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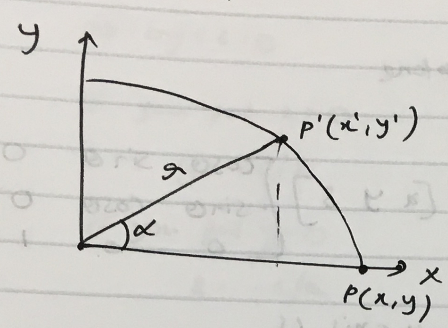
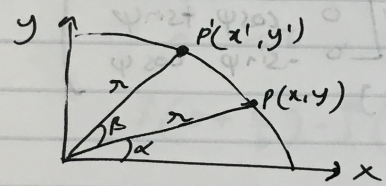
1) 2D rotation of a point on x-axis around Origin



$\cos\alpha = \frac{x'}{r} \Rightarrow x' = r\cos\alpha$

$\sin\alpha = \frac{y'}{r} \Rightarrow y' = r\sin\alpha$

2D rotation of arbitary point around origin



$x = r\cos\alpha$

$y = r\sin\alpha$

$x' = r * \cos(\alpha+\beta) \Rightarrow r(\cos\alpha\cos\beta - \sin\alpha\sin\beta) \Rightarrow r\cos\alpha\cos\beta - r\sin\alpha\sin\beta$

$= x\cos\beta - y\sin\beta$

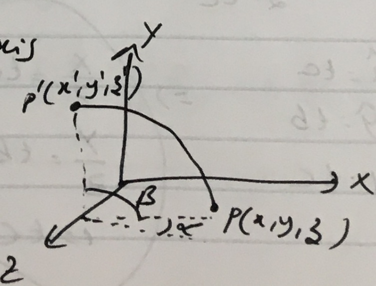
$y' = r * \sin(\alpha+\beta)$

$= r(\sin\alpha\cos\beta + \cos\alpha\sin\beta)$

$= y\cos\beta + x\sin\beta$

$$[x' \ y'] = [x \ y]\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

3D rotation around major axis



Rotation around z-axis is

z axis component remains same

$$[x' \; y' \; z'] = [x \; y \; z] \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So its same as rotating in x-y plane

$\beta = \theta$

$$[x' \; y' \; z'] = [x \; y \; z] \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Similarly rotation about y axis is

$$R_y(\varphi) = \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ +\sin\varphi & 0 & \cos\varphi \end{bmatrix}$$

$[x \; y \; z]$

Similarly rotation about x axis is

$$R_x(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & +\sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix}$$

$[x \; y \; z]$

2)a) parametric form 3D line is

$x = x_0 + ta$

$y = y_0 + tb$

$z = z_0 + tc$

We know that since it passes through $(0,0,0)$

$x = ta$

$y = tb$

$z = tc$

homogeneous coordinates

$\begin{pmatrix} \hat{x} \\ \hat{y} \\ 1 \end{pmatrix}$

$\hat{x} = ta$

$\hat{y} = tb$

$1 = tc$

$\Rightarrow$

$\frac{X}{Z} = ta$

$\frac{Y}{Z} = tb$

$1 = tc$

2D

2) b) equation of a line is

$ax+by+c=0$

if you put $x=0$

$y=-c/b$

if you put $y=0$

$x=-c/a$

So two points $(-c/a, 0)$ & $(0, -c/b)$ lie on line

converting to homogeneous coordinates

$(-c/a, 0, 1)$ $(0, -c/b, 1)$

cross product of 2 points

$$\begin{vmatrix} i & j & k \\ -c/a & 0 & 1 \\ 0 & -c/b & 1 \end{vmatrix}$$

$$\hat{i}\left(0+\frac{c}{b}\right)-\hat{j}\left(-\frac{c}{a}\right)+\hat{k}\left(\frac{c^2}{ab}-0\right)$$

$$=\frac{c}{b}\hat{i}+\frac{c}{a}\hat{j}+\frac{c^2}{ab}\hat{k}$$

cross product of 2 points gives ~~equatio~~ normal of the plane points lie in

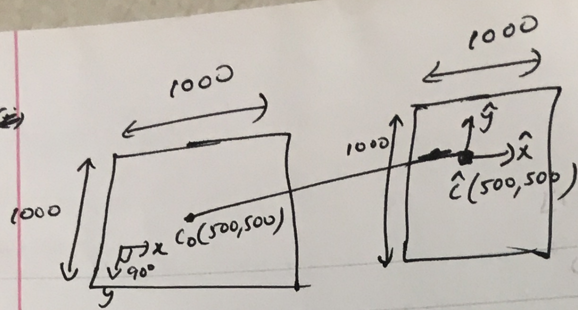
consider some point (x,y,z)

Since $(0,0,0)$ lies on plane

~~So~~ dot ~~product~~ of (x,y,z) & normal should give you 0

$$\left(\frac{c}{b}\hat{i}+\frac{c}{a}\hat{j}+\frac{c^2}{ab}\hat{k}\right)\cdot(x,y,z)=0$$

3) a)(i)



$\theta = -90°$

$(x, y) = 1000, 1000$

$c_0 = 500, 500$

$f = 50mm$

$x = \alpha \hat{x} - \alpha \cot\theta \hat{y} + x_0$

$y = \frac{\beta}{\sin\theta} \hat{y} + y_0$

$p = K\hat{p}$

$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$ $\quad K^{d4} = \begin{pmatrix} \alpha & -\alpha\cot\theta & x_0 \\ 0 & \beta/\sin\theta & y_0 \\ 0 & 0 & 1 \end{pmatrix}$

$\alpha = kf$

$\beta = lf$

$k = 1 \times \frac{1}{0.05} mm^{-1}$

$= 20mm^{-1}$

$l = 20mm^{-1}$

$\alpha = kf = 20mm^{-1} \times 50mm = 1000$

$\beta = lf = 20mm^{-1} \times 50mm = 1000$

$K = \begin{pmatrix} 1000 & -1000 \times \cot(-90) & 500 \\ 0 & 1000/\sin(-90) & 500 \\ 0 & 0 & 1 \end{pmatrix}$

$$\boxed{K = \begin{pmatrix} 1000 & 0 & 500 \\ 0 & -1000 & 500 \\ 0 & 0 & 1 \end{pmatrix}}$$

3)b)(i)

$M = K(R\ t)$

rotation of 30° clockwise about X_C

$$R_{X_C}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$\theta = 30°$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$t = (5, 4, 2)$

assuming O_w is $(0,0,0)$

O_c is $(5,4,2)$

$t = (-5, -4, -2)$

$M = K(R\ t)$

$$= \begin{pmatrix} 1000 & -1000\times\cot(-90) & 500 \\ 0 & 1000/\sin(90) & 500 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -5 \\ 0 & \sqrt{3}/2 & -1/2 & -4 \\ 0 & 1/2 & \sqrt{3}/2 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 1000 & 0 & 500 \\ 0 & -1000 & 500 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -5 \\ 0 & \sqrt{3}/2 & -1/2 & -4 \\ 0 & 1/2 & \sqrt{3}/2 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 1000 & 250 & 433 & -6000 \\ 0 & -616 & 933 & 3000 \\ 0 & 0.5 & 0.866 & -2 \end{pmatrix}$$

3)b)(ii) equation of a 3D line

$X(\lambda) = X_0 + \lambda D$

D is $(0, b, 0)$

$X_0 = (x, y, z)$

Converting to homogeneous coordinates

$$X_0 \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \qquad D = \begin{pmatrix} 0 \\ b \\ 0 \\ 0 \end{pmatrix}$$

$$X_0 + \lambda D = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ b \\ 0 \\ 0 \end{pmatrix}$$

$$M\left(\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ b \\ 0 \\ 0 \end{pmatrix}\right)$$

$$\underbrace{\begin{pmatrix} 1000 & 250 & 433 & -6000 \\ 0 & -616 & 933 & 3000 \\ 0 & 0.5 & 0.866 & -2 \end{pmatrix}}_{3\times4} \underbrace{\begin{pmatrix} x \\ y + \lambda b \\ z \\ 1 \end{pmatrix}}_{4\times1}$$

$\lambda \to \infty$

$$\vec{p}h = M\begin{pmatrix} \vec{d} \\ 0 \end{pmatrix}$$

$$x = \frac{250}{0.5} = 500$$

$$y = \frac{-616}{0.5} = -1232$$

(4)

3b)(iii) equation of a 3D line

$$X(\lambda) = X_0 + \lambda D$$

$$D \text{ is } \begin{pmatrix} a \\ 0 \\ c \\ 0 \end{pmatrix}$$

$$X_0 = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$X_0 + \lambda D = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} a \\ 0 \\ c \\ 0 \end{pmatrix}$$

$$M\left(\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} a \\ 0 \\ c \\ 0 \end{pmatrix}\right)$$

$$\underset{3\times 4}{\begin{pmatrix} 1000 & 250 & 433 & -6000 \\ 0 & -616 & 933 & 3000 \\ 0 & 0.5 & 0.866 & -2 \end{pmatrix}} \underset{4\times 1}{\begin{pmatrix} x+\lambda a \\ y \\ z+\lambda c \\ 1 \end{pmatrix}}$$

$\lambda \to \infty$

$$\vec{p}^h = M\begin{pmatrix} \vec{d} \\ 0 \end{pmatrix}$$

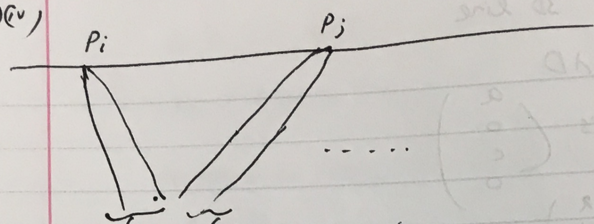
$$x = M\begin{pmatrix} a \\ 0 \\ c \\ 0 \end{pmatrix}$$

$$x = \frac{1000u + 433w}{0.866} \quad \text{if } u \,\&\, w = 1$$

$$= \frac{1433}{0.866} = 1654.73$$

$$y = \frac{933}{0.866} = 1077.36$$

3) b) (iv)



another set of parallel lines

one set of parallel lines

$$\vec{P}_k^h = M\begin{bmatrix}\vec{u}_k \\ 0\end{bmatrix}$$

tangent direction are coplanar in 3D

$$\vec{u}_k = \underbrace{a_k \vec{u}_1 + b_k \vec{u}_2}_{\text{constants}}$$

$$\vec{P}_k^h = M\begin{bmatrix}a_k\vec{u}_1 + b_k\vec{u}_2 \\ 0\end{bmatrix}$$

$$= M a_k \begin{bmatrix}\vec{u}_1 \\ 0\end{bmatrix} + M b_k \begin{bmatrix}\vec{u}_2 \\ 0\end{bmatrix}$$

$$\vec{P}_k^h = a_k \vec{P}_1^h + b_k \vec{P}_2 h$$

Bringing down from homogeneous to inge point

$$\vec{P}_k^h = a_k\left(\frac{P^h_{1,3}}{P^h_{k,3}}\right)\vec{P}_1 + \left(\frac{b_k P^h_{2,3}}{P^h_{k,3}}\right)\vec{P}_2$$

basically dividing by last coordinate

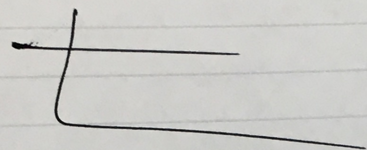
$$\vec{P}_k = \alpha_k \vec{P}_1 + \beta_k \vec{P}_2$$

$\vec{P}_k$ is combination of $\vec{P}_1$ & $\vec{P}_2$

horizon is line passing through 2 points
$\vec{P_1}$ & $\vec{P_2}$

if u look at 3(b)(iii)

y value is constant



so parallel to x axis

eq becomes

$$y = \frac{933}{0.866}$$