



PRIME1

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Problem and i/o

- **Inputs**

- For a given 't' number of test cases (ranges) where $t \leq 10$.
- Range is defined by m and n where $1 \leq m \leq n \leq 1000000000$, $n-m \leq 100000$.
- First line of input will give the number of test cases and subsequent lines will give m and n.

- **Problem**

- Find prime numbers in the given range.

- **Output**

- Enter each prime number in a new line
- There needs to be an extra newline between primes of ranges

Segmented sieve in Range

- Works on the basis of simple sieve. Simple sieve in itself is applied in it.
- Cuts down on calculation as simple sieve is only applied on a small segment.
- The small segment is calculated by taking floor of $\sqrt{\text{max integer in range} + 1}$.
- The primes of the rest of range are calculated using the primes derived from the result of simple sieve on the small segment.
- In all cases, evaluation starts from 2 as no primes exist below that number.
- Lets try implementing simple sieve.

Simple Sieve (till 24)

| | | | | |
|--------|--------|--------|--------|--------|
| [F] | [F] | [2,F] | [3,F] | [4,F] |
| [5,F] | [6,F] | [7,F] | [8,F] | [9,F] |
| [10,F] | [11,F] | [12,F] | [13,F] | [4,F] |
| [15,F] | [16,F] | [17,F] | [18,F] | [19,F] |
| [20,F] | [21,F] | [22,F] | [23,F] | [24,F] |

Simple Sieve (prime: 2)

| | | | | |
|--------|--------|--------|--------|--------|
| [0,F] | [1,F] | [2,F] | [3,F] | [4,T] |
| [5,F] | [6,T] | [7,F] | [8,T] | [9,F] |
| [10,T] | [11,F] | [12,T] | [13,F] | [14,T] |
| [15,F] | [16,T] | [17,F] | [18,T] | [19,F] |
| [20,T] | [21,F] | [22,T] | [23,F] | [24,T] |

Simple Sieve (prime: 3)

| | | | | |
|--------|--------|--------|--------|--------|
| [0,F] | [1,F] | [2,F] | [3,F] | [4,T] |
| [5,F] | [6,T] | [7,F] | [8,T] | [9,T] |
| [10,T] | [11,F] | [12,T] | [13,F] | [14,T] |
| [15,T] | [16,T] | [17,F] | [18,T] | [19,F] |
| [20,T] | [21,T] | [22,T] | [23,F] | [24,T] |

Simple Sieve (prime: 3)

| | | | | |
|--------|--------|--------|--------|--------|
| [0,F] | [1,F] | [2,F] | [3,F] | [4,T] |
| [5,F] | [6,T] | [7,F] | [8,T] | [9,T] |
| [10,T] | [11,F] | [12,T] | [13,F] | [14,T] |
| [15,T] | [16,T] | [17,F] | [18,T] | [19,F] |
| [20,T] | [21,T] | [22,T] | [23,F] | [24,T] |

Simple Sieve (prime: 5 and the rest)

| | | | | |
|--------|--------|--------|--------|--------|
| [0,F] | [1,F] | [2,F] | [3,F] | [4,T] |
| [5,F] | [6,T] | [7,F] | [8,T] | [9,T] |
| [10,T] | [11,F] | [12,T] | [13,F] | [14,T] |
| [15,T] | [16,T] | [17,F] | [18,T] | [19,F] |
| [20,T] | [21,T] | [22,T] | [23,F] | [24,T] |

Segmented Sieve (till 24)

- For segmented Sieve, we break the range in two parts
 - 1st part: $\sqrt{24} + 1 = 5$.
 - 2nd part: rest of the range we find primes in.
- To find primes in the rest of the range, we use the primes in the 1st part and find their multiples in the 2nd range.
- Everything that was not a multiple in the first part, is a prime number.

| | | | | | |
|--------|--------|--------|--------|--------|-------|
| [0,F] | [1,F] | [2,F] | [3,F] | [4,T] | [5,F] |
| [6,F] | [7,F] | [8,F] | [9,F] | [10,F] | |
| [11,F] | [12,F] | [13,F] | [14,F] | [15,F] | |
| [16,F] | [17,F] | [18,F] | [19,F] | [20,F] | |
| [21,F] | [22,F] | [23,F] | [24,F] | | |

Segmented Sieve (discard multiples)

| | | |
|-------|-------|-------|
| [2,F] | [3,F] | [5,F] |
|-------|-------|-------|

| | | | | |
|--------|--------|--------|--------|--------|
| [6,T] | [7,F] | [8,T] | [9,T] | [10,T] |
| [11,F] | [12,T] | [13,F] | [14,F] | [15,T] |
| [16,T] | [17,F] | [18,T] | [19,F] | [20,T] |
| [21,T] | [22,T] | [23,F] | [24,T] | |

Simple Sieve

| | | | | |
|--------|--------|--------|--------|--------|
| [0,F] | [1,F] | [2,F] | [3,F] | [4,T] |
| [5,F] | [6,T] | [7,F] | [8,T] | [9,T] |
| [10,T] | [11,F] | [12,T] | [13,F] | [14,T] |
| [15,T] | [16,T] | [17,F] | [18,T] | [19,F] |
| [20,T] | [21,T] | [22,T] | [23,F] | [24,T] |

Segmented Sieve

| | | | | |
|--------|--------|--------|--------|--------|
| [0,F] | [1,F] | [2,F] | [3,F] | [4,T] |
| [5,F] | [6,T] | [7,F] | [8,T] | [9,T] |
| [10,T] | [11,F] | [12,T] | [13,F] | [14,T] |
| [15,T] | [16,T] | [17,F] | [18,T] | [19,F] |
| [20,T] | [21,T] | [22,T] | [23,F] | [24,T] |

Simple Sieve

- Goes through entire range to find primes.
- Calculates multiples of every prime in range
- Easier to implement.
- Faster than brute force and dynamic programming combination.
- Time complexity: $n \cdot \log(\log n)$. Works the same way for small integers and ranges. But in larger numbers disk paging might be required, which slows down the process.

Segmented Sieve

- Goes through smaller range to find primes.
- Does not need to calculate multiples of every prime in range.
- More time consuming to implement.
- Faster than simple sieve and dynamic programming combination.
- Same time complexity but is better as it requires less space hence faster for larger ranges or bigger integers.

Segmented Sieve in given Range (main)

```
//main function that calls the primesinRnage function.
//takes care of the input from the user ie., the range to find primes in
int main()
{
    int count;
    vector<int> ip(2);
    map<int, vector<int>> target;

    //count is used to input the amount of ranges that will be input from the user.
    cin>>count;

    //vector is created to take multiple inputs from one line. this vector is then saved in a map.
    for (int j = 0; j < count; j++)
    {
        for (int i = 0; i < 2; i++){
            cin>>ip[i];
            if (ip[i]==1 || ip[i]==0)
                ip[i]=2;
        }
        target[j] = ip;
    }

    //loop is called to iterate map of inputs.
    //primesinRange function is called for each range and new line is used to format the output as per requirnmnts
    for(int z = 0; z<count; z++){
        primesInRange(target[z][0],target[z][1]);
        cout<<"\n";
    }
    return 0;
}
```

Segmented Sieve in given Range (Segment)

```
//primesInRange function is called from the main function.
//this function is what adds the segmented part to the normal sieve. the simple sieve function is called from here.
//it takes as input 2 integers, which has the range of the in which primes are to be found
void primesInRange(int low, int high)
{
    int limit = floor(sqrt(high)) + 1;
    vector<int> prime;
    simpleSieve(limit, prime);
    //the floor of the square root of the high number +1 is taken along with an integer vector. These two are sent to the simple sieve method
    //calling this function updates the prime vector with primes till the limit integer

    int n = high - low + 1;
    vector<bool> mark(n+1, false);
    //new integer is calculated to figure out the size of range in which primes are yet to be found
    //vector of boolean is created with range size and all indexes are marked false. false in this case means prime.

    for (int i = 0; i < prime.size(); i++) {
        int loLim = floor(low / prime[i]) * prime[i];
        //in the loop the known prime numbers are iterated to find the rest of the non primes in the list
        //new int is created so that it is the smallest multiple of prime in the given range
        if (loLim < low)
            loLim += prime[i];
        if (loLim == prime[i])
            loLim += prime[i];
        //the above two cases take care of the cases if in case the int is below the given range or is the prime number i itself
        for (int j = loLim; j <= high; j += prime[i])
            mark[j - low] = true;
        //the loop is started from the lowest multiple to high end of range and is iterated in multiples of the prime i
        //position [j-low] in mark is put to true. j-low as vector mark does not contain the entire range of primes. true here is non prime.
    }

    //mark is used in the given range to print the primes
    for (int i = low; i <= high; i++)
        if (!mark[i - low])
            cout << i << "\n";
    //if the position in mark is not true i.e., false. it is a prime and is printed.
    //new line is used to format the output as per requirements
}
```

Segmented Sieve in given Range (SimpleSieve)

```
//the basic sieve is called by primesinRange function.  
//for a given max integer, finds prime form 2 to the decided integer  
//sieve finds integers in an opposite of the conventional way, instead of marking an integer as prime,  
//it marks all multiples of known primes as non primes in the given range  
//simple sieve takes as input, max integer named "limit" to find primes below it and a reference to the vector "prime" that contains the primes  
void simpleSieve(int limit, vector<int>& prime)  
{  
    vector<bool> mark(limit+1,false);  
    //vector of type boolean is created with all values as false, in this case, when traversal begins, if an integer is false, it is prime.  
    for (int i = 2; i <= limit; ++i) {  
        //loop id started from two so as to avoid positions 0 and 1 as they can never be primes or no primes.  
        if (mark[i] == false) {  
            prime.push_back(i);  
            //as soon as a prime is found, it is added to the reference vector passed at the beginning  
            for (int j = i; j <= limit; j += i)  
                mark[j] = true;  
            //all multiples of the prime that was found are marked as not prime ie., true in this case.  
        }  
    }  
}
```



THANK YOU.