AE 219 Supervised Learning Project :
A Review of Analytical solution for Shape-Memory-polymer Euler–Bernoulli beams under bending for Morphing Application

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Introduction

Shape Memory materials allow a wide range of applications because of its shape memory effect which is its ability to remember the shape and returning to prior shape because of change in external environmental stimuli such as heat, chemical or pH. In Aerospace Applications such as Smart Wing morphing Shape Memory Polymers can be used, during its operation, it experiences bending loads, so bending characterisations of SMP are crucial for such applications. For this purpose, one of the approaches is the finite element formulation. Still, for specific objectives such as design and optimisation, this approach involves a large number of simulations which affects computational cost and power. In this Report, I will Investigate the Analytical Solution for bending of Shape-Memory-polymer Euler–Bernoulli beams. This Analytical Solution is an alternative approach for bending characterisation of SMP beam. Finally, The results and conclusions are discussed, thereby a deeper understanding of the topic is achieved.

Shape Memory Polymers and its Applications in Morphing [1]

Shape Memory refers to materials with one way, two-way or multi-shape memory effect. Two-way shape memory effect is a reversible transition between two different shapes of the material one of these occurring when the temperature is below transition temperature while the other shape is obtained by increasing the temperature above the Transition temperature.
Shape Memory Polymers allows large recoverable strain. Shape memory effect in polymers is normally achieved by relaxation of polymeric chains due to change in temperature. In general, SMPs are formed by two active phases. One is the fixity phase, and the other one acts as the ‘switch phase’. The fixity phase memorises the initial shape of the polymer while the switch phase allows fixing the temporary shape.

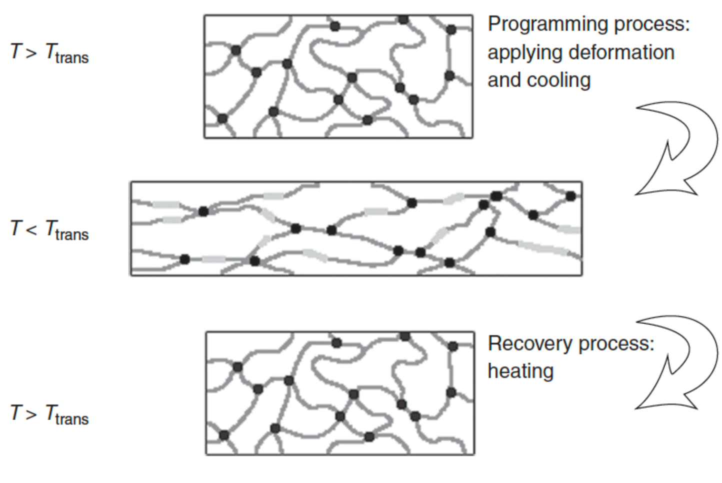

FIGURE 1. Shape Memory Effect in Polymers

Below the Transition Temperature of the switch phase, the chain movements are not allowed and if the temperature increases above T(trans) the chains can move. In this state, if stress is applied the chains will suffer an elastic deformation, releasing the stress the polymer is able to go back to original conformation. In contrast, when the temperature is below T(trans), the switch phase will fix the chains keeping the temporary shape.

The Usage of SMP and SMA based actuators in the morphing of aircraft [2] and drones has increased. Morphing (derived from metamorphosis) in Aerospace domain symbolises the technology/concept that involves substantial changes in state and geometry of the aircraft to adapt to changing mission environment and improve vehicle's performance, efficiency and capabilities.

SMP or SMA based actuators are utilised for the purpose of Camber morphing, out-of-plane morphing (twist bending, camber twist, and spanwise bending), and planform morphing (Sweep and span); thus the necessity of obtaining an accurate analytical and numerical solution for SMP devices is the motivation behind the research.

Characterisation of the Shape Memory Effect is carried out through mechanical tests like stress-strain or bending tests.

In this report on a review of the article by M. Baghani, H.Mohammadi, and R.Naghdabadi the behaviour of SMP in a full cycle for both "fixed strain stress recovery" and "stress-free strain recovery" processes are described.

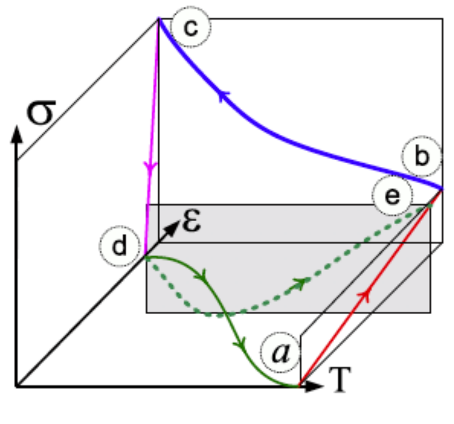

FIGURE 2. Stress-strain-temperature Diagram depicting the behaviour of a pre-Tensioned SMP under different stress recovery conditions

As observed in Figure 2. a typical SMP cycle involves four paths. The cycle starts at a high temperature, Th (a). At this point the material is deformed up to a point (b), then the material is held fixed and cooled down to a low-temperature Tl (point (c) temporary shape) at this step material is unloaded. Finally increasing the temperature to Th though two different paths one is stress-free strain recovery (a) and other fixed strain stress recovery (e)

Closed-form analytical solution

Governing equations on the behaviour of SMP beam during different steps are different, So the Problem of Bending characterisation of the beam is studied separately in each step.

SMP beam is considered as shown in the figure below with the assumption of the Euler-Bernoulli beam theory.

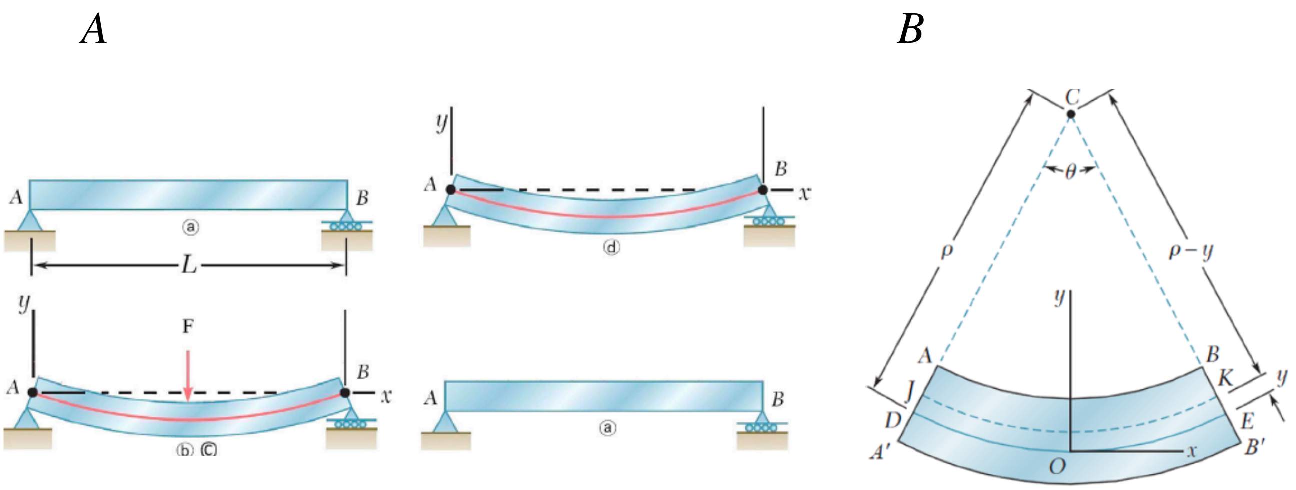

FIGURE 3. [A] The SMP Beam configuration in the thermo-mechanical cycle in correspondence to steps in Figure 2. **[B]** Longitudinal, vertical section of the beam (plane of symmetry)

The Closed form of Analytical solution is obtained employing the constitutive model for SMPs presented by Baghani et al. where Strain is decomposed into four parts as follows,

$$\epsilon = \phi_a \epsilon_a + \phi_f \epsilon_f + \epsilon_T$$

.. [1]

where, φa and φf are volume fractions of the active and frozen phases, respectively with φa + φf = 1. The total mechanical strain and thermal strains are represented by ϵ_a, and ϵ_f , respectively. Elastic strains in the active and frozen phases are denoted by ϵ_a and ϵ_f , respectively, while ϵ_{is} is the inelastic stored strain which evolves during thermally activated changes by the following evolution laws:

$$\varepsilon'_{is} = \varphi'_f \left(k_1 \varepsilon_a + k_2 \frac{\varepsilon_{is}}{\varphi_f} \right); \quad \begin{cases} k_1 = 1, & k_2 = 0; \quad \dot{T} < 0 \\ k_1 = 0, & k_2 = 1; \quad \dot{T} > 0 \\ k_1 = 0, & k_2 = 0; \quad \dot{T} = 0 \end{cases}$$

.. [2]

here in the above equation prime and dot stand for derivative with respect to the temperature and time, respectively. k1 and k2 are defined to identify the type of process(heating or cooling).

Applying the second law of Thermodynamics in the sense of Clausius-Dunham inequality, stress and strain relation is obtained as follows involving the free energy density function for frozen and active phases:

$$\boldsymbol{\sigma} = \boldsymbol{\lambda} = \frac{\partial \Psi_a}{\partial \boldsymbol{\varepsilon}_a} = \frac{\partial \Psi_f}{\partial \boldsymbol{\varepsilon}_f} = \mathbb{K}_a : \boldsymbol{\varepsilon}_a = \mathbb{K}_f : \boldsymbol{\varepsilon}_f$$

.. [3]

in which Ka and Kf are positive definite elasticity tensors of the active and frozen phases, respectively.

To find the response of the SMP Beam, equilibrium equations :
$\Sigma F_x = 0;\ \Sigma M_x = 0$ should be satisfied.

Using the equations mentioned above, the results are obtained through Mathematical formulation, and usage of intermediate terms along with solving differential equations. Finally the closed form analytical solution* is obtained which is inelastic stored strain, beam centre curvature and beam deflection in each step of the SMP cycle.

	$\epsilon_{\mathbf{is}}$ inelastic stored strain	**Kh (Kc)** Beam center curvature
Loading at T_h	0	$\frac{M}{E_a I}$
Cooling from T_h to T_l path b-c	$\frac{M}{E_a I}$	$\frac{M}{E_a I}\left(1+\frac{E_a}{E_f}\varphi\right)$
Unloading at T_l path c-d	$\epsilon_{is}^{cf} = -\frac{My}{E_a I}$	$\frac{M}{E_a I}(1+\frac{E_a}{E_f})$
Heating at T_l to T_h Path d-a or d-e	$\epsilon_{is}^{cf}\phi$	$\kappa_h = \frac{M_h}{I}\left(\frac{E_f - E_a}{E_f E_a}\right)(1-\varphi) + \frac{M}{E_a I}\varphi$

*Appendix A includes derivation and mathematical formulation intermediate steps

	Wh Beam Deflection
Tip Loading	$\lambda x^2 \frac{(x-3)}{6}$
Three point bending	$\lambda x \frac{4x^2-9}{48}$

To arrive at explicit expressions for deflection of an SMP beam after heating process, we note that y″=κ and apply appropriate boundary Conditions for tip loading and three point bending respectively.

$$\text{Here,} \quad \lambda = \frac{F_2(1-\frac{E_a}{E_f})}{E_a I}(1-\phi) + \frac{F\phi}{E_a I} \qquad \text{.. [4]}$$

Numerical Results and discussion

Two different boundary and loading conditions are studied in this section i.e, three point bending and tip loading.

Three Point Bending

Firstly inelastic stored strain in the beam is observed without any external loading during heating recovery step (d-a) for the three point bending test here The SMP beam has length of 1 m and cross sectional area of 25 cm²

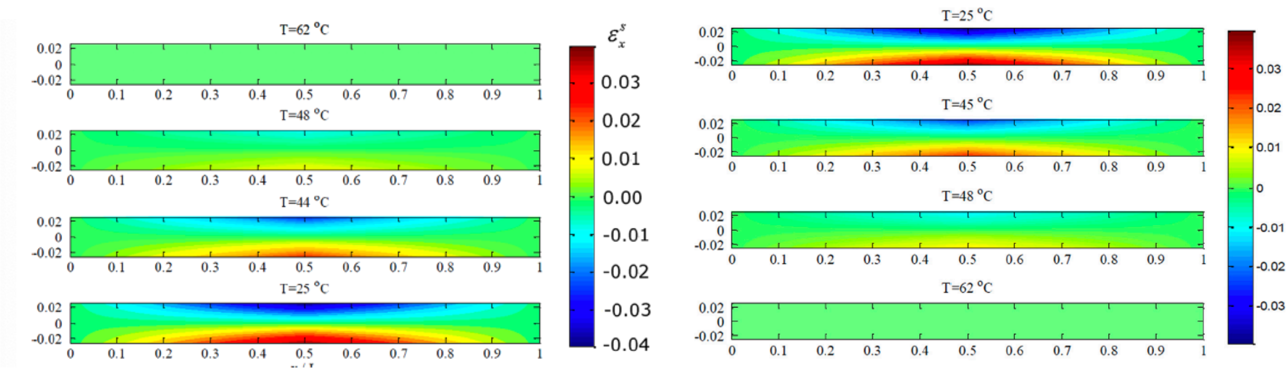

FIGURE 4. Strain storage and release at different temperatures during cooling and heating process in three-point bending condition

To investigate the effect of applying external force on SMP bending characteristics, beam centre curvature is plotted against temperature for different values of External forces F2 during heating process. Also beam deflection after heating is plotted to observe the beam shape at the end of the test.

A *B*

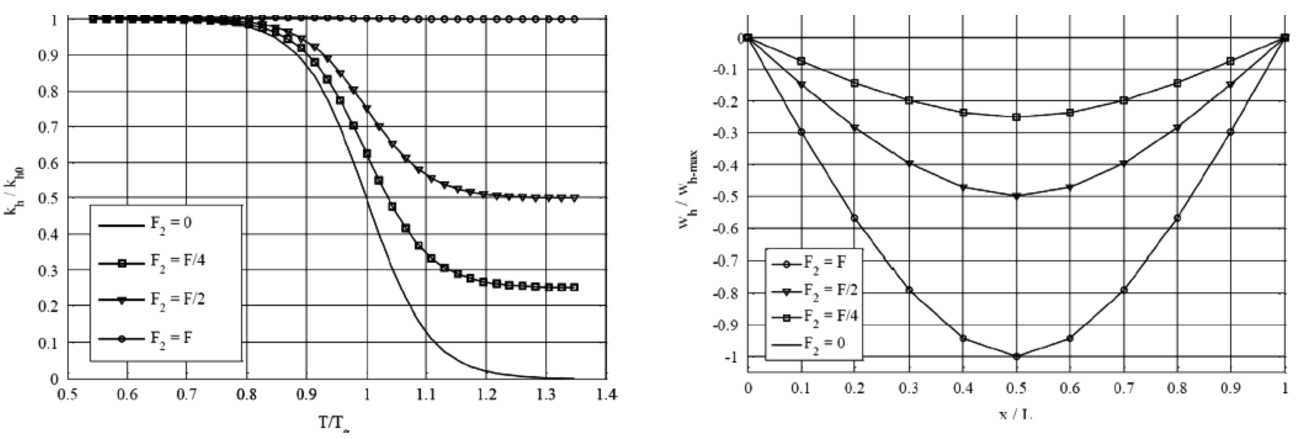

FIGURE 5. [*A*] Dimensionless beam centre curvature during the heating process. F denotes the maximum force applied during path a-b, and F_2 denotes the force applied at the beam centre in the heating process from d for three-point bending condition
[B] Final SMP beam configuration (after heating) W_h is the beam deflection after heating for three-point bending condition

<u>Observations:</u>

- The stored strain starts from zero and increases, reaching a maximum value at Tl, and again recovers to zero during the heating process. Here complete recovery is observed.
- A larger value of strain is observed in the centre of the beam where the force or load is applied.
- A considerable part of recovery occurs near the Transition temperature.
- During the cooling process beam centre curvature changes slightly demonstrating its ability to fix the temporary shape.
- When the external force is absent (not applied during recovery heating process) complete recovery to its permanent shape takes place.
- While increasing this external force results into smaller recovery ratio with the particular case of zero recovery when an external force is equal to force applied in step (a-b) of the SMP cycle during heating

Cantilever beam with force on free tip

Through tip loading test in addition to three-point bending test, capabilities and generality of the proposed analytical solution are demonstrated. Here the Cantilever beam of length 1m and 5 x 7.5 cm2 cross-sectional area is taken which is subjected to an external force on its tip. Similar to previous test beam centre curvature and deflection is plotted against Temperature and beam length respectively with a different set of values of external force on its tip.

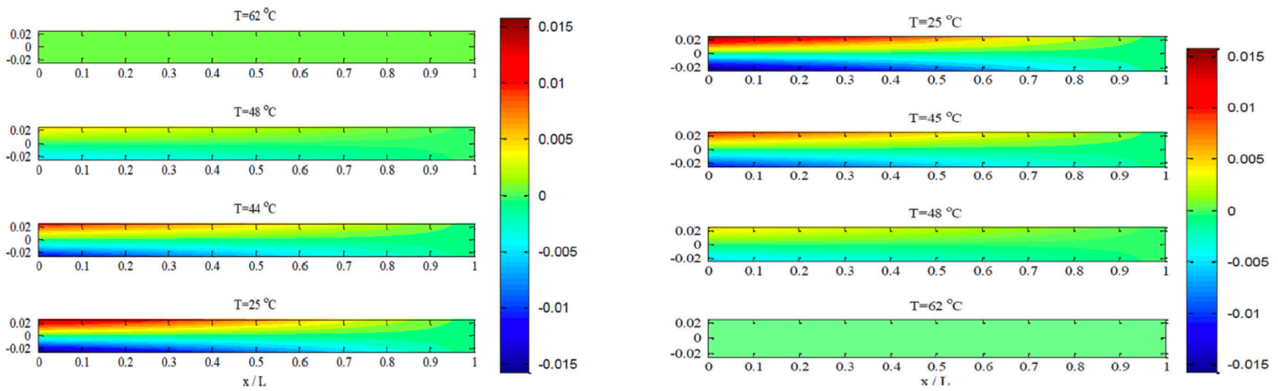

FIGURE 6. Strain storage and release at different temperatures during cooling and heating process for Cantilever beam with force on free tip

Center of curvature and deflection plots for Tip Loading condition are plotted characterising the behaviour of SMP with this change in loading conditions.

A B

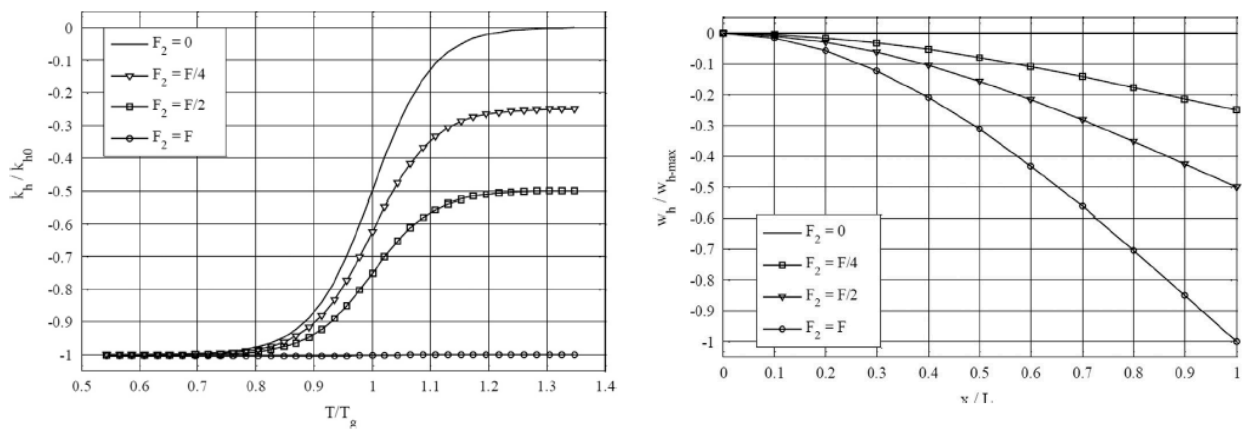

FIGURE 7. [*A*] Dimensionless beam centre curvature during the heating process. F denotes the maximum force applied during path a-b, and F_2 denotes the force applied at the beam centre in the heating process from d for Cantilever beam with force on free tip. **[B]** Final SMP beam configuration (after heating) W_h is the beam deflection after heating for Tip-Loading condition

Observations

- The stored strain starts to increase from zero at Th to reach its maximum value at Tl and then in the heating process it's fully released on condition of absence of external force during heating process during recovery.
- Effects of applying of external force during heating process in the recovery step (d-e) are observed on the beam centre curvature plot vs temperature.
- Beam response is observed in the deflection plot where SMP beam is observed to fully its initial configuration. While with larger amounts of external force, smaller shape recovery ratio is observed, which matches with the results obtained in the three-point bending.
- The author claims that the results obtained are in agreement with experimental observations made by Liu et al. and Tobushi et al.

Reproduction of the results

In the process of developing a deeper understanding of the research presented in the article and observing an application of the closed analytical solution in SMP beam with custom parameters, An algorithm in Python* was created incorporating the closed-form analytical solution and the plots were reproduced. The results obtained matched qualitatively with the results given in the article.

Conclusion

In this research closed-form expression for beam centre curvature, strain, stress and other internal variables are presented which is obtained employing constitutive model proposed for SMPs under the assumption of Euler-Bernoulli beam. It was observed that during the cooling process stored strain evolves to fit the temporary shape and then recover to permanent shape on heating. Effect of external force during heating recovery step of the SMP cycle is also investigated, and it was found that the shape recovery ratio becomes smaller with an increase in external force. While SMP beam fully recovers under no external loading. It is an important inference for morphing application where the ability to recover to prior shape under external loading is crucial. However, the present analytical solution can be used as an effective tool and alternative to Finite element simulation approach for SMP bending characterisation especially for the purpose of design and optimisation which involves a large number of simulations to be performed.

*Appendix B includes algorithm for reproduction of numerical results

References

1. Peponi, L., Navarro-Baena, I., & Kenny, J. M. (2014). Shape memory polymers: properties, synthesis and applications. *Smart Polymers and Their Applications*, 204–236.

2. Barbarino, S., Bilgen, O., Ajaj, R. M., Friswell, M. I., & Inman, D. J. (2011). A Review of Morphing Aircraft. *Journal of Intelligent Material Systems and Structures*, *22*(9), 823–877.

3. Baghani, M., Mohammadi, H., & Naghdabadi, R. (2014). An analytical solution for shape-memory-polymer Euler–Bernoulli beams under bending. *International Journal of Mechanical Sciences*, *84*, 84–90.

Appendix A

Mathematical intermediate steps towards derivation of the closed-form Analytical Solution

→ For one dimensional case,

$$\sigma = E_a \varepsilon_a = E_s \varepsilon_s$$

[Applying the second law of thermodynamics in the sense of Duhem-Clausius inequility]

→ For Bending of SMP beam in SMP cycle

$$\varepsilon = \varepsilon_m + \varepsilon_T \,, \quad \varepsilon_m = -ky$$

Euler-bernoulli beam Theory.

→ To find the response of the SMP beam, equilibrium equations should be satisfied.

viz, $\sum F_x = 0$ and $\sum M_z = 0$.

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Euler-bernoulli beam Theory.

→ To find the response of the SMP beam, equilibrium equations should be satisfied.

viz, $\sum F_x = 0$ and $\sum M_z = 0$.

Now $E = \left(\frac{1-\phi}{E_a} + \frac{\phi}{E_f}\right)^{-1}$

we can write eq (9) as

$$E\varepsilon_m' = \sigma_c' + \frac{E\phi'}{E_f}\sigma_c$$

which is differential equation of the form

$$\frac{dy}{dt} + ay = g(t)$$

its solution is $y = e^{-at}\int_{t_0}^{t} e^{at} g(s)\,ds + ce^{-at}$

So here solution is

$$\sigma_c = e^{-\int \frac{E\phi'}{E_f}dT}\left[\int E\varepsilon_m' e^{+\int \frac{E\phi'}{E_f}dT}\, dT + C\right] \qquad \boxed{10}$$

where $C = -\frac{M_0 y}{I}$ which σ_c at $T = T_h$.

Now taking $\psi_{c(\phi)} = \left[\phi\left(\frac{E_a}{E_f} - 1\right) + 1\right]^{E_a/(E_a - E_f)}$

$$\therefore \quad \sigma_c = \psi_c^{-1}\left[\int_{T_h}^{T} E\,\psi_c\,\varepsilon_m'\, dT - \frac{M_0 y}{I}\right] \qquad \boxed{11}$$

(3)

considering equation (5) and (6)

we can further write

$$M = -\int_A \psi_c^{-1} \left[\int_{Th}^{T} -E K' y \, \psi_c \, dT - \frac{M_0 y}{I} \right] y \, dA \quad \boxed{12}$$

since $M = -\int_A \sigma_c \, y \, dA$

during the cooling process $(M = M_0)$ and eq (12) is further written as.

$$\psi_c \frac{M}{\psi_c} = I \int_{Th}^{T} E K' \psi_c \, dT - M_0 \psi_c$$

$$\Rightarrow \int_{Th}^{T} E K' \psi_c \, dT = \frac{M}{I} (1 + \psi_c) \quad \boxed{13}$$

Now $\frac{M}{I}(1+\psi_c) = \frac{M}{I} \int_{Th}^{T} \dot{\psi}_c \, dT$.

$$\therefore \int_{Th}^{T} E K' \psi_c \, dT = \int_{Th}^{T} \frac{M}{I} \dot{\psi}_c \, dT$$

$$\therefore \int_{Th}^{T} K' \, dT = \int_{Th}^{T} \frac{M}{I} \frac{\dot{\psi}_c}{E \psi_c} \, dT$$

$$\therefore K_c - K_{Th} = \frac{M}{I} \int_{Th}^{T} \frac{\dot{\psi}_c}{E \psi_c} \, dT$$

$$K_c = \frac{M}{E_a I} + \frac{M}{I} \int_{Th}^{T} \frac{\dot{\psi}_c}{E \psi_c} \, dT \quad (14) \Leftarrow K_c = K_{Th} + \frac{M}{I} \int_{Th}^{T} \frac{\dot{\psi}_c}{E \psi_c} \, dT \quad \left[\text{here } K_{Th} = \frac{M}{E_a I} \right]$$

(4)

Therefore

$$\frac{M}{IE_a} + \frac{M}{I}\int_{T_h}^{T} \frac{\dot{\psi}_c}{E\psi_c}\, dT = \frac{M}{E_a I}\left(1+\frac{E}{E_f}\phi\right) \quad \text{—— (14)}$$

Since $\quad d\psi_c = \frac{E_a}{E_a-E_f}\left[1+\phi\left(\frac{E_a}{E_f}-1\right)\right]^{E_f/E_a-E_f} d\phi$

$$\int_{T_h}^{T} \frac{\dot{\psi}_c\, dT}{E\psi_c} = \int_0^{\phi} \frac{E_a}{E_a-E_f}\left(\frac{1-\phi}{E_a}+\frac{\phi}{E_f}\right)\left[1+\phi\left[\frac{E_a}{E_f}-1\right]\right]^{-1} d\phi$$

$$= \frac{\int_0^{\phi} d\phi}{E_f}$$

Now putting the values of ϕ & taking eq (11) / finding σ_c we get

$$\sigma_c = -\frac{My}{I}\psi_c^{-1}\left[\psi_c - \underbrace{\psi_c(0)}_{0} + 1\right]$$

$$\sigma_c = -\frac{My}{I} \qquad (K_1=1,\ K_2=0)$$

Substituting stress from eq (15) into eq (2) we get

$$\varepsilon_{is}^{c} = -\frac{My}{E_a I}\phi \ , \quad \text{—— (16)}$$

stored strain which evolves from 0 to this value

(5)

(3) unloading at const. T_l path c-d.

here external moment M is vanished.

$$\sigma_u = E_f \varepsilon_f = E_f(\varepsilon_m - \varepsilon_{is}^{cf}) = E_f\left(-K + \frac{M}{E_a I}\right)y$$

here $M_z = 0$ which is equilibrium condition hence

$$\sigma_u = 0 \quad \text{--- (18)}$$

(4) Heating from T_l to T_h path d-a or d-e

→ Since it is heating process $(K_1 = 0, K_2 = 1)$

and $$\varepsilon_{is}^{\prime h} = \phi_f' \frac{\varepsilon_{is}^c}{\phi_f}$$

$$\therefore \varepsilon_{is}^h = \varepsilon_{is}^c e^{\int \phi_f'/\phi_f \, dt} = \varepsilon_{is}^c \phi.$$

final value of stored strain

$$\cancel{\varepsilon_m' = \left(\frac{1-\phi}{E_a} + \frac{\phi}{E_f}\right)\sigma_h'} + \varepsilon_{is}^{\prime h}$$

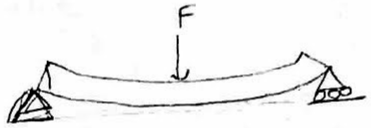
$$\cancel{\left(\frac{1}{E_f} - \frac{1}{E_a}\right)}$$

$$\varepsilon_m' = \left(\frac{1-\phi}{E_a} + \frac{\phi}{E_f}\right)\sigma_h' + \left(\frac{1}{E_f} - \frac{1}{E_a}\right)\phi'\sigma_h + \varepsilon_{is}^{cf}\phi' \quad \text{--- (20)}$$

(6)

3 Point Bending.

$$M = \frac{Fx}{2}$$



Assumming.

$$\frac{d^2y}{dx^2} = K$$

For heating process $K = Kh = \frac{Mh}{EaI}\left(1-\frac{Ea}{Ef}\right)(1-\phi) + \frac{M\phi}{EaI}$

$$\frac{d^2y}{dx^2} = \lambda\frac{x}{2} \quad \text{where } \lambda = \frac{Fa}{EaI}\left(1-\frac{Ea}{Ef}\right)(1-\phi) + \frac{F\phi}{EaI}$$

$$\int\frac{d^2y}{dx^2} = \int\frac{\lambda x}{2}$$

$$\frac{dy}{dx} = \frac{\lambda x^2}{4} + C_1$$

$$\left.\frac{dy}{dx}\right|_{x=L/2} = 0 = \frac{\lambda L^2}{16} + C_1 \Rightarrow C_1 = -\frac{\lambda L^2}{16}$$

$$\int\frac{dy}{dx} = \int\frac{\lambda x^2}{4} + \int -\frac{\lambda L^2}{16}$$

$$y = \frac{\lambda x^3}{12} - \frac{\lambda L^2}{16}x + D$$

$$y|_{x=0} = D = 0$$

Thus

$y(x, T)$

$$= \frac{\lambda}{48}x(-3L^2 + 4x^2).$$

TIP DEFLECTION

$$M = -F(L-x)$$

$$\frac{d^2y}{dx^2} = K \quad [\text{Assumming}] \,, \quad \left.\frac{dy}{dx}\right|_{x=0} = 0 \,, \quad \left. y \right)_{x=0} = 0$$

For heating process, $K = K_h = \lambda (L+x)$

where $\lambda = \frac{Fz}{E_a I}\left(1 - \frac{E_a}{E_f}\right)(1-\phi) + \frac{F\phi}{E_a I}$

$$\int \frac{d^2y}{dx^2} = \int \lambda (x-L)$$

$$\therefore \quad \frac{dy}{dx} = \frac{\lambda x^2}{2} - \lambda L x + C_1$$

Since $\left.\frac{dy}{dx}\right|_{x=0} = 0 \Rightarrow \quad C = 0$

Now $\int \frac{dy}{dx} = \int \frac{\lambda x^2}{2} - \lambda L \int x + 0$

$$\Rightarrow \quad y = \frac{\lambda x^3}{6} - \frac{\lambda L x^2}{2} + C_2$$

Now $y|_{x=0} = 0 \Rightarrow C_2 = 0$

$$\therefore \quad \boxed{y = \frac{\lambda x^2}{6}(x - 3L)}$$

Appendix B

Python Algorithm for reproduction the results from the Paper

1. Reproducing Numerical Results for an SMP beam under 3 point bending with Load F=50N

In [2]:

```
import numpy as np
import pandas as pd
from matplotlib import colors
import matplotlib.pyplot as plt
```

In [3]:

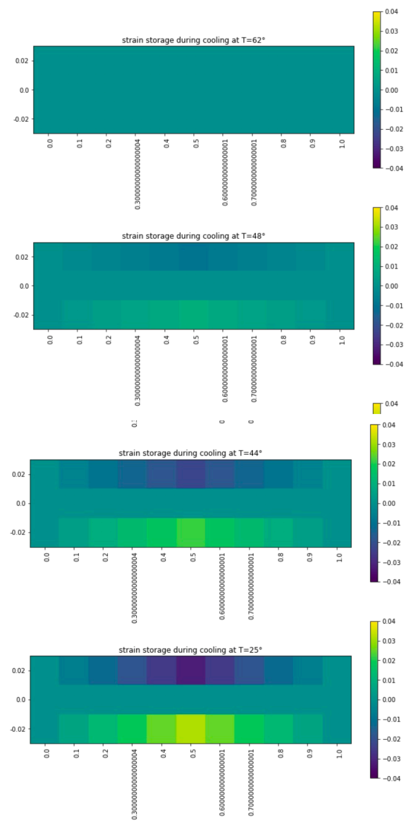
```
X=np.arange(0,1.1,0.1);
Y=np.array([0.02,0,-0.02]);
T=[25,44,48,62];
```

In [4]:

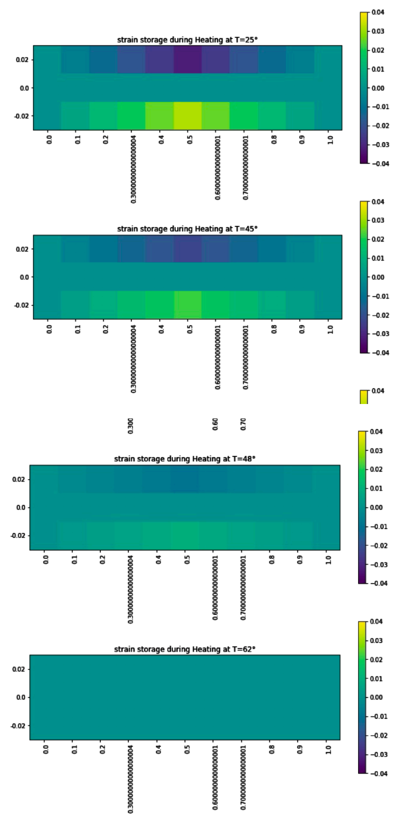
```
def inelastic_strain(x,y,t):
    phi=(np.tanh((62-46)/4.817)-np.tanh((t-46)/4.817))/(np.tanh((62-46)/4.817)-np.tanh((25-46)/4.817))
    inelastic_strain=(50*(-x)*y*phi)/(15.2*(10**6)*((0.05)**4)/6);
    return(inelastic_strain)
```

In [5]:

```
Eis_t=list([]);
for t in range(len(T)):
    Eis_t.append([])
    for y in range(len(Y)):
        Eis_t[t].append([])
        for x in range(len(X)):
            if(X[x]<0.5):
                l=X[x];
            else:
                l=1-X[x];
            Eis_t[t][y].append(inelastic_strain(l,Y[y],T[t]))
```



Strain Storage during Cooling process



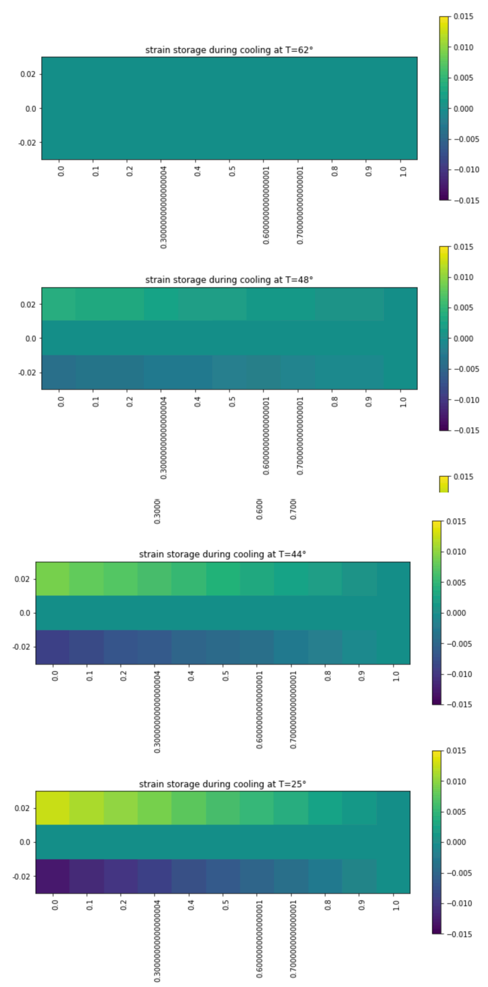
Strain Storage during Heating Process

2. Reproducing Numerical Results for an SMP beam bending under Tip Load of F=10N

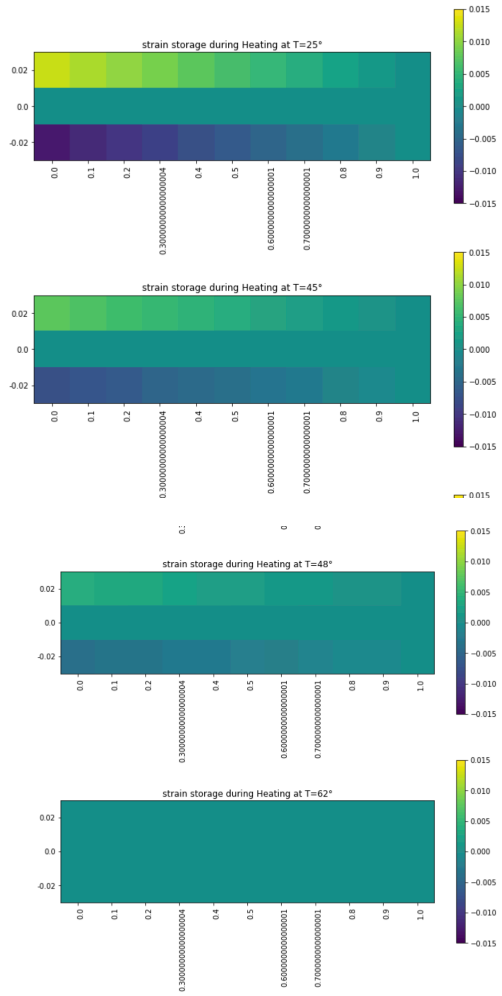
```
def inelastic_strain_tipload(x,y,t):
    phi=(np.tanh((62-46)/4.817)-np.tanh((t-46)/4.817))/(np.tanh((62-46)/4.817)-np.tanh((25-46)
/4.817))
    inelastic_strain=(10*(1-x)*y*phi)/(15.2*(10**6)*((0.05)**4)/6);
    return(inelastic_strain)
```

```
T=[25,44,48,62];
```

```
Eis_t=list([]);
for t in range(len(T)):
    Eis_t.append([])
    for y in range(len(Y)):
        Eis_t[t].append([])
        for x in range(len(X)):
            Eis_t[t][y].append(inelastic_strain_tipload(X[x],Y[y],T[t]))
```

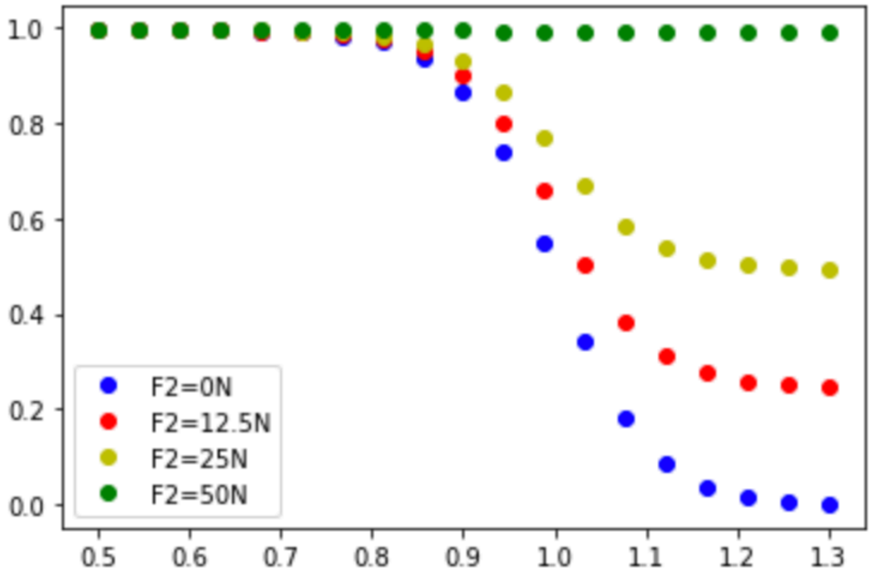


Strain Storage during Cooling Process When Load is applied at Tip



Strain Storage during Heating Process When Load is applied at Tip

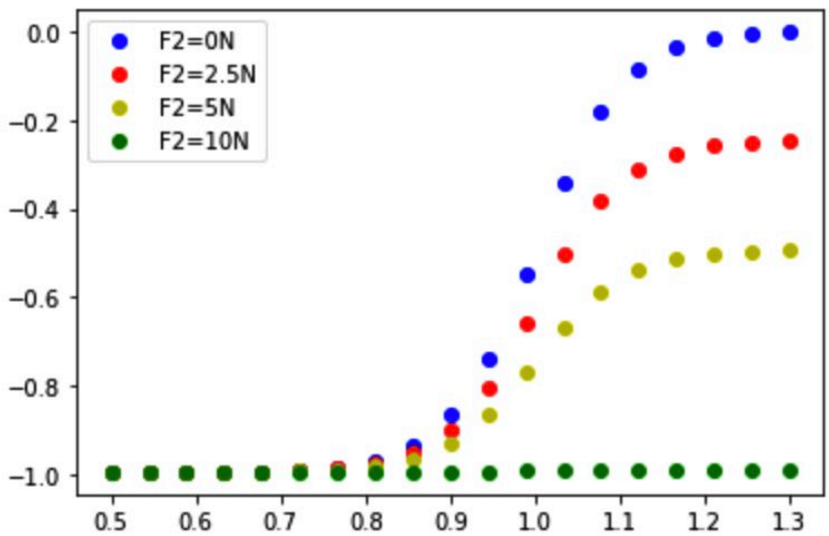
3. Plotting **Kh/Kho v/s T/Tg** for Tip Loading with variable F2, keeping F=10N



Code

```
def curvature(x,t,F,F2):
    phi=(np.tanh((62-46)/4.817)-np.tanh((t*46-46)/4.817))/(np.tanh((62-46)/4.817)-np.tanh((25-46)/4.817))
    Kh=(F2*(-1+x)*(1-phi)*((2600-15.2)/(1000000*2600*15.2)))/(((0.075)**3)*(0.05)/12)+(F*(-1+x)*phi)/(15.2*1000000*(((0.075)**3)*(0.05)/12))
    return Kh

def K(F2):
    k=list([]);
    T=np.linspace(0.5,1.3,num=19);
    for t in T:
        k.append(curvature(0.5,t,10,F2)/0.1882)
    return k

T=np.linspace(0.5,1.3,num=19);
plt.plot(T,K(0),'bo',label='F2=0N')
plt.plot(T,K(2.5),'ro',label='F2=2.5N')
plt.plot(T,K(5),'yo',label='F2=5N')
plt.plot(T,K(10),'go',label='F2=10N')
plt.legend()
```

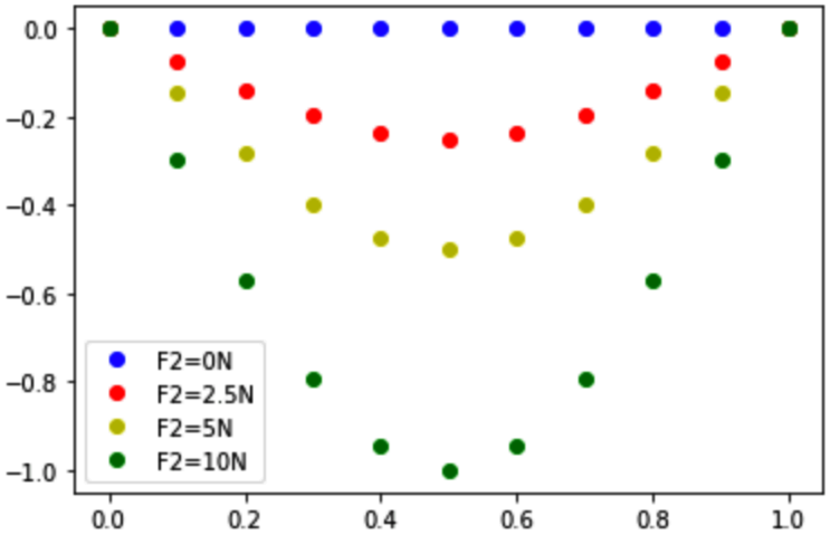
4. Plotting **Kh/Kho v/s T/Tg** for 3 Point bending with variable F2, keeping F=50N



Code

```
def curvature(x,t,F,F2):
    phi=(np.tanh((62-46)/4.817)-np.tanh((t*46-46)/4.817))/(np.tanh((62-46)/4.817)-np.tanh((25-46)/4.817))
    Kh=(F2*(x/2)*(1-phi)*((2600-15.2)/(1000000*2600*15.2)))/(((0.05)**4)/12)+(F*(x/2)*phi)/(15.2*1000000*((0.05)**4)/12)
    return Kh

def K(F2):
    k=list([]);
    T=np.linspace(0.5,1.3,num=19);
    for t in T:
        k.append(curvature(0.5,t,50,F2)/1.589)
    return k

T=np.linspace(0.5,1.3,num=19);
plt.plot(T,K(0),'bo',label='F2=0')
plt.plot(T,K(12.5),'ro',label='F2=12.5')
plt.plot(T,K(25),'yo',label='F2=25')
plt.plot(T,K(50),'go',label='F2=50')
plt.legend()
```

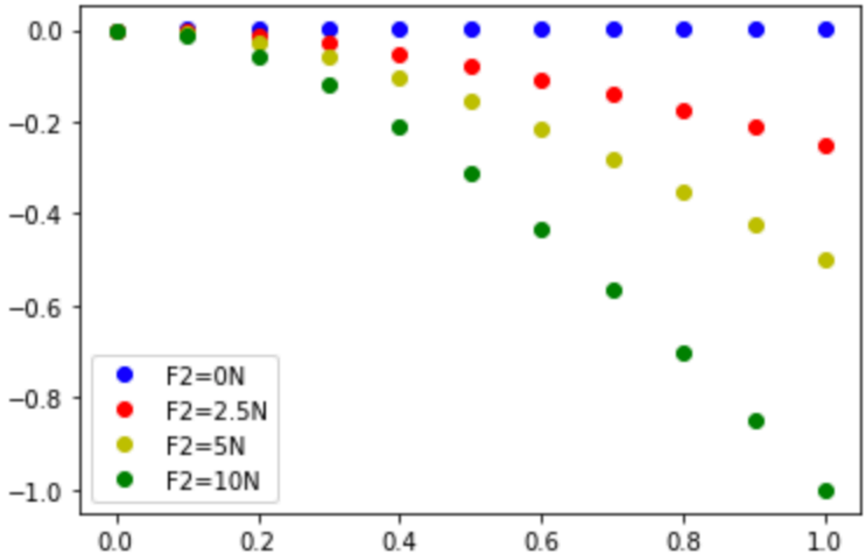
5. Plotting **Wh/Wh_max v/s X/L** for 3 point bending after heating with variable F2, keeping F=10N



Code

```
def curvature_h(x,t,F,F2):
    phi=(np.tanh((62-46)/4.817)-np.tanh((t-46)/4.817))/(np.tanh((62-46)/4.817)-np.tanh((25-46)/4.817))
    Kh=(F2*(1-phi)*((2600-15.2)/(1000000*2600*15.2)))/(((0.05)**4)/12)+(F*phi)/(15.2*1000000*(((0.05)**4)/12))
    return Kh

def deflection(x,F2):
    w_h=(curvature_h(x,62,10,F2)*x*(4*(x**2)-3))/48;
    return w_h

w_max=abs(deflection(0.5,10));

def D(F2):
    d=list([]);
    x=np.linspace(0,1,num=11);
    d=deflection(x,F2)
    return d/w_max

X=np.linspace(0,1,num=11);
plt.plot(X,D(0),'bo',label='F2=0N')
plt.plot(X,D(2.5),'ro',label='F2=2.5N')
plt.plot(X,D(5),'yo',label='F2=5N')
plt.plot(X,D(10),'go',label='F2=10N')
plt.legend()
```

6. Plotting Wh/Wh_max v/s X/L for Tip Loading after heating with variable F2, keeping F=10N



Code

```
def curvature_h(x,t,F,F2):
    phi=(np.tanh((62-46)/4.817)-np.tanh((t*46-46)/4.817))/(np.tanh((62-46)/4.817)-np.tanh((25-46)/4.817))
    Kh=(F2*(1-phi)*((2600-15.2)/(1000000*2600*15.2)))/(((0.075)**3)*(0.05)/12)+(F*phi)/(15.2*1000000*(((0.075)**3)*(0.05)/12))
    return Kh

def deflection(x,F2):
    w_h=(-1*(curvature_h(x,62,10,F2))*(x**2)*(3-x)/6);
    return w_h

def D(F2):
    d=list([]);
    x=np.linspace(0,1,num=11);
    d=deflection(x,F2)
    return d

wh_max=abs(D(10)[10]);
```