Assignment 1, SML Winter 2021

Q1. Suppose that the two variables x and z are statistically independent. Show that the mean and variance of their sum satisfies [1+1]

$$E[x + z] = E[x] + E[z]$$
$$var[x + z] = var[x] + var[z].$$

Q2. Compute covariance for [.5+1.5]

$X = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & 6 \end{bmatrix}$, read X as dxn matrix where d is the dimension.

Q3. Using mean and covariance obtained from Q2, compute the probability of point x = $[0\ \ 1]^T$. Assume a multivariate Gaussian distribution. [1]

Q4. Suppose two equally probable one-dimensional densities (Laplacian pdf) are of the form $p(x|\omega_i) = \frac{1}{2b} e^{-\frac{|x-a_i|}{b}}$ for i=1,2 and $a_1 = 0, a_2 = 1, b = 1$.

a. Calculate likelihood ratio and plot against x. [2]
b. Compute optimal/minimum error rate decision boundary for zero-one loss. Note: This can be obtained by equating both posterior densities. Using a sketch indicate how posterior densities will appear and also mark the decision boundary. [2]
c. Compute average probability of error. [3]