HOMEWORK ASSIGNMENT #1

DUE: 5PM, Thursday, September 13, 2018
CSCI 677: Advanced Computer Vision, Prof. Nevatia
Fall Semester, 2018

Submit assignment on the DEN class page. You may choose to write solutions by hand; in that case, please submit a scanned copy.

1. A rotation matrix applied to a point vector yields the coordinates of the point in a rotated coordinate system. An example is given in slide 6, lecture 4 and equation (1.8) of the FP. Write out the terms of rotation matrices for three different cases: rotation θ about the *z*-axis, φ about the *y*-axis and ψ about the *x*-axis. Assume that the rotations are in counter clock-wise direction when pointing along the direction of the rotation axes. Note that you are not being asked to derive the rotation matrix for a combination of these rotations, just for one rotation at a time.

2. For this problem, assume that the image coordinates are specified in the normalized coordinate system and 3-D points are specified in the camera coordinate system.
   a) Given a point $\hat{p}$ $(\hat{x}, \hat{y})$ in the image, we know that the corresponding 3-D point casting this image lies on a 3-D line. Derive the equation of this line (in the camera coordinate system).
   b) Consider a line, *l*, in the image, given by parameters (a, b, c). We know that the corresponding 3-D line casting this image lies in a plane. Derive the equation of this plane (in the camera coordinate system).

3. Suppose that we have a right-handed camera coordinate system (X_c, Y_c, Z_c) associated with origin at the lens center (or the pin hole), as in the examples discussed in class. Suppose that the imaging plane is at a distance of 50 millimeters from the lens center (i.e. focal length is 50 millimeters), the imaging surface (a planar patch) is 1000 x 1000 pixels, each pixel is .05 millimeters in each dimension and that the principal ray intersects the imaging surface in the center. Let the image (or retinal) coordinate system have its origin at the upper left corner of the imaging sensor, the *x*-axis along the top-row and the *y*-axis points downward (at an angle of -90 degrees to the *x*-axis) as is common in a digital image. Assume that the x-axis in the image plane is parallel to the $\hat{x}$-axis in the normalized image plane.
   a) For these conditions, derive the intrinsic matrix, *K*, which helps map a point, specified in the normalized image coordinate frame to the image coordinates $(x, y, 1)^T$ expressed in pixel units (ignore the issue of rounding off pixel coordinates to integers). ~~Choose a convenient alignment of the axes of the camera coordinate frame.~~

b) Now suppose that the camera is placed in a world coordinate system (O_w, X_w, Y_w, Z_w) such that O_c is at location (5,4,2) in the world coordinate system (all distances expressed in meters); X_c is parallel to X_w and then the camera is rotated by 30 degrees about the X_c axis in a clockwise direction (visualize as a person taking a picture with camera pointing down slightly). All questions below correspond to this configuration.

i) Compute the final projection matrix, M (as used in equation 1.16 of the FP book, or slide 7 of lecture 4).

ii) Compute the vanishing point of vertical lines in the world (i.e. lines parallel to the Y_w axis) in the image coordinates. You may use any equations given in the book or in class but derive any other equations, if any, that you make use of. You are encouraged, but not required, to use projective geometry formulation.

iii) Consider a set of parallel lines in the horizontal plane (i.e. the X_w, Z_w plane). Find the vanishing point, in the image coordinates, of this set of lines in terms of the direction of the lines. Again, you may use any equations given in the book or in class but derive any other equations, if any, that you make use of. You are encouraged, but not required, to use projective geometry formulation.

iv) Show that the vanishing points of the horizontal lines lie on a line in the image plane. Derive the equation of this line, also called the horizon line. Again, you may use any equations given in the book or in class but derive any other equations, if any, that you make use of. You are encouraged, but not required, to use projective geometry formulation.