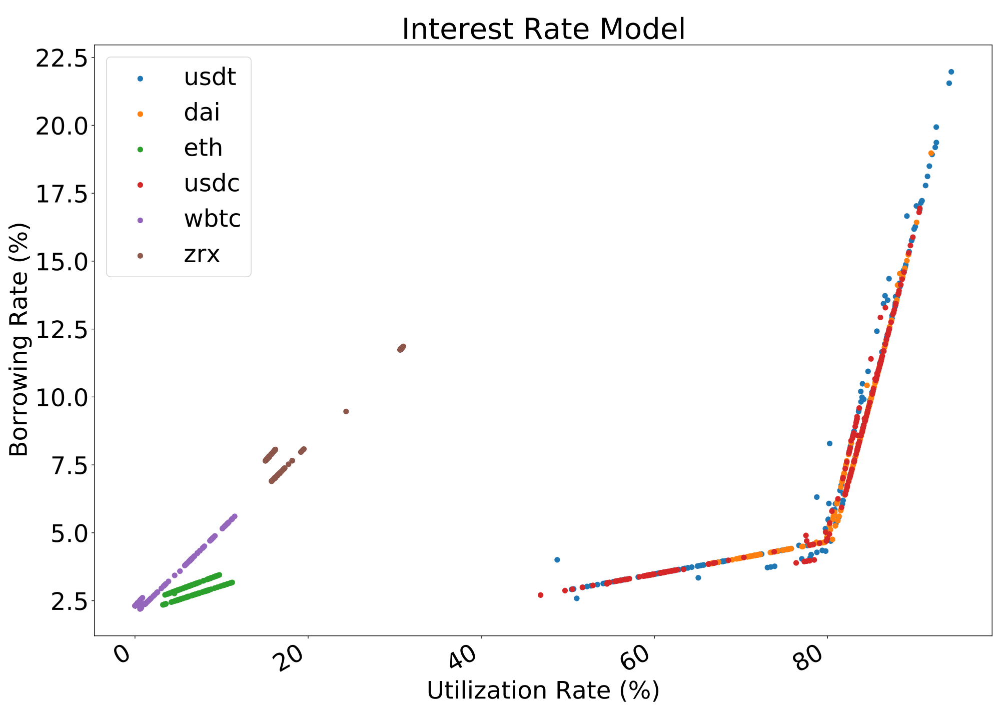
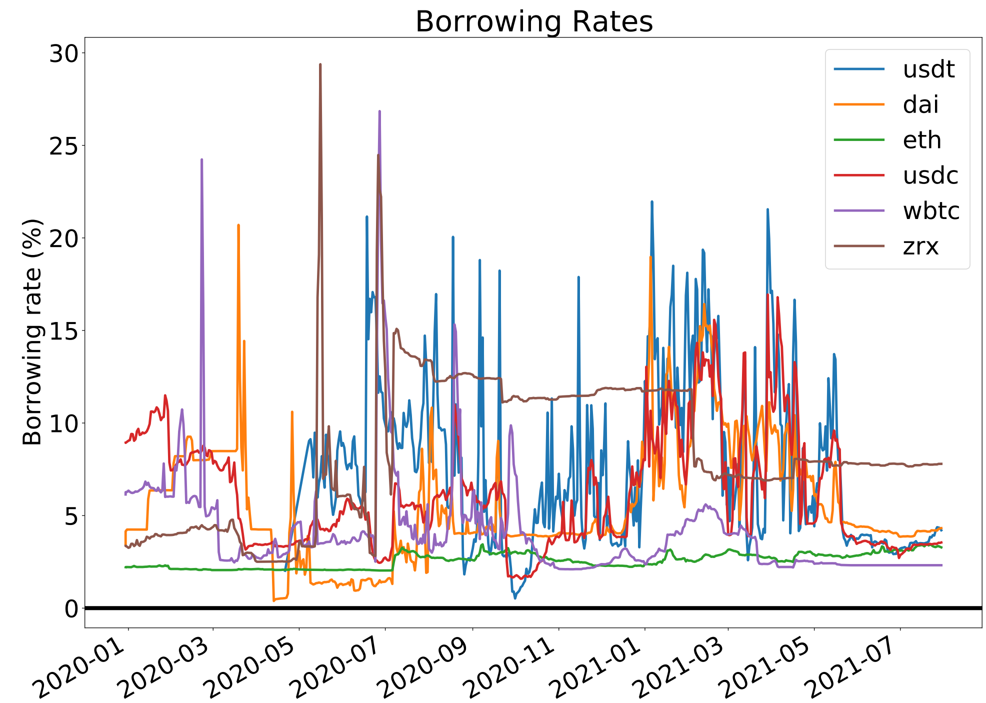
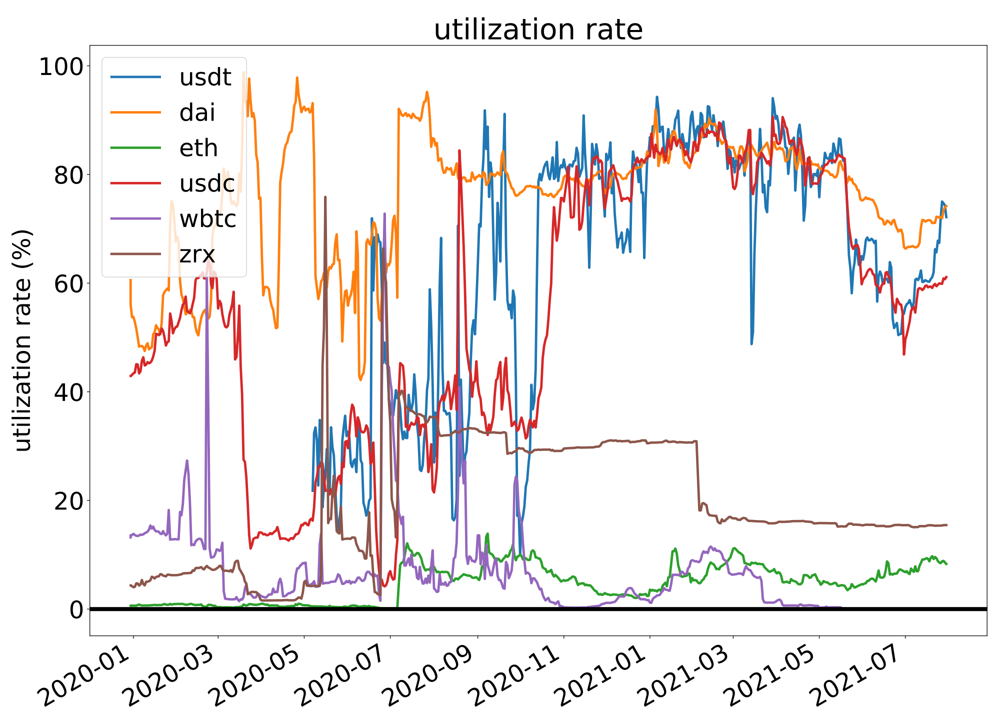


Figure 2: Borrowing Rates and Utilization Rates





Model Setup

- Consider a single currency case, with a fixed amount of deposits D. Choice variable is amount of loans L borrowed.
- DeFi lending protocol sets interest rates based on utilization $u = \frac{L}{D}$, which is the fraction of deposits that is borrowed. A high utilization indicates a low reserve buffer and means the bank is susceptible to a bank run (if, for example depositors decide to redeem a fraction (1-u) of deposits).
- Therefore the algorithm is set to adjust interest rates upwards when there is high utilization, in order to reduce utilization and increase the reserve buffer.
- Borrowing rates of the protocol are set as follows:

$$r_L = a + b\frac{L}{D}$$

- Consider a two period model [0,1] and risk-neutral borrower maximizing value of the loan less borrowing costs and cost of liquidation.
- The borrower needs to post sufficient collateral, $C = \theta L$, with $\theta > 1$ requiring borrowers to post collateral greater than the size of the loan.
- Define $\bar{L}$ as the maximum borrowings, i.e. the leverage limit the borrower can take. For example, we can assume that it is governed by the ratio of borrowings to collateral, so it is $\bar{L} = \theta L$, i.e. the loan must be over-collateralized.
- If the loan revalues due to appreciation of the currency, which we define as the relative price of the currency (in a numeraire eg. USD), then the borrower must pay a liquidation penalty. Formally, this is when the valuation of the loan exceeds the leverage limit, so when $L\frac{P_1}{P_0} - \bar{L} > 0$.
- The liquidation penalty r_P is approximately 5%.
- The maximization problem can be written as follows:

$$\max \quad V_1 = L(1 - r_L - E[\frac{P_1}{P_0}]) - r_P max(0, L\frac{P_1}{P_0} - \bar{L})$$

- subject to: $\bar{L} \leq \theta L$

- The solution is given as follows, by first substituting the algorithm for lending rates set by the protocol, $r_L = a + bL$

$$\max \quad V_1 = L(1 - a - b\frac{L}{D} - E[\frac{P_1}{P_0}]) - r_P max(0, L\frac{P_1}{P_0} - \bar{L})$$

$$\frac{\partial V}{\partial L} = 1 - a - 2b\frac{L}{D} - E[\frac{P_1}{P_0}] - r_P \times Prob\left(L\frac{P_1}{P_0} > \bar{L}\right) = 0$$

$$1 - a - 2b\frac{L}{D} - E[\frac{P_1}{P_0}] = r_P \times Prob\left(L\frac{P_1}{P_0} > \bar{L}\right)$$

$$r_L = b\frac{L}{D} = \frac{1 - a - E[\frac{P_1}{P_0}] - r_P \times Prob\left(L\frac{P_1}{P_0} > \bar{L}\right)}{2}$$

- We can think about the effect of an increase in liquidation risk on borrowing rates in the protocol.

- The intuition is simple, assuming the return follows a normal distribution, so $\frac{P_1}{P_0} \sim N(E[\frac{P_1}{P_0}], \sigma^2)$, we can write

$$\begin{aligned} r_L &= \frac{1 - a - r_P \times Prob\left(L\frac{P_1}{P_0} > \bar{L}\right)}{2} \\ &= \frac{1 - a - r_P \times Prob\left(\frac{P_1}{P_0} > \theta\right)}{2} \\ &= \frac{1 - a - r_P \times Prob\left(\frac{P_1}{P_0} > \theta\right)}{2} \\ &= \frac{1 - a - r_P \times Prob\left(\frac{\frac{P_1}{P_0} - E[\frac{P_1}{P_0}]}{\sigma} > \frac{\theta - E[\frac{P_1}{P_0}]}{\sigma}\right)}{2} \\ &= \frac{1 - a - r_P \times (1 - \Phi\left(\frac{\theta - E[\frac{P_1}{P_0}]}{\sigma}\right))}{2} \end{aligned}$$

- An increase in volatility σ increases the probability of a liquidation event, and reduces equilibrium borrowing L and therefore utilization $u = \frac{L}{D}$. Given the algorithm sets interest rates proportional to utilization, interest rates are lower for more risky currencies!

- Currencies that are expected to appreciate ($E[\frac{P_1}{P_0}] > 0$) also increase the probability of liquidation. This is a **debt deflation** problem, as investors have borrowed in a currency that has appreciated, the value of debt exceeds the value of the collateral, which for simplicity we keep fixed.[1]

UIP deviations and Carry trade

- The model considered one currency type. However, generalizing it to borrowing in multiple currencies (yes, Roman, I'm taking short-cuts here). We can consider borrowing rates in currencies i and j, assuming that $r_L^i = a + b\frac{L_i}{D_i}$ and $r_L^j = a + b\frac{L_j}{D_j}$, i.e. the parameters a and b are the same for each currency.

$$r_L^i = \frac{1 - a - E[\frac{P_1^i}{P_0^i}] - r_P \times Prob\left(L_i \frac{P_1^i}{P_0^i} > \bar{L}\right)}{2}$$

$$r_L^j = \frac{1 - a - E[\frac{P_1^j}{P_0^j}] - r_P \times Prob\left(L_j \frac{P_1^j}{P_0^j} > \bar{L}\right)}{2}$$

- Taking the difference in interest rates we obtain:

$$\begin{aligned} r_L^i - r_L^j &= \frac{-E[\frac{P_1^i}{P_0^i}] - r_P \times Prob\left(L_i \frac{P_1^i}{P_0^i} > \bar{L}\right) + E[\frac{P_1^j}{P_0^j}] + r_P \times Prob\left(L_j \frac{P_1^j}{P_0^j} > \bar{L}\right)}{2} \\ &= \frac{E[\frac{P_1^j}{P_0^j}] - E[\frac{P_1^i}{P_0^i}] + r_P \times \left(Prob\left(L_j \frac{P_1^j}{P_0^j} > \bar{L}\right) - Prob\left(L_i \frac{P_1^i}{P_0^i} > \bar{L}\right)\right)}{2} \end{aligned}$$

- The first term on the RHS reflects differences in expected appreciation of the two currencies. The second term reflects differences in liquidation risk across the two currencies.

- What about UIP deviations? Accounting for exchange rate changes, we get the following, where we approximate the exchange rate change between the two currencies as

$$r_L^i - r_L^j + E\left[\frac{P_1^i P_0^j}{P_0^i P_1^j}\right] = E\left[\frac{P_1^i P_0^j}{P_0^i P_1^j}\right] + \frac{E[\frac{P_1^j}{P_0^j}] - E[\frac{P_1^i}{P_0^i}] + r_P \times \left(Prob\left(L_j \frac{P_1^j}{P_0^j} > \bar{L}\right) - Prob\left(L_i \frac{P_1^i}{P_0^i} > \bar{L}\right)\right)}{2}$$

- Approximating $E\left[\frac{P_1^i P_0^j}{P_0^i P_1^j}\right] \approx E[\frac{P_1^i}{P_0^i}] - E[\frac{P_1^j}{P_0^j}]$ we obtain

$$r_L^i - r_L^j + E\left[\frac{P_1^i P_0^j}{P_0^i P_1^j}\right] = \frac{E[\frac{P_1^i}{P_0^i}] - E[\frac{P_1^j}{P_0^j}] + r_P \times \left(Prob\left(L_j \frac{P_1^j}{P_0^j} > \bar{L}\right) - Prob\left(L_i \frac{P_1^i}{P_0^i} > \bar{L}\right)\right)}{2}$$

[1] Think of the collateral as a stablecoin, i.e. fixed in terms of USD

- We get an interesting result. The first term is the expected appreciation of currency i with respect to currency j. The second term is the excess liquidation risk of currency j. If the low interest rate currency j has higher liquidation risk and is expected to appreciate with respect to currency i, we get an ambiguous effect on UIP deviations. It depends on the relative strength of the appreciation. If the appreciation of currency j is really high, then the UIP deviation as defined above is negative. If the appreciation of currency j is zero over a sufficiently long horizon, then the UIP deviation is equivalent to the interest rate wedge, which is just a liquidation risk differential.

- A final point is on the carry trade. We have been neglecting carry trade deviations as it requires borrowing in a low interest rate currency and lending in a high interest rate currency. The lending protocol defines lending rates as the borrowing rate multiplied by the utilization rate. Therefore $r_D = r_L \times \frac{L}{D}$. Given that $r_L = a + b\frac{L}{D}$, we can write $r_D = r_L \frac{r_L - a}{b}$. The carry trade return is equal to

$$r_D^i - r_L^j + E\left[\frac{P_1^i P_0^j}{P_0^i P_1^j}\right] = \frac{E[\frac{P_1^i}{P_0^j}] - E[\frac{P_1^j}{P_0^j}] + r_P \times \left(Prob\left(L_j \frac{P_1^j}{P_0^j} > \bar{L}\right) - Prob\left(L_i \frac{P_1^i}{P_0^i} > \bar{L}\right)\right)}{2} - r_L^i \times (1 - \frac{L}{D})$$

- The carry trade return is the UIP wedge plus an additional negative term that is equal to the spread between deposit and lending rates for currency i. Therefore for the carry trade to be positive, liquidation risk of currency j has to be sufficiently high.

- The model as it stands delivers some intuition behind what drives interest rates, UIP and the carry trade, but we cannot determine the sign conclusively without placing more structure. In particular, we have assumed risk-neutrality, but a risk averse investor may place a stronger weight on liquidation risk and therefore result in a UIP wedge and positive carry trade return.

Model Discussion

Things to expand on in Model

- Current result is partial equilibrium setup. Need to expand to include a pool of assets and deriving optimal weights of borrowing in each currency. First order conditions should generate the liquidity risk results of the single currency case.

- Right now lending is fixed. But lending can be endogenized as well, I am unsure about the value-added as the emphasis is on borrowers internalizing liquidation risk leading to lower interest rates on volatile currencies.

- Right now borrower is risk-neutral, but could extend it to include a risk averse case. Hunch is that as risk aversion increases, UIP spread should increase as borrowers are more averse to the liquidation penalty.

- The difference between lending and borrowing rates is assumed constant. However in practice the spread is non-constant.

- Can also look at effects of a liquidation event: as utilization increases, lending and borrowing rates increase. There are arguments in literature about how a quadratic function (so $r_l = a + bu^2$) is able to raise rates more aggressively to prevent a liquidation spiral.

- Can also look at different rules for setting rates and spreads/profits/welfare under different algorithms. This is a bit different to understanding liquidation risk but could be an interesting contribution in itself...i.e. how to make the protocol more efficient. This is similar to introduction of USDC collateral for DAI peg efficiency but this time we are making a recommendation ex ante instead of ex post.

Can we predict the future spot exchange rate?

- If CIP and UIP holds true: Forward/Futures market should be the unbiased predictor of future spot exchange rates
- In cryptocurrencies interest rate differentials are governed by Defi systems like Compound and are exposed to currency risks

$$\textbf{\textit{Excess return}} = i\text{-}i^{*} - (risk : liquidation/counterparty) - \Delta s(exchange\ rate)$$

$$\textbf{\textit{If CIP holds}} : forward\ premium = i\text{-}i^{*} - (risk : liquidation/counterparty)$$

$$\underbrace{\Delta s_{t+1}}_{\text{\% change in exchange rate}} = \alpha + \beta \underbrace{\left(\frac{f_t - s_t}{s_t}\right)}_{\text{forward/futures premium/basis}} + \varepsilon_t \qquad \text{(eq.1)}$$

- Risk premium can arises from the liquidations and counterparty risk

Can we predict the future spot exchange rate?

- Two equations to test the UIP

$$\Delta s_{t+1} = \alpha + \beta\left(\frac{f_t - s_t}{s_t}\right) + \varepsilon_t \qquad \text{(eq.1)}$$

$$\Delta s_{t+1} = \alpha + \beta^o\,(i - i^*) + \varepsilon_t \quad \text{(eq. 2)}$$

% change in exchange rate

$$\left(\frac{f_t - s_t}{s_t}\right) \cong i - i^* + Z$$

Z is related to deviations from CIP due to borrowing risk on Compound

- Results from currency markets in literature shows that $\beta < 1$ so carry trading strategy can generate excess returns
- We can estimate the above equation using cryptocurrency futures
  - I took ETH-USDT pair data from Binance exchange Ether futures and spot exchange rates while interest rate of ETH-USDT pair are from Compound

Linear regression results

- Forward/futures premium exchange rates (eq. 1)

OLS: dependent variable: Ether exchange rate price change

	(1) changes exhange rate	(2) changes exhange rate	(3) changes exhange rate	(4) changes exhange rate
Forward discount	-11.01***	-4.470	-26.27***	8.738***
	(3.048)	(2.877)	(1.001)	(2.834)
Future	Future 1	Future 2	Future 3	Future 4
N	43	134	171	137
R2	0.241	0.0180	0.803	0.0658

Standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

Estimates of β are routinely negative in traditional currency market (Forward premium puzzle) but β of future 4 is positive.

Linear regression results

- Compound interest : borrowing rate for ETH and supply rate for USDT

OLS: dependent variable: Ether exchange rate price change

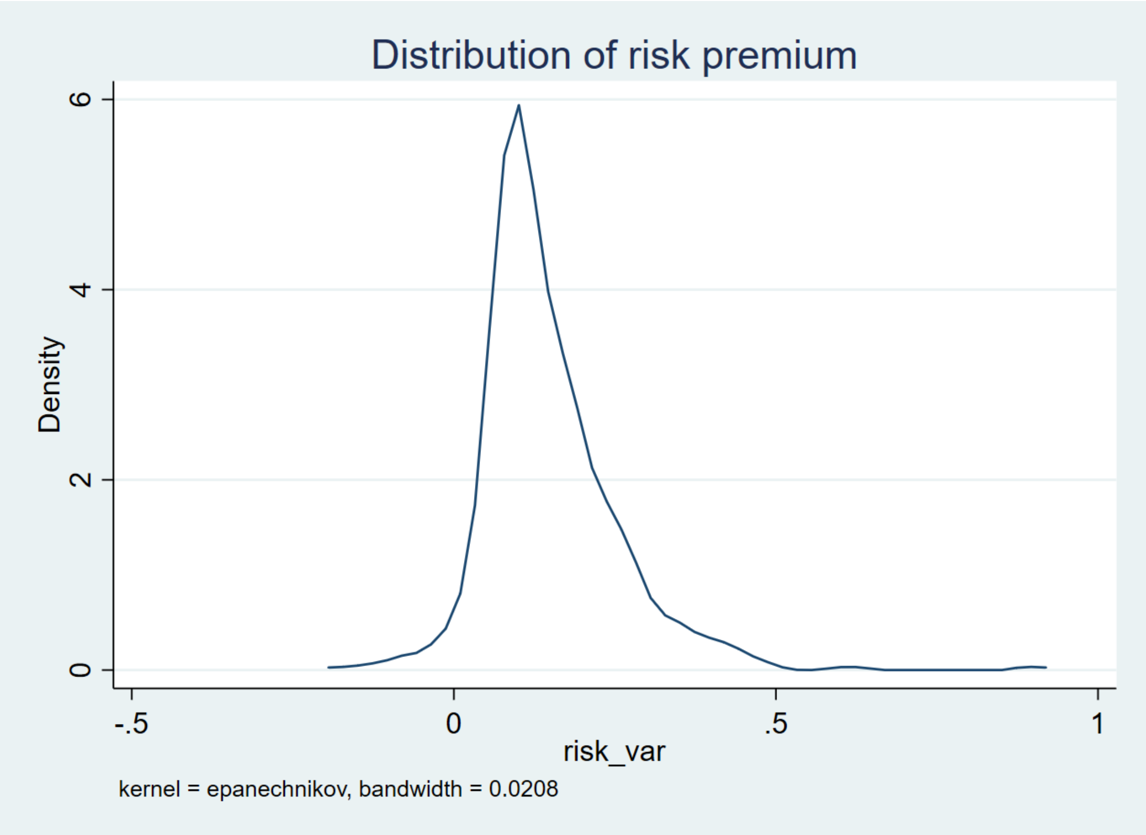
	(1) changes exhange rate	(2) changes exhange rate	(3) changes exhange rate	(4) changes exhange rate
i_diff	18.60	-6.248	-54.07***	-41.01***
	(14.66)	(6.300)	(4.653)	(6.298)
Future	Future 1	Future 2	Future 3	Future 4
N	43	134	171	137
R2	0.0378	0.00739	0.444	0.239

Standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

Estimates of β are negative and coefficients are different from forward discount rate. This indicates the importance of time varying risk

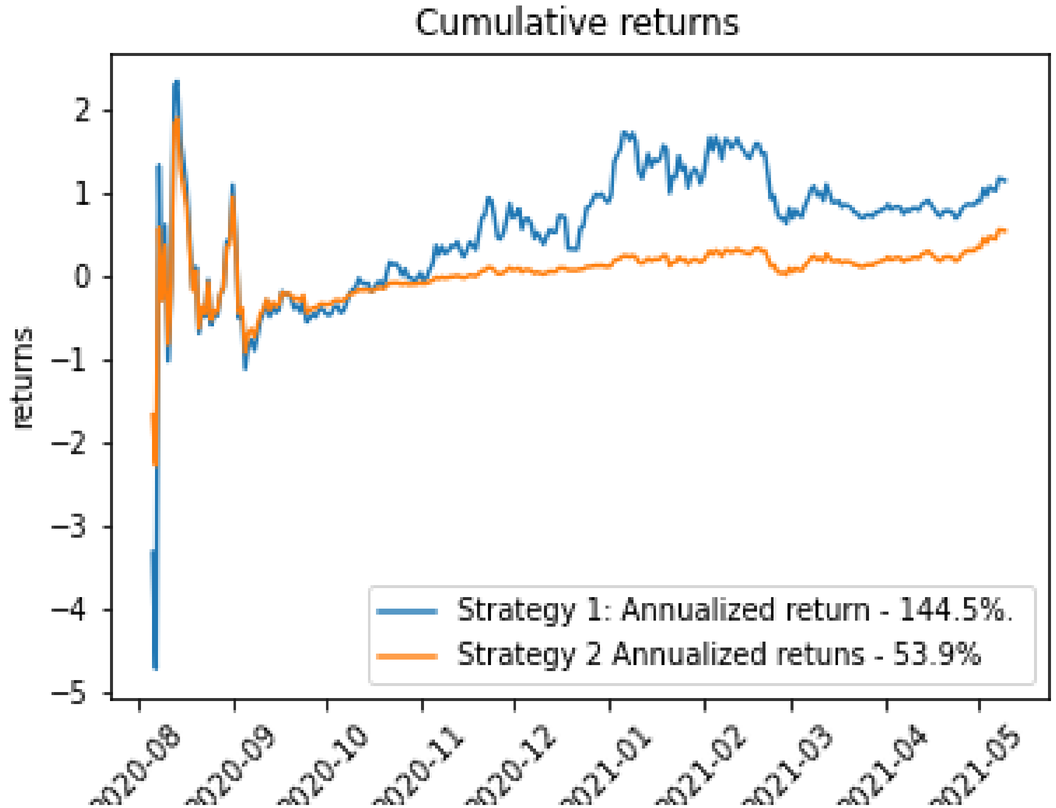
Carry trading using Ethereum futures

- Risk premium as the signal in the trading strategy
- Risk premium = (forward premium) – (Compound interest rate : i-i*)
- Implemented a cut-off based strategy



Performance of strategy

- Signals are based on cut-offs of risk premium
  - Strategy 1 : long if risk premium > 0% ; short if risk premium < 0%
  - Strategy 2 : long if risk premium > mean of signal (12%); short if risk premium <0%
- Portfolio can have multiple futures , rollover (1 week before expiry) and daily portfolio balancing.



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