

PROOF FOR NUMBER OF FPC CALLS using APD

Given,

Plans in space = $\{P_1, P_2, P_3, \dots, P_n\}$

Total points in space be $S_{Total} = S_1 + S_2 + S_3 + \dots S_N$

FPC Calls for Plan P_1

$$S_2 + S_3 + \dots + S_N = S_{Total} - S_1$$

So,

$$\begin{aligned} \text{Total \# FPC Calls} &= \sum_{k=1}^n (S_{Total} - S_k) \\ &= n * S_{Total} + \sum_{k=1}^n S_k = (n - 1) * S_{Total} \end{aligned}$$

Let, Δ_{now} and Δ_{avg} be immediate Δ , and average Δ

Contribution using Simple Exponential Average

$$\Delta_{avg} = \alpha * \Delta_{now} + (1 - \alpha) * \Delta_{avg}$$

for some, $0 < \alpha < 1$

Let, " S " be the step size in exponential jump

Then, impact of Δ_{now} with S size step is as follows

$$\begin{aligned} &\alpha * \Delta_{now} + \alpha * (1 - \alpha) * \Delta_{now} + \dots + \alpha * (1 - \alpha)^{S-1} * \Delta_{now} \\ &= \left(\alpha * \frac{(1 - (1 - \alpha)^S)}{1 - (1 - \alpha)} \right) * \Delta_{now} = (1 - (1 - \alpha)^S) * \Delta_{now} \end{aligned}$$

Contribution using Adaptive Exponential Average

$$\Delta_{avg} = (1 - (1 - \alpha)^S) * \Delta_{now} + ((1 - \alpha)^S) * \Delta_{avg}$$