Percolation on the complete graph

- Complete graph K_n : vertices $1, \ldots, n$ and all possible edges.
- Denote by $G_{n,p}$ the graph obtained by only keeping edges of K_n of weight at most p.
- In $G_{n,p}$, each edge is independently present with probability p.

Erdős & Renyi, 1960

- For $p = (1 - \epsilon)/n$, a.a.s. every component of $G_{n,p}$ has size $O(\log n)$.
- For $p = (1 + \epsilon)/n$, a.a.s. one component of $G_{n,p}$ has size $\Theta(n)$, all others have size $O(\log n)$.
- For $p = 1/n$, a.a.s. the largest component of $G_{n,p}$ has size $\Theta(n^{2/3})$ and there may be many components of this order.