1D Mixed Layer Code Description
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This 1D mixed layer code mixed_layer.m simulates the evolution of a water column under specified wind and heat forcing. Allowance is also made for currents forced via a depth-independent pressure acceleration. There are 6 physical state variables: *U* and *V* are the east to west and south to north current velocities, q^2 is a turbulent quantity equal to $2\times$ the turbulent kinetic energy, ℓ is a turbulent length scale, and *T* and *S* are the temperature and salinity. The evolution of these quantities are governed according to:

$$\frac{\partial U}{\partial t} - fV = -\frac{1}{\rho_o}\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}K_M\frac{\partial U}{\partial z} - \varepsilon U \tag{1}$$

$$\frac{\partial V}{\partial t} + fU = -\frac{1}{\rho_o}\frac{\partial p}{\partial y} + \frac{\partial}{\partial z}K_M\frac{\partial V}{\partial z} - \varepsilon V \tag{2}$$

$$\frac{\partial q^2}{\partial t} = \frac{\partial}{\partial z}K_q\frac{\partial q^2}{\partial z} + 2K_M\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] + \frac{2g}{\rho_o}K_H\frac{\partial \rho}{\partial z} - \frac{2q^3}{B_1\ell} \tag{3}$$

$$\frac{\partial q^2\ell}{\partial t} = \frac{\partial}{\partial z}K_q\frac{\partial q^2\ell}{\partial z} + E_1\ell\left(K_M\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] + E_3\frac{g}{\rho_o}K_H\frac{\partial \rho}{\partial z}\right) - \frac{q^3}{B_1}W \tag{4}$$

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}K_H\frac{\partial T}{\partial z} + s/s \tag{5}$$

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial z}K_H\frac{\partial S}{\partial z} + s/s \tag{6}$$

Boundary forcing for *U* and *V* are set by the wind and bottom stress (see below). The depth-independent pressure acceleration terms is specified as part of the model forcing. The bottom stress generated by currents arising from this term help prevent the bottom layers of the water column from stagnating. A dissipation term is included in eqs. (1) and (2) as a surrogate for horizontal momentum divergence. This term removes energy from past storm events over a specified time-scale as though energy was being transferred to more quiescent surrounding waters. Energy tends to accumulate unrealistically in 1D water columns without this effect (e.g., Mellor, 2001). The value of ε can be tuned such that the energy in the modeled currents is consistent with that observed. Values comparable to the time scales of storm events (0.2-1 day^{-1}) seem to yield reasonable results.

The turbulent quantities q^2 and $q^2\ell$ in equations (3) and (4) are used to derive the mixing coefficients for momentum (K_M), turbulence (K_q), and tracers (K_H) using the Mellor-Yamada 2.5 turbulence closure scheme (Mellor and Yamada, 1982). Terms on the right hand side of these equations represent the transport of turbulence, generation of turbulence via shear, and suppression of turbulence via stratification and dissipation. B_1, E_1, E_2, and E_3 are constants equal to 16.6, 1.8, 1.33 and 1.0 respectively. g is the acceleration due to gravity, and W is a "wall proximity function" which limits the eddy length scale near boundaries. Further details of the turbulence closure and boundary conditions for the turbulent quantities are described below.

In the equations (5) and (6), *s/s* represents a variety of source and sink terms for temperature and salinity. Temperature changes are driven by incident radiation and sensible, latent, and longwave heat fluxes. A net heat transport via advection can also be specified. Salinity changes are presently driven via relaxation to specified values. These sources and sinks are described in detail in the sections that follow. T and S are translated to a density using a routine from Phil Morgan's CSIRO seawater toolbox (UNESCO 1983 polynomial). Water is modeled as incompressible for this calculation.

Wind Forcing

Wind should be entered in the format:

[year month day hour minute second u-speed v-speed]

Where u and v are in m/s. These are translated to wind stresses using Large and Pond (Large and Pond, 1981). The stress (N m^{-2}) is calculated as:

$$\tau_x = \rho_{air} C_d |W_{10}| U_{10}$$
$$\tau_y = \rho_{air} C_d |W_{10}| V_{10}$$

Where W_{10} is the wind speed at 10m, C_d is a dimensionless drag coefficient, and ρ_{air} is the air density (which is assumed to be 1.25 kg m^{-3}). W_{10} is calculated from the measured wind speed assuming a logarithmic velocity profile. C_d is an empirical function of the wind speed:

$$C_d = 1.2 \times 10^{-3} \text{ for } W_{10} < 11\, m\, s^{-1}$$
$$C_d = 1 \times 10^{-3}(0.49 + 0.065 W_{10}) \text{ for } W_{10} \geq 11\, m\, s^{-1}$$

The wstress.m routine from Rich Signell's matlab toolbox is used for this calculation.

Bottom Stresses

The Princeton Ocean Model Formulation from of Mellor (2004) is used to calculate the bottom stress:

$$\tau_{x,b} = \rho C_{bot}\left(U^2 + V^2\right)^{1/2} U$$

$$\tau_{y,b} = \rho C_{bot}\left(U^2 + V^2\right)^{1/2} V$$

Where C_{bot} is a bottom drag coefficient calculated as:

$$C_{bot} = MAX\left[\frac{\kappa^2}{\left[\ln\left(0.5dz/z_{ob}\right)\right]^2}, 0.0025\right]$$

Where κ is Von-Karman's constant (=0.4, dimensionless), *dz* is the width of the bottom grid cell (m), and z_{ob} is a bottom roughness coefficient. The basic form of this relationship comes from a logarithmic boundary layer solution. z_{ob} is given a value of 0.01 by default - which is indicative of a rather smooth boundary. The lower C_{bot} limit comes into play only when dz resolution is very large relative to the roughness length scale. It was inserted to be consistent with the POM formulation, but can be avoided as long as dz/z_{ob} ~ 5000.

The Turbulence Closure Sub-model

Vertical mixing is determined using the Mellor-Yamada 2.5 turbulence closure scheme (Mellor and Yamada, 1982) with adjustments and simplifications described in Galperin et al. (1988) and Mellor (2004). Mixing coefficients are derived from the turbulent quantities q^2 and ℓ which are tracked prognostically via eqs. (3) and (4). The boundary condition for q^2 accounts for waves and is taken from equation (10) of Mellor and Blumberg (2004):

$$q^2(0) = (15.8\alpha_{CB})^{2/3} u_\tau^2(z=0)$$

Where u_τ is the surface "friction velocity" driven by the wind (=$\sqrt{\tau_{wind}/\rho_w}$, where ρ_w is the surface water density). Mellor and Blumberg (2004) suggest using α_{CB} = 100. A minimum value of 0.0001 is applied, which corresponds to a wind ~1 m/s. The bottom boundary condition for q^2 is calculated from equation 16a of Mellor (2004):

$$q^2(z = z_{bot}) = B_1^{2/3} u_\tau^2(z = z_{bot})$$

Where $u_\tau(z = z_{bot})$ is the bottom friction velocity. The length scale ℓ at the surface is from equations 5a and 6a of Mellor and Blumberg (2004):

$$\ell(z=0)=\kappa z_w=\kappa\beta\frac{u_\tau^2(z=0)}{g}$$

where β is 2×10^5 and κ is von Karman's constant (0.41). The non-zero length scale at the ocean surface reflects the surface wave roughness. A minimum z_w value of 0.02 was imposed. This corresponds to waves generated by a wind of 1 m s^{-1} and is meant to mimic the persistence of some wave energy through calm periods.

ℓ at the bottom boundary was set to 0.

The wall proximity function W limits ℓ near the boundaries by enhancing the dissipation. The formulation used is:

$$1+E_2\left(\frac{\ell}{\kappa L}\right)^2$$

where E_2 is a constant (1.33) and L is the distance from the surface or the bottom of the water column:

$$L=\frac{1}{abs(z)}+\frac{1}{abs(z_{bot}-z)}$$

Various wall proximity functions can be found in the literature and model codes. This one is very similar to those found in various versions of the Princeton Ocean Model and was taken from Williams (2006).

ℓ is limited in stably stratified flows in accordance with eqs. (22), (26) and (30) of Galperin et al. (1988). This is done using the quantity[1]:

$$G_H=\frac{\ell^2 g}{q^2\rho_o}\frac{\partial\rho}{\partial z}$$

Which is a ratio of potential $\left(=\frac{g}{\rho_o}\frac{\partial\rho}{\partial z}\right)$ to kinetic energy $\left(=\frac{\ell^2}{q^2}\right)$. Since $\frac{\partial\rho}{\partial z}$ is negative in stable water columns (i.e., $z=0$ at the water surface and becomes more negative with depth), G_H is negative in stable water columns. Galperin et al. (1988) found it necessary to limit ℓ in highly stable flows such that G_H stayed below 0.28. Minimum values of 1e-9 are imposed on both q^2 and $q^2\ell$ in accordance with the POM2K code.

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[1] Note that this expression assumes that water is incompressible. If compressibility is considered: $G_H=\frac{\ell^2 g}{q^2\rho_o}\left[\frac{\partial\rho}{\partial z}-\frac{1}{c_s^2}\frac{\partial p}{\partial z}\right]$, where c_s is the speed of sound.

The diffusivities are calculated from q^2 and ℓ following Mellor and Yamada (1982) and Galperin et al. (1988):

$$K_M = q\ell S_M$$
$$K_H = q\ell S_H$$
$$K_q = 0.41 q\ell S_H$$

The factors S_M and S_H all depend on G_H according to:

$$S_H = \frac{A_2\left[\left(1-6A_1/B_1\right)\right]}{1-\left(3A_2B_2-18A_1A_2\right)G_H} \sim \frac{0.49}{1-34.68G_H}$$

A_1, A_2, B_1, and B_2 are empirical constants that are 0.92, 0.74, 16.6, and 10.1. Because $S_H \to \infty$ for G_H values approaching 0.028 (unstable, high energy conditions), values of G_H are capped at 0.028. $S_H \to 0$ as $G_H \to -\infty$ (highly stable and or low energy conditions). For S_M:

$$S_M = \frac{A_1\left[1-3C_1-6A_1/B_1\right]+S_H\left[\left(18A_1^2+9A_1A_2\right)G_H\right]}{1-9A_1A_2G_H} \sim \frac{0.39+21.36S_HG_H}{1-6.13G_H}$$

Where C_1 is 0.08. This factor also approaches zero as $G_H \to -\infty$.

Heat Forcing

The mixed layer model can handle two formats for heat input. The first calls for direct specification of the incident, sensible, latent, and longwave radiation fluxes at the ocean surface. The second calls for the incident flux, air temperature, and the dew point temperature. In this latter case, sensible, latent, and longwave fluxes are calculated using various bulk formulae described below. This has the advantages of relying on commonly measured quantities and allowing feedbacks between observed atmosphere conditions and the calculated ocean temperature. Such feedbacks reduce the probability of developing large, persistent biases over long model runs. The bulk formulae enlisted are those used by Mountain et al. (1996) to model heat fluxes in the Gulf of Maine.

Incident radiation (Q_I)

The incident irradiance is entered in watts m^{-2} and the model assumes that data represents energy over the wavelength range measured by most pyranometers (300-5000 nanometers). This is adjusted downward according to the albedo specified in the input data. 45% of the Q_I is assumed to be from the photosynthetically available range (350-700 nm, Baker and Frouin, 1987). By default, this energy is assumed to have an attenuation coefficient of 0.15 m^{-1}. The remaining fraction is attributed to infra-red wavelengths and is absorbed rapidly (1.67 m^{-1}). There are presently no feedbacks between the biology and light absorption.

Sensible Heat (Q_S)

The Friehe and Schmitt (1976) formulae are used to estimate the sensible heat flux (Q_S, watts m^{-2}):

$$Q_S = -\rho_{air} C_p \overline{\omega\theta}$$

Where ρ_{air} is the air density (assumed to be ~ 1.2 kg m^{-2}), C_p is the specific heat capacity of air (1000 J kg^{-1} K^{-1}), and $\overline{\omega\theta}$ is the mean vertical velocity, temperature covariance (m K s^{-1}). This latter term is estimated as a function of wind speed at 10m (W_{10}) and the sea-air temperature difference ($\Delta T = T_{ocean} - T_{air}$):

$$\begin{aligned}
&\overline{\omega\theta} = 0.002 + \left(0.97 \times 10^{-3}\right) W_{10}\Delta T && 0 < W_{10}\Delta T < 25 && \text{(unstable)} \\
&\overline{\omega\theta} = \left(1.46 \times 10^{-3}\right) W_{10}\Delta T && W_{10}\Delta T \geq 25 && \text{(highly unstable)} \\
&\overline{\omega\theta} = 0.0026 + \left(0.86 \times 10^{-3}\right) W_{10}\Delta T && W_{10}\Delta T \leq 0 && \text{(stable)}
\end{aligned}$$

The sensible heat goes exclusively into the top grid cell.

The Latent Heat Flux (Q_L)

The formulae of Friehe and Schmitt (1976) are also used to estimate the latent heat flux (Q_L, watts m^{-2}):

$$Q_L = L_e \overline{wq}$$

Where $\overline{wq}$ is the mean vertical velocity, water vapor density covariance (m s^{-1} g m^{-3}) and L_e is the latent heat of evaporation (= 2250 J g^{-1}). This is estimated with the formula:

$$Q_L = L_e\left(1.32 \times 10^{-3}\right) W_{10}\left(q_s - q_a\right)$$

Where q_s is the water vapor density (g m^{-3}) at the sea surface, and q_a is the water vapor density (g m^{-3}) at a reference height above the ocean surface (~10m). q_s is calculated as the saturation humidity at the sea surface temperature (following the approach used in the Princeton Ocean Model). q_a is determined from the measure dew point temperature in the model forcing. The approximation $e_s = Ae^{\beta T}$ is used to calculate the saturated vapor pressure at temperature T for these calculations, where A = 611 Pa and β = 0.067 ^{o}C^{-1}. This was taken from Marshall and Plumb (2008, p. 6) and gives of the saturated vapor pressure at typical atmospheric conditions.

The Longwave Heat flux (Q_{LW})

The longwave flux is calculated using the Efimova formula as reported by Simpson and Paulson (1979). This calculates the clear-sky longwave radiation flux as:

$$Q_{LW,clear} = \varepsilon\sigma T_{srf}^{4}\left(0.254 - 0.00495e_a\right)$$

Where ε is the emissivity of the surface (= 0.97), σ is the Stefan-Boltzman constant (= 5.67e-8 W m^2 K^{-4}), T_{srf} is the surface ocean temperature (**in Kelvin)**, and e_a is the atmospheric vapor pressure (in mb = 100 Pascals). The clear sky longwave radiation loss is adjusted downward by a cloud correction factor of the form:

$$Q_{LW} = Q_{LW,clear}\left(1 - 0.8C\right)$$

Where C is the ratio of the observed incident radiation (Q_I) over 24 hours and the clear-sky value estimated using the Smithsonian formulas as reported in Reed (1977). These calculations are done during the model initialization. The clear-sky irradiance is calculated as a function of latitude and time of year. As of the writing of this description, only formulae good for latitudes from 20°S-60°N are included. The details can be found in the routine clearsky.m.

Advective heat fluxes (Q_{adv}).

A net heat flux due to advection can also be considered. This heat is added as a source/sink term over a depth scale (δ) following the approach of Umoh and Thompson (1994):

$$Q_{adv}(z) = Q_{adv,o} \times \frac{\exp(z/\delta)}{\delta(1 - \exp(-h/\delta)}$$

This assumed that the majority of the depth-integrated heat flux ($Q_{adv,o}$) is distributed over the depth scale δ - which is effectively and e-folding scale for the heat transport. h in the above expression it the total depth of the water column.

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