1

UNIT

Set Theory and Real & Complex Number

Set Theory

1.1 Introduction

The idea of a set theory is fundamental in mathematics. All mathematical objects and constructions go back to set theory. It was developed by German mathematician George Cantor (1845 – 1915).

A set is defined as a well defined collection of objects. A collection of objects is said to be well defined if we can definitely say that whether a particular object belongs to the collection or not. The objects are called the elements of the set. Words like collection, group, family, aggregate are often used to convey the idea of set in everyday life.

The examples of sets are given below:

(a) The set of BCA students of Tribhuvan University.

(b) The set of letters of the word "COMPUTER".

(c) The set of prime numbers = $\{2, 3, 5, 7, 11 \ldots\}$.

(d) The set of square numbers = $\{1, 4, 9, 16, 25 \ldots\}$.

(e) The set of vowels in English alphabet = $\{a, e, i, o, u\}$.

The collections given below are not sets as they are not well defined.

(a) Collection of smart students of a college.

(b) Collection of rich people of the world.

(c) Collection of good computer programmings.

1.2 Notation

The objects of a set are called elements or members of the set. Sets are usually denoted by capital letters A, B, C ... X, Y, Z but the elements of sets are usually denoted by small letters $a, b, c, \ldots x, y, z$.

1.3 Meaning of Symbol

$\in$ an element of or belongs to or is a member of

$\notin$ not an element of or does not belong to or is not a member of

$\Rightarrow$ implies that (conditional)

$\Leftrightarrow$ if and only if (Biconditional)

/or : such that

ϕ null or empty or void set

$\subseteq$ subset

$\subset$ proper subset

1.4 Specification of Sets

The following are the method of representing a set.

(a) Listing method or Roster method or Tabulation method.

(b) Descriptive phrase method.

(c) Rule method or set builder method.

1. **Listing Method or Roster Method or Tabulation Method**

   This method is used in listing each element of the set within the brackets.

   For example

   A = {1, 2, 3, 4, 5}.

   The order in which the elements are listed is not important in sets. Therefore, B = $\{a, b, c, d\}$ and C = $\{a, d, b, c\}$ represent the same set. Note that an element of a set is not written more than once.

2. **Descriptive Phrase Method**

   This method consists in placing a phrase describing the elements of the set.

   For examples

   A = The set of odd numbers between 1 and 20.

   B = The set of stars in the sky.

   This method may be used when there is a large number of elements or when all the elements cannot be named.

3. **Rule Method or Set Builder Method**

In this method, set is described by defining the property that characterizes or specifies all elements of the set.

For example

V = $\{a, e, i, o, u\}$ can be written as V = $\{x : x$ is a vowel in English alphabet$\}$.

1.5 Types of Sets

1. **Finite and Infinite Sets**

A set having finite number of elements is called finite set.

For examples

A = $\{1, 2, 3\}$.

B = $\{x : x$ is a month of the year$\}$.

A set having infinite number of elements is called infinite set.

For examples

A = $\{x : x$ is a star in the sky$\}$.

B = $\{x : x$ is a prime number$\}$ = $\{2, 3, 5, 7, \ldots\}$.

2. **Null Set or Empty Set or Void Set**

A set having no element is called a null or empty or void set. It is denoted by ϕ (phi) or $\{\ \}$.

For examples

A = $\{x : x$ is a prime number between 25 and 27$\}$ = ϕ.

$B = \{x : x^2 + 1 = 0, x \in \mathbb{R}\} = \phi$.

3. **Singleton Set or Unit Set**

A set having only one element is called singleton set.

For examples

A = $\{0\}$.

B = The set of even prime numbers = $\{2\}$.

Universal Set

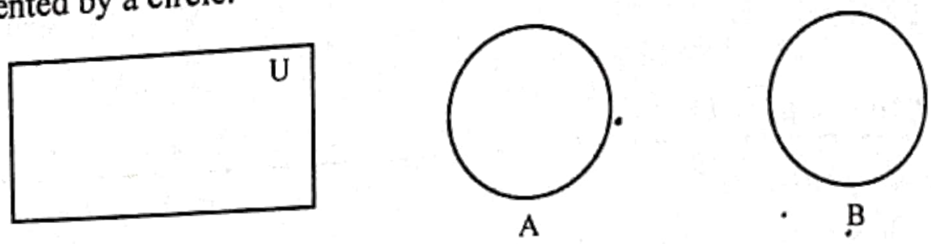
Universal set is the totality of elements under the consideration as elements of any set. It is denoted by the letter U.

For example

The set of all real numbers is the universal set in the real number system.

1.6 Venn Diagram

The diagramatic representation of sets is called Venn diagram. It was developed by British mathematician John Venn (1834 – 1923). The universal set U is usually represented by rectangle and any other given set is represented by a circle.



1.7 Relations between Sets

1. Subsets

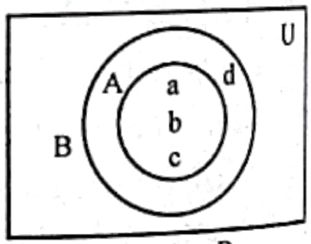
If A and B are two sets such that each element of A is also an element of B then A is called a subset of B. It is denoted by $A \subseteq B$.

Here, B is called **superset** of A.

For example

Let A = $\{a, b, c\}$

B = $\{a, b, c, d\}$. Here, all the elements of A are in B. So, $A \subseteq B$.



$A \subseteq B$

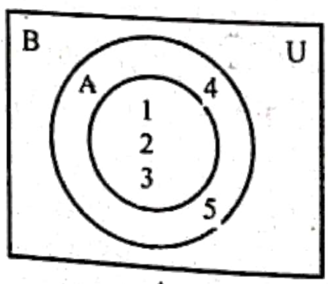
Proper Subset

If each element of set A is an element of B but at least one element of B does not belong to A (i.e. $A \neq B$), then A is called a **proper subset** of B. It is denoted by $A \subset B$.

For example

Let A = $\{1, 2, 3\}$

B = $\{1, 2, 3, 4, 5\}$. Here, each elements of A are in B but $4, 5 \in B$ are not in A. So, $A \subset B$.



$A \subset B$

Note:

(i) *Every set is a subset of itself.*

(ii) *The null set ϕ is the subset of every set.*

(iii) *Every set is a subset of universal set U.*

(iv) *The number of subsets of a set having n distinct elements is 2^n,*
number of proper subsets = $2^n - 1$
and number of proper non–empty subsets= $2^n - 2$.

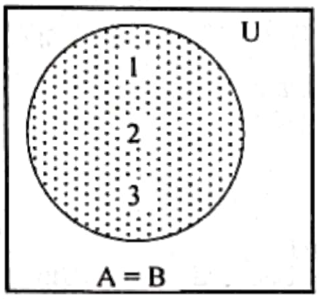
2. Equal and Equivalent Sets

Two given sets A and B are called equal sets if they have the same elements. It is denoted by A = B.

For example

Let A = {1, 2, 3}, B = {3, 2, 1}.

Then, A = B



Two sets A and B are said to be equivalent if they have equal number of elements. It is denoted by A ~ B.

For example

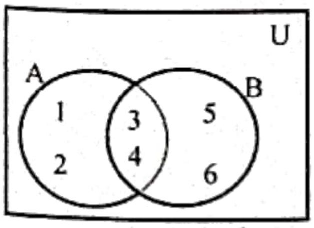
A = {*a*, *b*, *c*} and B = {1, 2, 3} are equivalent sets.

3. Joint and Disjoint Sets

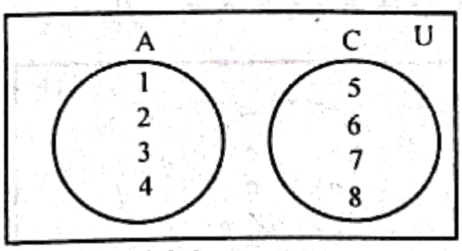
Two given sets are said to be joint (overlapping) sets if they have at least one element in common. Otherwise the sets are disjoint.

For example

Let A = {1, 2, 3, 4}
B = {3, 4, 5, 6}
C = {5, 6, 7, 8}.



Joint sets



Disjoint sets

Here, A and B are joint sets and A and C are disjoint sets.

1.8 Power Set

The set of all the subsets of the given set S is called power set of S. It is denoted by 2^S or P(S).

Example: Write the power set of the set $S = \{a, b, c\}$.

Solution

No. of elements in set S, $n = 3$

Total no. of possible subsets $= 2^n = 2^3 = 8$

Then, the possible subsets of S are

$\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}$.

Hence, the power set of S,

$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$.

Note:

Set	*No. of elements*	*No. of subsets*
ϕ	0	$2^0 = 1$
$\{a\}$	1	$2^1 = 2$
$\{a, b\}$	2	$2^2 = 4$
$\{a, b, c\}$	3	$2^3 = 8$
$\vdots$	$\vdots$	$\vdots$
$\{a, b, c, \ldots, \text{to } n \text{ elements}\}$	n	2^n

1.9 Operations on Sets

We can combine given sets to produce new set by using operations on sets defined as follows:

1. **Union**

The union of two sets A and B is defined as the set of all elements that belong either to A or B or both. It is denoted by $A \cup B$.

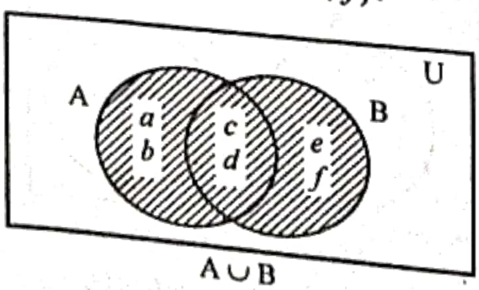
In set builder form, $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

For examples

(i) If $A = \{a, b, c, d\}$ and

$B = \{c, d, e, f\}$ then .

$A \cup B = \{a, b, c, d, e, f\}$.

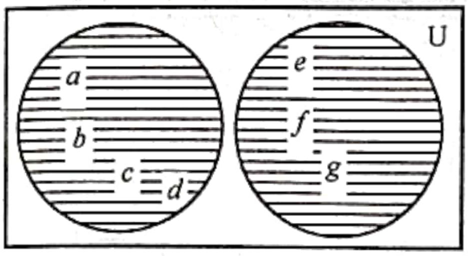


$A \cup B$

(ii) If $A = \{a, b, c, d\}$ and

$B = \{e, f, g\}$ then

$A \cup B = \{a, b, c, d, e, f, g\}$.



$A \cup B$

Note: The words like either ··· or, at least one refer to union.

2. Intersection

The intersection of two sets A and B is defined as the set of all elements that belong to both A and B. It is denoted by $A \cap B$.

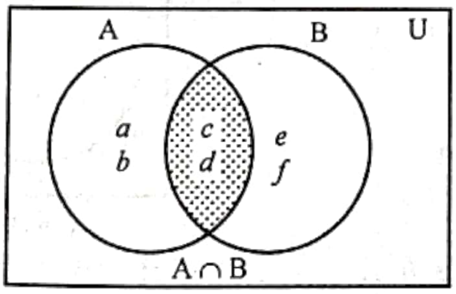
In set builder form, $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

For examples

(i) If $A = \{a, b, c, d\}$

$B = \{c, d, e, f\}$ then

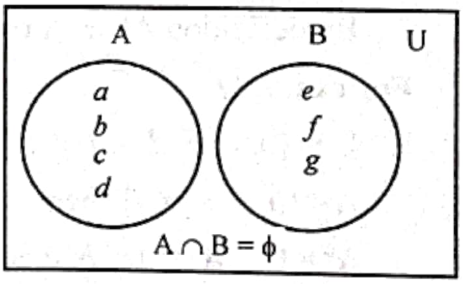
$A \cap B = \{c, d\}$.



(ii) If $A = \{a, b, c, d\}$

$B = \{e, f, g\}$ then

$A \cap B = \phi$.



Note: The words like and, both, common refer to intersection.

3. Difference

The difference of two sets A and B denoted by $A - B$ is the set of all the elements that belong to A but not to B.

In set builder form, $A - B = \{x : x \in A, x \notin B\}$.

Similarly, $B - A = \{x : x \in B, x \notin A\}$.

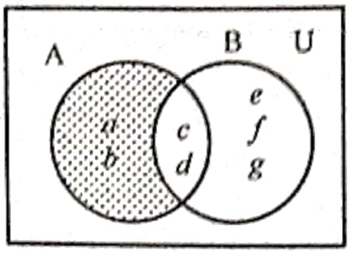
Note that $A - B$ is read as 'A difference B'.

For examples

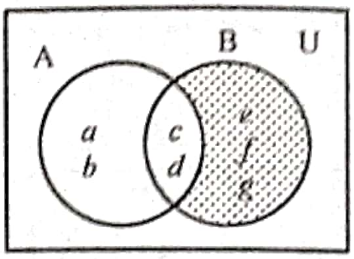
(i) If $A = \{a, b, c, d\}$

$B = \{c, d, e, f, g\}$ then

$A - B = \{a, b\}$ and $B - A = \{e, f, g\}$.



A – B



B – A

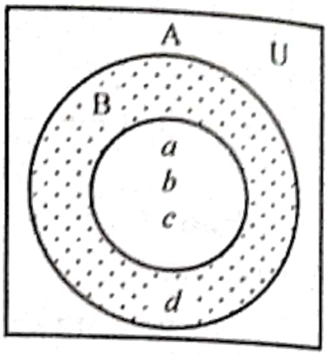
(ii) If A = $\{a, b, c, d\}$

B = $\{a, b, c\}$ then

A – B = $\{d\}$

Similarly, B – A = ϕ.

Note: The word only refers to difference.



A – B

4. Complement

If U be universal set and $A \subseteq U$, then the complement of A is the set of all elements that belong to U but not to A. It can be written as A' or $\bar{A}$ or A^c.

In set builder form,

$\bar{A} = U - A = \{x : x \in U, x \notin A\}$

$\therefore$ By definition A' or $\bar{A}$ or $A^c = U - A$.

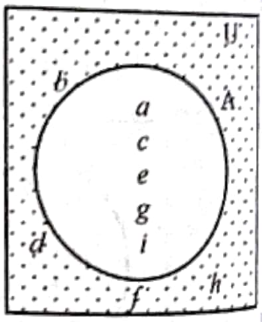
For example

If U = $\{a, b, c, d, e, f, g, h, i\}$ and

A= $\{a, c, e, g, i\}$ then

$A^c = U - A = \{a, b, c, d, e, f, g, h, i\} - \{a, c, e, g, i\}$

$= \{b, d, f, h\}$.



$A^c = U - A$

Note: The words like–none of them, neither, nor refer to complement.

5. Symmetric Difference

The symmetric difference of two sets A and B, denoted by $A\Delta B$ (read as A delta B) is defined by $A \Delta B = (A - B) \cup (B - A)$.

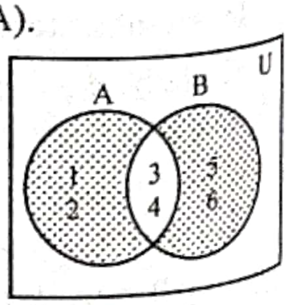
For example

If A = $\{1, 2, 3, 4\}$ and B = $\{3, 4, 5, 6\}$, then

$A\Delta B = (A - B) \cup (B - A)$

$= \{1, 2\} \cup \{5, 6\}$

$= \{1, 2, 5, 6\}$.



A Δ B

1.10 Laws of Algebra of Sets

There are various laws (properties) of the operations on sets which are given below.

Let A, B and C be the subsets of a universal set U.

1. **Idempotent laws**

   (a) $A \cup A = A$ (b) $A \cap A = A$.

2. **Identity laws**

   (a) $A \cup U = U$ (b) $A \cap U = A$

   (c) $A \cup \phi = A$ (d) $A \cap \phi = \phi$.

3. **Commutative laws**

   (a) $A \cup B = B \cup A$ (b) $A \cap B = B \cap A$.

4. **Associative laws**

   (a) $A \cup (B \cup C) = (A \cup B) \cup C$ (b) $A \cap (B \cap C) = (A \cap B) \cap C$.

5. **Distributive laws**

   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

6. **Complementation laws**

   (a) $(A^c)^c = A$ (b) $\phi^c = U$

   (c) $U^c = \phi$ (d) $A \cup A^c = U$

   (e) $A \cap A^c = \phi$.

7. **De–Morgan's laws**

   (a) $(A \cup B)^c = A^c \cap B^c$ (b) $(A \cap B)^c = A^c \cup B^c$.

1.11 Cardinality of a Set

The number of elements in a finite set A is called the cardinality of the set A. It is denoted by $n(A)$ or $|A|$.

For example

If $A = \{1, 2, 3\}$ then $n(A) = 3$.

The cardinality of the null set ϕ is zero. The cardinality of singleton set is one.

Note: We can say that the cardinality of an infinite set is undefined.

List of Important Formulae

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ if A and B are joint sets.

(ii) $n(A \cup B) = n(A) + n(B)$ if A and B are disjoint sets.

(iii) $n(A \cup B) = n_0(A) + n_0(B) + n(A \cap B)$, where $n_0(A)$ and $n_0(B)$ denote the number of elements in A only and B only respectively.

(iv) $n(A^c) = n(U) - n(A)$

(v) $n_0(A) = n(A) - n(A \cap B) = n(A \cup B) - n(B)$

(vi) $n(A - B) = n(A) - n(A \cap B)$

(vii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - (C \cap A) + n(A \cap B \cap C)$

* *Besides these formulae, we can find the number of elements in sets using Venn diagrams.*

WORKED OUT EXAMPLES

Example 1. Rewrite the following in set notation form.

(a) A is a subset of B.

(b) x belongs to set A.

(c) a is not an element of set A.

(d) x belongs to G implies that x belongs to H.

Solution

(a) $A \subseteq B$

(b) $x \in A$

(c) $a \notin A$

(d) $x \in G \Rightarrow x \in H$.

Example 2. Interpret the following.

(a) $a \notin A$

(b) $A \subseteq B$

(c) $M \not\subset N$

(d) $x \in R \Rightarrow x \in H$.

Solution

(a) a does not belong to A.

(b) A is a subset of B.

(c) M is not a proper subset of N.

(d) x belongs to R implies that x belongs to H.

Example 3. Write the following sets in tabular form

(a) Set of vowels in English alphabets.

(b) Set of single digit even positive numbers.

Solution

(a) $\{a, e, i, o, u\}$

(b) $\{2, 4, 6, 8\}$.

Example 4. Write the following sets in set builder form

(a) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(b) $B = \{3, 6, 9, 12, 15, 18\}$

(c) $C = \{-9, -8, -7, -6, -5\}$.

Solution

(a) $A = \{x : x$ is a set of natural numbers less than $10\}$.

(b) $B = \{x : x$ is a set of natural numbers less than 19 and divisible by $3\}$.

(c) $C = \{x : -9 \le x \le -5, x \in \mathbb{Z}\}$.

Example 5. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{7, 8, 9\}$, find

(a) $A \cup B$ (b) $A \cap B$

(c) $A \cup C$ (d) $A \cap C$

(e) $A - B$ (f) $A \Delta B$.

Solution

(a) $A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$
$= \{1, 2, 3, 4, 5, 6\}$.

(b) $A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\}$
$= \{3, 4\}$.

(c) $A \cup C = \{1, 2, 3, 4\} \cup \{7, 8, 9\}$
$= \{1, 2, 3, 4, 7, 8, 9\}$.

(d) $A \cap C = \{1, 2, 3, 4\} \cap \{7, 8, 9\}$
$= \phi$.

(e) $A - B = \{1, 2, 3, 4\} - \{3, 4, 5, 6\}$
$= \{1, 2\}$.

(f) $A \Delta B = (A - B) \cup (B - A)$
$= \{1, 2\} \cup \{5, 6\}$
$= \{1, 2, 5, 6\}$.

Example 6. If $A = \{a, b, c, d, e\}$, $B = \{c, d, e, f, g\}$ and $C = \{a, c, e, g\}$, show that

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(b) $A - (B \cup C) = (A - B) \cap (A - C)$.

Solution

(a) $A \cup B = \{a, b, c, d, e\} \cup \{c, d, e, f, g\}$
$= \{a, b, c, d, e, f, g\}$

$A \cup C = \{a, b, c, d, e\} \cup \{a, c, e, g\}$
$= \{a, b, c, d, e, g\}$

$B \cap C = \{c, d, e, f, g\} \cap \{a, c, e, g\}$
$= \{c, e, g\}$

Now, $A \cup (B \cap C) = \{a, b, c, d, e\} \cup \{c, e, g\}$
$= \{a, b, c, d, e, g\}$

And,

$(A \cup B) \cap (A \cup C) = \{a, b, c, d, e, f, g\} \cap \{a, b, c, d, e, g\}$

$= \{a, b, c, d, e, g\}$

$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(b) $B \cup C = \{c, d, e, f, g\} \cup \{a, c, e, g\}$

$= \{a, c, d, e, f, g\}$

$A - B = \{a, b, c, d, e\} - \{c, d, e, f, g\}$

$= \{a, b\}$

$A - C = \{a, b, c, d, e\} - \{a, c, e, g\}$

$= \{b, d\}$

Now, $A - (B \cup C) = \{a, b, c, d, e\} - (a, c, d, e, f, g\}$

$= \{b\}$

And,

$(A - B) \cap (A - C) = \{a, b\} \cap \{b, d\}$

$= \{b\}$

$\therefore A - (B \cup C) = (A - B) \cap (A - C)$.

Example 7. If $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 4, 6\}$ and $B = \{2, 3, 5, 7\}$, veri that $(A \cap B)^c = A^c \cup B^c$.

Solution

$(A \cap B) = \{1, 3, 4, 6\} \cap \{2, 3, 5, 7\}$

$= \{3\}$

$A^c = U - A = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 3, 4, 6\}$

$= \{2, 5, 7\}$

$B^c = U - B$

$= \{1, 2, 3, 4, 5, 6, 7\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6\}$

And,

$(A \cap B)^c = U - (A \cap B)$

$= \{1, 2, 3, 4, 5, 6, 7\} - \{3\}$

$= \{1, 2, 4, 5, 6, 7\}$

Then,

$A^c \cup B^c = \{2, 5, 7\} \cup \{1, 4, 6\}$

$= \{1, 2, 4, 5, 6, 7\}$

$(A \cap B)^c = A^c \cup B^c$.

Example 8. Given $U = \{1, 2, 3 \ldots, 15\}$; $A = \{x : x \geq 8\}$; $B = \{x : x \leq$ $C = \{x : 4 < x < 12\}$. Find $(A \cap B)$, $(A \cup B)$, $(\overline{A \cup C})$, $(A - C)$.

Solution

$U = \{1, 2, 3, \ldots, 15\}$

$A = \{x : x \geq 8\} = \{8, 9, 10, \ldots, 15\}$

$B = \{x : x \leq 4\} = \{1, 2, 3, 4\}$

$C = \{x : 4 < x < 12\} = \{5, 6,7, \ldots, 11\}$

Now,

$A \cap B = \{8, 9, 10, 11, 12, 13, 14, 15\} \cap \{1, 2, 3, 4\}$

$= \phi$

$A \cup B = \{8, 9, 10, 11, 12, 13, 14, 15\} \cup \{1, 2, 3, 4\}$

$= \{1, 2, 3, 4, 8, 9, 10, \ldots, 15\}$

$A \cup C = \{8, 9, 10, 11, 12, 13, 14, 15\} \cup \{5, 6, 7, 8, 9, 10, 11\}$

$= \{5, 6, 7, 8, 9, \ldots, 15\}$

$\overline{A \cup C} = U - (A \cup C)$

$= \{1, 2, 3, \ldots, 15\} - \{5, 6, 7, \ldots, 15\}$

$= \{1, 2, 3, 4\}$

$A - C = \{8, 9, 10, \ldots 15\} - \{5, 6, 7, \ldots 11\}$

$= \{12, 13, 14, 15\}$.

Example 9. In a statistical investigation of 500 families in certain town, it was found that 40 families had neither a radio nor a TV, and 320 families had a radio and 190 a TV. How many families in that group had both radio and TV?

Solution

Total number of families, $n(U) = 500$

Number of families having radio, $n(R) = 320$

Number of families having TV, $n(T) = 190$

Number of families having neither radio nor TV, $n(\overline{R \cup T}) = 40$

Number of families having both radio and TV, $n(R \cap T) = ?$

We have,

$n(R \cup T) = n(U) - n(\overline{R \cup T})$

$= 500 - 40$

$= 460$

Also,

$n(R \cap T) = n(R) + n(T) - n(R \cup T)$

$= 320 + 190 - 460$

$= 50$.

Example 10. In a class containing 100 students, 50 study mathematics, 40 study computer and 25 study both. Find out (i) how many students study at least one subject (ii) how many students study mathematics only (iii) how many students study computer only (iv) how many students study neither mathematics nor computer.

Solution

Total no. of students, $n(U) = 100$

No. of mathematics students, $n(M) = 50$

No. of computer students, $n(C) = 40$

No. of students who read both the subjects, $n(M \cap C) = 25$

Example 12. In a town of 25,000 population in Kathmandu, 14000 use motorbike, 2500 use bicycle and 500 use both. What percentage use neither motorbike nor bicycle?

Solution

Total population, $n(U) = 25{,}000$

No. of people who use motorbike, $n(M) = 14000$

No. of people who use bicycle, $n(B) = 2500$

No. of people who use both motorbike and bicycle,

$n(M \cap B) = 500$

No. of people who use either motorbike or bicycle,

$$\begin{aligned} n(M \cup B) &= n(M) + n(B) - n(M \cap B) \\ &= 14000 + 2500 - 500 \\ &= 16000 \end{aligned}$$

No. of people who use neither motorbike nor bicycle,

$$\begin{aligned} n(\overline{M \cup B}) &= n(U) - n(M \cup B) \\ &= 25000 - 16000 \\ &= 9000 \end{aligned}$$

$\therefore$ Percentage of people who use neither motorbike nor bicycle

$$= \frac{9000}{25000} \times 100$$

$$= 36\%.$$

Example 13. In a class of 100 students, 40 students failed in mathematics, 70 failed in English and 20 failed in both subjects. Find

a. How many students passed in both subjects?

b. How many students passed in mathematics only?

c. How many students failed in mathematics only?

Solution:

Let U be the set of all students, M be the set of students who failed in Mathematics and E be the set of students who failed in English. Then, by question

$n(U) = 100$

$n(M) = 40$

$n(E) = 70$

$n(M \cap E) = 20$

a. Number of students passed in both subjects $n(\overline{M \cup E}) = ?$

We have,

$$\begin{aligned} n(M \cup E) &= n(M) + n(E) - n(M \cap E) \\ &= 40 + 70 - 20 \\ &= 90. \end{aligned}$$

Again,

$$n(\overline{M \cup E}) = n(U) - n(M \cup E)$$
$$= 100 - 90$$
$$= 10$$

Hence, 10 students passed in both subjects.

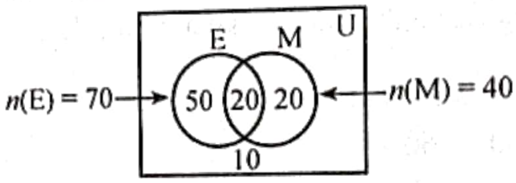
b. Number of students passed in Mathematics only = Number of studen failed in English only = $n_0(E)$

$$= n(E) - n(M \cap E)$$
$$= 70 - 20$$
$$= 50.$$

c. Number of students failed in Mathematics only $= n_0(M) = n(M) - n(M \cap E$

$$= 40 - 20$$
$$= 20$$

The above information in Venn-diagram



Example 14. If A and B be two subsets of universal set U such that $n(U) = 35$

$n(A) = 100$, $n(B) = 150$ and $n(A \cap B) = 50$, then find $n(\bar{A} \cap \bar{B})$.

Solution

From De-Morgan's law of set theory, we have,

$$\bar{A} \cap \bar{B} = \overline{A \cup B}$$

$$\therefore\ n(\bar{A} \cap \bar{B}) = n(\overline{A \cup B})$$

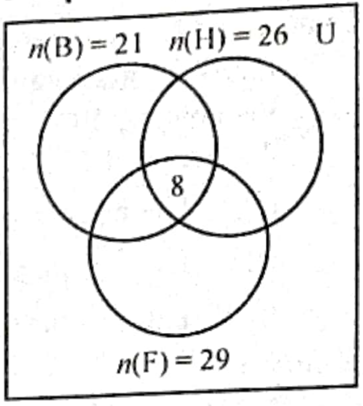
Now, $n(\bar{A} \cap \bar{B}) = n(\overline{A \cup B})$

$$= n(U) - n(A \cup B)$$
$$= n(U) - [n(A) + n(B) - n(A \cap B)]$$
$$= 350 - (100 + 150 - 50)$$
$$= 350 - 200 = 150.$$

Example 15. Of the number of three athletic teams, 21 are in the basketball tea 26 in hockey team and 29 in football team, 14 play hockey and basketball, play hockey and football, 12 play football and basketball and 8 play all games. How many members are there in all?

Solution

Let B, H, F be the sets of members playing basketball, hockey and footb respectively.



Then,

$n(B)$ = No. of members playing basketball = 21

$n(H)$ = No. of members playing hockey = 26

$n(F)$ = No. of members playing football = 29

$n(H \cap B)$ = No. of members playing hockey and basketball = 14

$n(H \cap F)$ = No. of members playing hockey and football = 15

$n(F \cap B)$ = No. of members playing football and basketball = 12

$n(B \cap H \cap F)$ = No. of members playing all three games = 8

$n(U) = n(B \cup H \cup F)$ = Total no. of members = ?

We know,

$$n(B \cup H \cup F) = n(B) + n(H) + n(F) - n(B \cap H) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$$
$$= 21 + 26 + 29 - 14 - 15 - 12 + 8 = 84 - 41$$
$$= 43.$$

$\therefore$ Total number of members = 43.

Example 16. Out of 500 people 285 like tea, 195 like coffee, 115 like lemon juice, 45 like tea and coffee, 70 like tea and juice, 50 like juice and coffee. If 50 do not like any drinks.

(i) How many of people like all three drinks?

(ii) How many people like only one drink? **[TU BCA 2020]**

Solution

Let U denote the set of all people, T denote the set of people who like tea, C denote the set of people who like coffee and L denote the set of people who like lemon juice. By question,

$n(U) = 500$	$n(T) = 285$	$n(C) = 195$
$n(L) = 115$	$n(T \cap C) = 45$	$n(T \cap L) = 70$
$n(L \cap C) = 50$	$n(\overline{T \cup C \cup L}) = 50$	

(i) We have,

$$n(T \cup C \cup L) = n(U) - n(\overline{T \cup C \cup L}) = 500 - 450 = 50$$

Again, we have

$$n(T \cup C \cup L) = n(T) + n(C) + n(L) - n(T \cap C) - n(L \cap C) - n(T \cap L) + n(T \cap C \cap L)$$

or, $450 = 285 + 195 + 115 - 45 - 50 - 70 + n(T \cap C \cap L)$

or, $450 = 430 + n(T \cap C \cap L)$

$\therefore$ $n(T \cap C \cap L) = 450 - 430 = 20$

(ii) $n_0(T) = n(T) - n(T \cap C) - n(T \cap L) + n(T \cap C \cap L)$
$= 285 - 45 - 70 + 20 = 190$

$n_0(C) = n(C) - n(T \cap C) - n(L \cap C) + n(T \cap C \cap L)$
$= 195 - 45 - 50 + 20 = 120$

$n_0(L) = n(L) - n(L \cap C) - n(T \cap L) + n(T \cap C \cap L)$
$= 115 - 50 - 70 + 20 = 15$

$\therefore$ $n_0(T) + n_0(C) + n_0(L) = 190 + 120 + 15 = 325$

Example 17. In an examination conducted by T.U., 55% failed in English, 35% failed in Account and 30% failed in Economics, 16% failed in English and Economics, 10% failed in Economics and Account, 15% failed in English and Account and 7% failed in all three subjects. Find

(i) The pass percentage in all subjects

(ii) The fail percentage in one subject

(iii) The fail percentage in exactly two subjects.

Solution

Let the total number of students be 100.

If E, A, E_c represent the sets of students failed in English, Account and Economics respectively. Then

$n(E) = 55$, $n(A) = 35$,

$n(E_c) = 30$, $n(E \cap E_c) = 16$

$n(E_c \cap A) = 10$, $n(E \cap A) = 15$,

$n(E \cap A \cap E_c) = 7$

(i) We have

$$n(E \cup A \cup E_c) = n(E) + n(A) + n(E_c) - n(E \cap A) - n(A \cap E_c) - n(E_c \cap E) + n(E \cap A \cap E_c)$$
$$= 55 + 35 + 30 - 15 - 10 - 16 + 7$$
$$= 86$$

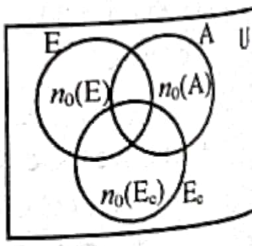
$\therefore$ 86% failed in at least one subject. Thus the percentage of students pass in all subjects $= n(U) - n(E \cup A \cup E_c)$

$= 100 - 86 = 14.$

(ii) No. of students who failed in economics only,

$n_0(E_c) = n(E_c) - n(E \cap E_c) - n(E_c \cap A) + n(E \cap A \cap E_c)$

$= 30 - 16 - 10 + 7 = 11$



No. of students who failed in Account only,

$n_0(A) = n(A) - n(E_c \cap A) - n(E \cap A) + n(E \cap A \cap E_c)$

$= 35 - 10 - 15 + 7 = 17$

No. of students who failed in English only,

$n_0(E) = n(E) - n(E \cap A) - n(E \cap E_c) + n(E \cap A \cap E_c)$

$= 55 - 15 - 16 + 17 = 31$

Thus, total number of students who failed in exactly one subject

$= n_0(E) + n_0(A) + n_0(E)$

$= 11 + 17 + 31 = 59$

$\therefore$ % failed in exactly one subject is 59%.

(iii) No. of students who failed in English and Economics only

$n_0(A \cap E_c) = n(E \cap E_c) - n(E \cap A \cap E_c)$

$= 16 - 7 = 9$

No. of students who failed in English and Account only

$n_0(E \cap A) = n(E \cap A) - n(E \cap A \cap E_c)$
$= 15 - 7 = 8$

No. of students who failed in Account and Economics only

$n_0(A \cap E_c) = n(A \cap E_c) - n(E \cap A \cap E_c)$
$= 10 - 7 = 3$

∴ The total no. of students who failed in exactly two subjects

$n_0(E \cap E_c) + n_0(E \cap A) + n_0(A \cap E_c) = 9 + 8 + 3 = 20$

∴ 20% students failed in exactly two subjects.

EXERCISE – 1 A

1. (a) Given $U = \{1, 2, 3, 4, \ldots, 12\}$, $A = \{2, 3, 5, 6, 8, 10\}$, $B = \{1, 4, 6, 9, 10\}$ and $C = \{2, 3, 6, 8, 12\}$. Find:

   (i) $A \cup B$ (ii) $A - B$

   (iii) $\overline{A \cup C}$ (iv) $\overline{B \cap C}$ (v) $A \Delta B$

   (b) Given $U = \{1, 2, 3, 4, \ldots, 10\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find:

   (i) $(A \cup B) \cup C$ (ii) $(A \cup B) - C$ (iii) $(A - B) \cap C$

   (c) Given $U = \{1, 2, 3, \ldots, 20\}$, $A = \{x : x \geq 10\}$, $B = \{x : x \leq 14\}$ where, A and B are subsets of the universal set U. Find:

   (i) $A - B$ (ii) $\overline{A} \cup \overline{B}$ (iii) $\overline{B - A}$

   (d) If $U = \{x : 2 \leq x + 1 \leq 11, x \text{ is an integer}\}$ $A = \{x : x \text{ is an even number}\}$ and $B = \{x : x \text{ is a prime number}\}$; find the followings.

   (i) $A \cap B$ (ii) $\overline{A \cap C}$ (iii) $\overline{A \cup B}$

2. (a) If $A = \{a, b, x, y\}$ and $B = \{c, d, x, y\}$ then find the following by Venn diagram (i) $B - A$ (ii) $A \cup B$.

   (b) Given set $U = \{x : x \text{ is a positive integer less than } 11\}$, $A = \{2, 3, 6, 7, 9\}$, $B = \{2, 4, 6, 8\}$ and $C = \{2, 3, 4, 5, 9\}$. Find by Venn-diagram.

   (i) $B \cup C$ (ii) $A - B$

   (iii) $A \cup (B \cup C)$ (iv) $\overline{A \cap (B \cap C)}$.

3. Let $U = \{a, b, c, d, e\}$; $A = \{a, b, c\}$; $B = \{c, d, e\}$; and $C = \{b, c\}$. Verify that

   (a) $(A \cap B)^c = A^c \cup B^c$

   (b) $(A \cup B)^c = A^c \cap B^c$

   (c) $A - (B \cap C) = (A - B) \cup (A - C)$.

4. If $n(U) = 200$, $n(A) = 150$, $n(B) = 80$, $n(A \cup B) = 160$ find $n(A \cap B)$, $n(A - B)$, $n(\overline{A \cup B})$.

5. If $n(U) = 100$, $n(A) = 70$ and $n(B) = 40$, find:
   (a) maximum value of $n(A \cup B)$ (b) maximum value of $n(A \cap B)$
   (c) minimum value of $n(A \cup B)$ (d) minimum value of $n(A \cap B)$.

6. 20 students play football and 15 students play hockey. It is found that 5 students play both games. Find the number of students playing at least one game.

7. In a survey of a city market, it was found that 143 families used Colgate toothpaste, 135 used Everest toothpaste and 70 families used both. Find the number of families using at least one type of toothpaste.

8. In a college of 500 students, 400 use Facebook, 300 use Twitter and 50 use neither of them. Find the number of students who use both Facebook and Twitter.

9. 32 students play basketball and 25 students play volleyball. It is found that 20 students play both the games. Find the number of students playing at least one game. Also, find total number of students if 13 students play none of these games.

10. In a city of 26000 populations, 5000 read English newspaper, 12000 read Nepali newspaper and 1000 read both. What percentage read neither English nor Nepali newspaper?

11. In a survey of a city market, it was noted that 300 families were randomly selected, out of which 142 used Laptop and 139 used Desktop computers and 70 families used both. Find the number of families who used exactly one of these types of computers.

12. In an examination, 60% of the candidates failed in Science and 45% failed in Mathematics. If 15% failed in both subjects, find the percentage of those who passed in both the subjects.

13. In a market survey of 1000 consumers of tea, it was found that 500 purchased Soktim Tea, 400 purchased Tokla Tea and 150 purchased both brands. How many purchased (a) Soktim only (b) Tokla only (c) exactly one of these brands and (d) neither of them.

14. Of the number of three athletic teams, 25 are in the basketball team, 30 in hockey team and 28 in football team, 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the games. How many members are there in all?

15. In a group of twenty eight teachers of a school, 15 teach English, 15 teach Maths, 14 teach Nepali, 7 teach English and Maths, 6 teach English and Nepali, 5 teach Maths and Nepali. Find how many teach (a) all three subjects, (b) Maths only (c) Nepali only.

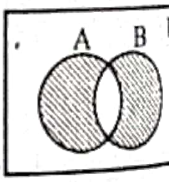
16. In a group of students, 24 study Maths, 30 study Biology, 22 study Physics, 8 study Math only, 14 study Biology only, 6 study Biology and physics only and 2 study Maths and Biology only. Find:
    (a) How many study all three subjects?
    (b) How many students were in the group?
17. In a city of 50,000 population, 20,000 read The Rising Nepal, 25,000 read The Kathmandu Post, 30,000 read The Annapurna, 10,000 read none of these newspapers, 5,000 read The Rising Nepal and The Kathmandu Post 15,000 read The Rising Nepal and The Annapurna and 20,000 read The Kathmandu Post and The Annapurna. Find
    (a) The number of readers reading all newspapers.
    (b) The number of readers reading The Rising Nepal only.
    (c) The number of readers reading The Kathmandu Post only.
    (d) The number of readers reading The Kathmandu Post and The Annapurna only.

Answers

1. (a) (i) {1, 2, 3, 4, 5, 6, 8, 9, 10} (ii) {2, 3, 5, 8}
   (iii) {1, 4, 7, 9, 11} (iv) {1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12}
   (v) {1, 2, 3, 4, 5, 8, 9}.
   (b) (i) {1, 2, 3, 4, 5, 6, 8} (ii) {1, 2, 8} (iii) {3}.
   (c) (i) {15, 16, 17, 18, 19, 20}
   (ii) {1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 16, 17, 18, 19, 20}
   (iii) {10, 11, 12, 13, ..., 20}.
   (d) (i) {2} (ii) {1, 3, 4, 5, 6, 7, 8, 9, 10} (iii) {1, 9}.
2. (a) (i) {c, d} (ii) {a, b, c, d, x, y}.
   (b) (i) {2, 3, 4, 5, 6, 8, 9} (ii) {3, 7, 9}
   (iii) {2, 3, 4, 5, 6, 7, 8, 9} (iv) {1, 3, 4, 5, 6, 7, 8, 9, 10}.
4. 70, 80, 40
5. (a) 100 (b) 40 (c) 70 (d) 10
6. 30 **7.** 208 **8.** 250
9. 37, 50 **10.** 38.46% **11.** 141
12. 10%
13. (a) 350 (b) 250 (c) 600 (d) 250
14. 50 **15.** (a) 2 (b) 5 (c) 5
16. (a) 8 (b) 46
17. (a) 5,000 (b) 5,000 (c) 5,000 (d) 15,000

Objective Questions

1. Which of the following collection is not a set?
   (a) $\{x : x^2 + 1 = 0, x \in \mathbb{R}\}$
   (b) collection of smart BCA students of Tribhuvan University.
   (c) collection of rivers of Nepal.
   (d) collection of prime numbers.
2. The set $\{x : x^2 + 4 = 0, x \in \mathbb{R}\}$ is a / an
   (a) null set (b) singleton set
   (c) infinite set (d) set having two elements.
3. The number of subsets of set $\{a, b, c\}$ is
   (a) 3 (b) 4
   (c) 6 (d) 8
4. The number of proper subsets of $A = \{a, b, c, d\}$ is
   (a) 15 (b) 16
   (c) 31 (d) 32
5. If $A = \{1, 2, 3\}$ then $n(P(A)) =$
   (a) 3 (b) 6
   (c) 8 (d) 12
6. The shaded region in the Venn-diagram is



   (a) $A \cup B$ (b) $A \cap B$
   (c) $A - B$ (d) $A \Delta B$
7. Which of the following is true?
   (a) $A - B = B - A$ (b) $A - B \subseteq A \cup B$
   (c) $A - B \subseteq A \cap B$ (d) $A \cup B \subseteq A \cap B$
8. If A and B are two subsets of universal set U, then $\overline{A \cup B} = \bar{A} \cap \bar{B}$ called
   (a) Identity laws (b) Associate laws
   (c) Distributive laws (d) De-Morgans' law
9. Let $A = \{x : x^2 = 16, x \in \mathbb{R}\}$ and $B = \{x : 2x = 8\}$ then $A \cap B =$
   (a) ϕ (b) $\{4\}$
   (c) $\{-4, 4\}$ (d) $\{8, 16\}$
10. If $A = \{x : x = 2n + 1, n \leq 4, n \in \mathbb{N}\}$ and $B = \{7, 9, 11\}$ then $A - B =$
    (a) $\{3, 5\}$ (b) $\{3, 5, 7\}$
    (c) $\{7\}$ (d) $\{9, 11\}$

11. If $n(A) = 30$, $n(B) = 32$ and $n(A \cap B) = 12$ then $n(A \cup B) =$

(a) 30 (b) 32
(c) 2 (d) 50

12. If $n(U) = 150$, $n(A) = 90$, $n(B) = 70$, then the maximum value of $n(A \cup B)$ is

(a) 70 (b) 90
(c) 150 (d) 160

13. In a group of 130 people, 80 people like football, 20 like both football and cricket. How many like cricket and not football?

(a) 50 (b) 40
(c) 30 (d) 15

14. If A and B are two subsets of universal set U such that $n(U) = 350$, $n(A) = 100$, $n(B) = 150$ and $n(A \cap B) = 50$, then $n(\overline{A} \cap \overline{B}) =$

(a) 200 (b) 250
(c) 300 (d) 150

15. If $A \cap B = \phi$ then $A - B =$

(a) A (b) B
(c) $B - A$ (d) $A \cup B$

Answer Sheet									
1	2	3	4	5	6	7	8	9	10
b	a	d	a	c	d	b	d	b	a
11	12	13	14	15					
d	c	a	d	a					