1. <u>Likelihood func^n</u> :

Let $\theta_1 = \mu$, $\theta_2 = \sigma^2$

$$L(\theta_1, \theta_2 \mid x_1, \ldots, x_n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_1-\mu)^2/2\sigma^2} \times \cdots \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_n-\mu)^2}{2\sigma^2}}$$

Take log on both the sides -

$$\ln[L(\theta_1, \theta_2 \mid x_1, \ldots x_n) = \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \times \cdots \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_n-\mu)^2}{2\sigma^2}}\right)$$

$$= \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right) + \cdots + \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_n-\mu)^2}{2\sigma^2}}\right)$$

$$= -\frac{1}{2}\ln(2\pi) - \ln(\sigma) - \frac{(x_1-\mu)^2}{2\sigma^2} - \cdots - \frac{1}{2}\ln(2\pi) - \ln(\sigma) \frac{-(x_n-\mu)^2}{2\sigma^2}$$

$$\ln[L(\theta_1, \theta_2 \mid x_1 \ldots x_n)] = -\frac{n}{2}\ln(2\pi) - n\ln(\sigma) - \frac{(x_1-\mu)^2}{2\sigma^2} - \cdots \frac{-(x_n-\mu)^2}{2\sigma^2}$$

$$\frac{\partial}{\partial\mu} \ln[L(\theta_1, \theta_2 \mid x_1 \ldots x_n)] = 0 - 0 + \left(\frac{x_1-\mu}{\sigma^2}\right) + \cdots + \left(\frac{x_n-\mu}{\sigma^2}\right)$$

$$= \frac{1}{\sigma^2}[(x_1 + \cdots + x_n) - n\mu] \quad -(1)$$

$$\frac{\partial}{\partial \sigma} \ln[L(\theta_1, \theta_2 | x_1 \ldots x_n)] = 0 - \frac{n}{\sigma} + \frac{1}{\sigma^3}[(x_1-\mu)^2 + \ldots + (x_n-\mu)^2] \quad -(2)$$

$$\frac{\partial}{\partial \mu} \ln[L(\theta_1, \theta_2 | x_1 \ldots x_n)] = 0$$

$$x_1 + x_2 + \ldots + x_n = n\mu$$

$$\boxed{\theta_1 \theta_2 \mu = \frac{x_1 + x_2 + \ldots + x_n}{n}}$$

$$\frac{\partial}{\partial \sigma} \ln[L(\theta_1, \theta_2 | x_1 \ldots x_n)] = 0$$

$$\sigma^2 n = (x_1-\mu)^2 + \ldots + (x_n-\mu)^2$$

$$\boxed{\begin{gathered}\theta_2 (\theta_2) \sigma^2 = \frac{(x_1-\mu)^2 + \ldots + (x_n-\mu)^2}{n} \\ \sigma = \sqrt{\frac{(x_1-\mu)^2 + \ldots + (x_n-\mu)^2}{n}}\end{gathered}}$$

(2) Likelihood funct :

$$L(\theta | x_1 \ldots x_n) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

$$\ln[L(\theta | x_1 \ldots x_n)] = \sum_{i=1}^{n} \left(\log\binom{m}{x_i} + x_i \log\theta + (m-x_i)\log(1-\theta) \right)$$

$$\frac{\partial}{\partial\theta} \ln [L(\theta | x_1 \dots x_n)] = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} - \frac{\sum_{i=1}^{n} (m - x_i)}{1-\theta} = 0$$

$$\frac{1-\theta}{\theta} = \frac{nm - \sum x_i}{\sum x_i}$$

$$\frac{1}{\theta} = \frac{nm}{\sum x_i}$$

$$\boxed{\theta = \frac{\sum x_i}{nm}}$$