Assignment 1
BITH212

Question 1

a) A cafeteria offers a choice of two soups, five sandwiches, three desserts and three drinks. How many different lunches, each consisting of a soup, a sandwich, a dessert and a drink are possible? **[4]**

b) The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 2 girls be formed? **[4]**

c) There are 50 baskets of apples. Each basket contains no more than 24 apples. Show by the pigeon hole principle that there are at least 3 baskets containing the same number of apples. **[4]**

d) Prove by Induction that $10^{3n} + 13^{n+1}$ is divisible by 7 for all n>=1 **[8]**

Question 2

a) Show by induction that $n^2 - 3n + 4$ is even. **[5]**

b) Prove that for all integer a, b and c, if *a divides b* and *a divides c*, then *a divides (b+c).* **[5]**

c) If *a* and *b* are real numbers such that the product *ab* is an irrational number, then either *a* or *b* must be an irrational number. Prove by contraposition. **[5]**

d) Prove by cases that if *x* is an integer then $x^2 + 3x + 1$ is odd. **[5]**

e) Prove that $2n + 1 \leq 2n$ for $n \geq 3$. **[5]**

Question 3

a) Prove by contradiction that for all integers *x* and *y* , $x^2 \neq 4y + 2$ **[6]**

b) Determine integers x and y such that gcd(421, 111) = 421x + 111y **[6]**

c) Find the number of distinct permutations that can be formed using the letters of the word "BENZENE". **[4]**

d) To prove by mathematical induction that $\mathbf{5^n - 1}$ is divisible by 4 for all positive integers. **[5]**

e) To win the small county lottery, one must correctly select 3 numbers from 30 numbers. The order in which the selection is made does not matter. How many different selections are possible? **[4]**