Identificación – Basic Econometrics

Lecturer: Luis Chávez

Agradecimientos a Angelo Nepo

Problema 1

SUBIDENTIFICACIÓN

Sea el modelo de oferta y demanda:

$$Q_t^d = \alpha_0 + \alpha_1 P_t + u_{1t} \quad (\alpha_1 < 0)$$

$$Q_t^s = \beta_0 + \beta_1 P_t + u_{2t} \quad (\beta_1 > 0)$$

Se igualan para hallar el precio y cantidad de equilibrio:

$$\alpha_0 + \alpha_1 P_t + u_{1t} = \beta_0 + \beta_1 P_t + u_{2t}$$

Precio de Equilibrio:

$$\alpha_1 P_t - \beta_1 P_t = \beta_0 - \alpha_0 + u_{2t} - u_{1t}$$

$$P_t(\alpha_1 - \beta_1) = \beta_0 - \alpha_0 + u_{2t} - u_{1t}$$

$$P_t = \frac{\beta_0 - \alpha_0 + u_{2t} - u_{1t}}{\alpha_1 - \beta_1}$$

$$P_t = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1}$$

Tendríamos:

$$\Pi_0 = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1}$$

$$v_t = \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1}$$

Entonces, el precio de equilibrio es:

$$P_t = \Pi_0 + v_t$$

Se reemplaza para hallar la cantidad de equilibrio:

$$Q_t = \alpha_0 + \alpha_1 P_t + u_{1t}$$

$$Q_t = \alpha_0 + \alpha_1(\Pi_0 + v_t) + u_{1t}$$

$$Q_t = \alpha_0 + \alpha_1.\Pi_0 + \alpha_1.v_t + u_{1t}$$

Se tiene los parametros:

$$\Pi_1 = \alpha_0 + \alpha_1(\frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1})$$

$$\Pi_1 = \alpha_0 + (\frac{\alpha_1\beta_0 - \alpha_1\alpha_0}{\alpha_1 - \beta_1})$$

$$\Pi_1 = \frac{\alpha_0(\alpha_1 - \beta_1) \, + \, (\alpha_1\beta_0 \, - \, \alpha_1\alpha_0)}{\alpha_1 \, - \, \beta_1}$$

$$\Pi_1 = \frac{\alpha_1.\beta_0 \, - \, \alpha_0.\beta_1}{\alpha_1 \, - \, \beta_1} \quad \ldots \quad (19.2.6)$$

Se despeja para el error:

$$w_t = u_{1t} \, + \, \alpha_1(\frac{u_{2t} \, - \, u_{1t}}{\alpha_1 \, - \, \beta_1})$$

$$w_t = u_{1t} \, + \, (\frac{\alpha_1 u_{2t} \, - \, \alpha_1 u_{1t}}{\alpha_1 \, - \, \beta_1})$$

$$w_t = \frac{u_{1t}(\alpha_1 \, - \, \beta_1) \, + \, (\alpha_1 u_{2t} \, - \, \alpha_1 u_{1t})}{\alpha_1 \, - \, \beta_1}$$

$$w_t = \frac{\alpha_1.u_{2t} \, - \, \beta_1.u_{1t}}{\alpha_1 \, - \, \beta_1}$$

La forma alterna de considerar el problema de identifiación, multiplicando la ecuasión (18.2.1) y la ecuasión (18.2.2) por lo siguiente:
Demanda:

$$(Q_t = \alpha_0 \, + \, \alpha_1 P_t \, + u_{1t}) \, \lambda$$

$$\lambda Q_t = \lambda\alpha_0 \, + \, \lambda\alpha_1 P_t \, + \lambda u_{1t}$$

Oferta:

$$(Q_t = \beta_0 \, + \, \beta_1 P_t \, + u_{2t}) \, (1 - \lambda)$$

$$(1 - \lambda)Q_t = (1 - \lambda)\beta_0 \, + \, (1 - \lambda)\beta_1 P_t \, + (1 - \lambda)u_{2t}$$

Donde tendriamos:

$$\lambda Q_t \, + \, (1 - \lambda)Q_t = Q_t$$

$$\gamma_0 = \lambda\alpha_0 \, + \, (1 - \lambda)\beta_0$$

$$\gamma_1 = \lambda\alpha_1 \, + \, (1 - \lambda)\beta_1$$

$$w_t = \lambda u_{1t} \, + \, (1 - \lambda)u_{2t}$$

Problema 2

IDENTIFICACIÓN EXACTA
Sea el modelo de oferta y demanda:

$$Q_t^d = \alpha_0 \, + \, \alpha_1 P_t \, + \, \alpha_2 I_t \, + \, u_{1t} \quad (\alpha_1 < 0 \, , \, \alpha_2 > 0) \quad \ldots \quad (19.2.12)$$

$$Q_t^s = \beta_0 \, + \, \beta_1 P_t \, + u_{2t} \quad (\beta_1 > 0) \quad \ldots \quad (19.2.13)$$

Igualamos para hallar el precio y cantidad de equilibrio.

$$\alpha_0 \, + \, \alpha_1 P_t \, + \, \alpha_2 I_t \, + u_{1t} \, = \, \beta_0 \, + \, \beta_1 P_t \, + u_{2t} \quad \ldots \quad (19.2.14)$$

Hallamos ahora Precio de Equilibrio:

$$\alpha_1 P_t \, - \, \beta_1 P_t \, = \, \beta_0 \, - \, \alpha_0 \, + u_{2t} \, - \, u_{1t} \, - \, \alpha_2 I_t$$

$$P_t(\alpha_1 - \beta_1) \, = \, \beta_0 \, - \, \alpha_0 \, + u_{2t} \, - \, u_{1t} \, - \, \alpha_2 I_t$$

$$P_t \, = \, \frac{\beta_0 \, - \, \alpha_0 \, + u_{2t} \, - \, u_{1t} \, - \, \alpha_2 I_t}{\alpha_1 - \beta_1}$$

$$P_t \, = \, \frac{\beta_0 \, - \, \alpha_0}{\alpha_1 - \beta_1} + \frac{u_{2t} \, - \, u_{1t}}{\alpha_1 - \beta_1} + \frac{- \, \alpha_2 I_t}{\alpha_1 - \beta_1}$$

Tendríamos:

$$\Pi_0 \, = \, \frac{\beta_0 \, - \, \alpha_0}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.16)$$

$$\Pi_1 \, = \, - \frac{\alpha_2}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.16)$$

$$v_t \, = \, \frac{u_{2t} \, - \, u_{1t}}{\alpha_1 - \beta_1}$$

Entonces nuestro Precio de Equilibrio es:

$$P_t \, = \, \Pi_0 \, + \, \Pi_1 I_t \, + \, v_t$$

Reemplazamos para hallar la Cantidad de Equilibrio

$$Q_t = \beta_0 \, + \, \beta_1 P_t \, + \, u_{2t}$$

$$Q_t = \beta_0 \, + \, \beta_1 (\, \Pi_0 \, + \, \Pi_1 I_t \, + \, v_t) \, + \, u_{1t}$$

$$Q_t = \beta_0 \, + \, \beta_1 . \Pi_0 \, + \, \beta_1 . \Pi_1 I_t \, + \, \beta_1 . v_t \, + \, u_{1t}$$

Despejamos para los parametros

$$(\Pi_2)$$

:

$$\Pi_2 = \beta_0 \, + \, \beta_1 (\frac{\beta_0 \, - \, \alpha_0}{\alpha_1 \, - \, \beta_1})$$

$$\Pi_2 = \beta_0 \, + \, (\frac{\beta_1 \beta_0 \, - \, \beta_1 \alpha_0}{\alpha_1 \, - \, \beta_1})$$

$$\Pi_2 = \frac{\beta_0 (\alpha_1 - \beta_1) \, + \, (\beta_1 \beta_0 \, - \, \beta_1 \alpha_0)}{\alpha_1 \, - \, \beta_1}$$

$$\Pi_2 = \frac{\alpha_1 . \beta_0 \, - \, \alpha_0 . \beta_1}{\alpha_1 \, - \, \beta_1} \quad \ldots \quad (19.2.18)$$

$$(\Pi_3)$$

:

$$\Pi_3 = \beta_1 (- \frac{\alpha_2}{\alpha_1 - \beta_1})$$

$$\Pi_3 = - \frac{\alpha_2 \beta_1}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.18)$$

Despejamos para el error:

$$w_t = u_{2t} \, + \, \beta_1 (\frac{u_{2t} \, - \, u_{1t}}{\alpha_1 \, - \, \beta_1})$$

$$w_t = u_{2t} + (\frac{\beta_1 u_{2t} - \beta_1 u_{1t}}{\alpha_1 - \beta_1})$$

$$w_t = \frac{u_{1t}(\alpha_1 - \beta_1) + (\beta_1 u_{2t} - \beta_1 u_{1t})}{\alpha_1 - \beta_1}$$

$$w_t = \frac{\alpha_1 . u_{2t} - \beta_1 . u_{1t}}{\alpha_1 - \beta_1}$$

La forma alterna de considerar el problema de identifiación, multiplicando la ecuasión (19.2.12) y la ecuasión (19.2.13) por lo siguiente:
Demanda:

$$(Q_t = \alpha_0 + \alpha_1 P_t + \alpha_2 I_t + u_{1t})\,\lambda$$

$$\lambda Q_t = \lambda\alpha_0 + \lambda\alpha_1 P_t + \lambda\alpha_2 I_t + \lambda u_{1t}$$

Oferta:

$$(Q_t = \beta_0 + \beta_1 P_t + u_{2t})\,(1-\lambda)$$

$$(1-\lambda)Q_t = (1-\lambda)\beta_0 + (1-\lambda)\beta_1 P_t + (1-\lambda)u_{2t}$$

Donde tendriamos:

$$\lambda Q_t + (1-\lambda)Q_t = Q_t$$

$$\gamma_0 = \lambda\alpha_0 + (1-\lambda)\beta_0$$

$$\gamma_1 = \lambda\alpha_1 + (1-\lambda)\beta_1$$

$$\gamma_2 = \lambda\alpha_2$$

$$w_t = \lambda u_{1t} + (1-\lambda)u_{2t}$$

Ahora supongamos que se considera el siguiente modelo: Demanda:

$$Q_t^d = \alpha_0 + \alpha_1 P_t + \alpha_2 I_t + u_{1t} \quad (\alpha_1 < 0\,,\, \alpha_2 > 0) \quad \ldots \quad (19.2.12)$$

Oferta:

$$Q_t^s = \beta_0 + \beta_1 P_t + \beta_2 P_{t-1} + u_{2t} \quad (\beta_1 > 0\,,\, \beta_2 > 0) \quad \ldots \quad (19.2.22)$$

Igualamos para hallar el precio y cantidad de equilibrio.

$$\alpha_0 + \alpha_1 P_t + \alpha_2 I_t + u_{1t} = \beta_0 + \beta_1 P_t + \beta_2 P_{t-1} + u_{2t} \quad \ldots \quad (19.2.23)$$

Hallamos ahora Precio de Equilibrio:

$$\alpha_1 P_t - \beta_1 P_t = \beta_0 - \alpha_0 + u_{2t} - u_{1t} - \alpha_2 I_t + \beta_2 P_{t-1}$$

$$P_t(\alpha_1 - \beta_1) = \beta_0 - \alpha_0 + u_{2t} - u_{1t} - \alpha_2 I_t + \beta_2 P_{t-1}$$

$$P_t = \frac{\beta_0 - \alpha_0 + u_{2t} - u_{1t} - \alpha_2 I_t + \beta_2 P_{t-1}}{\alpha_1 - \beta_1}$$

$$P_t = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1} + \frac{-\alpha_2 I_t}{\alpha_1 - \beta_1} + \frac{\beta_2 P_{t-1}}{\alpha_1 - \beta_1}$$

Tendríamos:

$$\Pi_0 = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.25)$$

$$\Pi_1 \; = \; -\frac{\alpha_2}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.25)$$

$$\Pi_2 \; = \; \frac{\beta_2}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.25)$$

$$v_t \; = \; \frac{u_{2t} \; - \; u_{1t}}{\alpha_1 - \beta_1}$$

Entonces nuestro Precio de Equilibrio es:

$$P_t \; = \; \Pi_0 \; + \; \Pi_1 I_t \; + \; \Pi_2 P_{t-1} \; + \; v_t$$

Reemplazamos para hallar la Cantidad de Equilibrio

$$Q_t = \alpha_0 \; + \; \alpha_1 P_t \; + \; \alpha_2 I_t \; + \; u_{2t}$$

$$Q_t = \alpha_0 \; + \; \alpha_1 (\, \Pi_0 \; + \; \Pi_1 I_t \; + \; \Pi_2 P_{t-1} \; + \; v_t) \; + \; u_{1t} \; + \; \alpha_2 I_t$$

$$Q_t = \alpha_0 \; + \; \alpha_1 . \Pi_0 \; + \; \alpha_1 . \Pi_1 I_t \; + \; \alpha_1 \Pi_2 P_{t-1} \; + \alpha_1 . v_t \; + \; u_{1t} \; + \; \alpha_2 I_t$$

Despejamos para los parametros

$$(\Pi_3)$$

:

$$\Pi_3 = \alpha_0 \; + \; \alpha_1 (\frac{\beta_0 \; - \; \alpha_0}{\alpha_1 \; - \; \beta_1})$$

$$\Pi_3 = \alpha_0 \; + \; (\frac{\alpha_1 \beta_0 \; - \; \alpha_1 \alpha_0}{\alpha_1 \; - \; \beta_1})$$

$$\Pi_3 = \frac{\alpha_0 (\alpha_1 - \beta_1) \; + \; (\alpha_1 \beta_0 \; - \; \alpha_1 \alpha_0)}{\alpha_1 \; - \; \beta_1}$$

$$\Pi_3 = \frac{\alpha_1 . \beta_0 \; - \; \alpha_0 . \beta_1}{\alpha_1 \; - \; \beta_1} \quad \ldots \quad (19.2.27)$$

$$(\Pi_4)$$

:

$$\Pi_4 = \alpha_1 (-\frac{\alpha_2}{\alpha_1 - \beta_1}) + \alpha_2$$

$$\Pi_4 = -\frac{\alpha_2 \alpha_1}{\alpha_1 - \beta_1} + \alpha_2$$

$$\Pi_4 = -\frac{\alpha_2 \alpha_1}{\alpha_1 - \beta_1} + \alpha_2$$

$$\Pi_4 = \frac{\alpha_2 (\alpha_1 - \beta_1) \; - \; (\alpha_2 \alpha_1)}{\alpha_1 \; - \; \beta_1}$$

$$\Pi_4 = -\frac{\alpha_2 \beta_1}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.27)$$

$$(\Pi_5)$$

:

$$\Pi_5 = \alpha_1(\frac{\beta_2}{\alpha_1 - \beta_1})$$

$$\Pi_5 = \frac{\alpha_1 \beta_2}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.27)$$

Despejamos para el error:

$$w_t = u_{1t} + \alpha_1(\frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1})$$

$$w_t = u_{1t} + (\frac{\alpha_1 u_{2t} - \alpha_1 u_{1t}}{\alpha_1 - \beta_1})$$

$$w_t = \frac{u_{1t}(\alpha_1 - \beta_1) + (\alpha_1 u_{2t} - \alpha_1 u_{1t})}{\alpha_1 - \beta_1}$$

$$w_t = \frac{\alpha_1 . u_{2t} - \beta_1 . u_{1t}}{\alpha_1 - \beta_1}$$

Problema 3

SOBREIDENTIFICACIÓN

Sea el modelo de oferta y demanda:

$$Q_t^d = \alpha_0 + \alpha_1 P_t + \alpha_2 I_t + \alpha_3 R_t + u_{1t} \quad \ldots \quad (19.2.28)$$

$$Q_t^s = \beta_0 + \beta_1 P_t + \beta_2 P_{t-1} + u_{2t} \quad \ldots \quad (19.2.22)$$

Se iguala para hallar el precio y cantidad de equilibrio.

$$\alpha_0 + \alpha_1 P_t + \alpha_2 I_t + \alpha_3 R_t + u_{1t} = \beta_0 + \beta_1 P_t + \beta_2 P_{t-1} + u_{2t}$$

Hallamos ahora Precio de Equilibrio:

$$\alpha_1 P_t - \beta_1 P_t = \beta_0 - \alpha_0 + u_{2t} - u_{1t} - \alpha_2 I_t - \alpha_3 R_t + \beta_2 P_{t-1}$$

$$P_t(\alpha_1 - \beta_1) = \beta_0 - \alpha_0 + u_{2t} - u_{1t} - \alpha_2 I_t - \alpha_3 R_t + \beta_2 P_{t-1}$$

$$P_t = \frac{\beta_0 - \alpha_0 + u_{2t} - u_{1t} - \alpha_2 I_t - \alpha_3 R_t + \beta_2 P_{t-1}}{\alpha_1 - \beta_1}$$

$$P_t = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{u_{2t} - u_{1t}}{\alpha_1 - \beta_1} + \frac{-\alpha_2 I_t}{\alpha_1 - \beta_1} + \frac{\beta_2 P_{t-1}}{\alpha_1 - \beta_1} + \frac{-\alpha_3 R_t}{\alpha_1 - \beta_1}$$

Tendríamos:

$$\Pi_0 = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.31)$$

$$\Pi_1 = -\frac{\alpha_2}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.31)$$

$$\Pi_2 = -\frac{\alpha_3}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.31)$$

$$\Pi_3 = \frac{\beta_2}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.31)$$

$$v_t \;=\; \frac{u_{2t}\;-\;u_{1t}}{\alpha_1 - \beta_1}$$

Entonces nuestro Precio de Equilibrio es:

$$P_t \;=\; \Pi_0 \;+\; \Pi_1 I_t + \Pi_2 R_t + \Pi_3 P_{t-1} + v_t$$

Reemplazamos para hallar la Cantidad de Equilibrio

$$Q_t = \alpha_0 \;+\; \alpha_1 P_t \;+\; \alpha_2 I_t \;+\; \alpha_3 R_t \;+\; u_{1t}$$

$$Q_t = \alpha_0 \;+\; \alpha_1(\,\Pi_0 \;+\; \Pi_1 I_t + \Pi_2 R_t + \Pi_3 P_{t-1} + v_t) \;+\; \alpha_2 I_t \;+\; \alpha_3 R_t \;+\; u_{1t}$$

$$Q_t = \alpha_0 \;+\; \alpha_1\,\Pi_0 \;+\; \alpha_1 \Pi_1 I_t + \alpha_1 \Pi_2 R_t + \alpha_1 \Pi_3 P_{t-1} + \alpha_1 v_t \;+\; \alpha_2 I_t \;+\; \alpha_3 R_t \;+\; u_{1t}$$

Despejamos para los parametros:

$$(\Pi_4)$$

:

$$\Pi_4 = \alpha_0 \;+\; \alpha_1(\frac{\beta_0\;-\;\alpha_0}{\alpha_1\;-\;\beta_1})$$

$$\Pi_4 = \alpha_0 \;+\; (\frac{\alpha_1\beta_0\;-\;\alpha_1\alpha_0}{\alpha_1\;-\;\beta_1})$$

$$\Pi_4 = \frac{\alpha_0(\alpha_1 - \beta_1)\;+\;(\alpha_1\beta_0\;-\;\alpha_1\alpha_0)}{\alpha_1\;-\;\beta_1}$$

$$\Pi_4 = \frac{\alpha_1.\beta_0\;-\;\alpha_0.\beta_1}{\alpha_1\;-\;\beta_1} \quad \ldots \quad (19.2.31)$$

$$(\Pi_5)$$

:

$$\Pi_5 = \alpha_1(-\frac{\alpha_2}{\alpha_1 - \beta_1}) + \alpha_2$$

$$\Pi_5 = -\frac{\alpha_2\alpha_1}{\alpha_1 - \beta_1} + \alpha_2$$

$$\Pi_5 = -\frac{\alpha_2\alpha_1}{\alpha_1 - \beta_1} + \alpha_2$$

$$\Pi_5 = \frac{\alpha_2(\alpha_1 - \beta_1)\;-\;(\alpha_2\alpha_1)}{\alpha_1\;-\;\beta_1}$$

$$\Pi_5 = -\frac{\alpha_2\beta_1}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.31)$$

$$(\Pi_6)$$

:

$$\Pi_6 = \alpha_1(-\frac{\alpha_3}{\alpha_1 - \beta_1}) + \alpha_3$$

$$\Pi_6 = -\frac{\alpha_3\alpha_1}{\alpha_1 - \beta_1} + \alpha_3$$

$$\Pi_6 = \frac{\alpha_3(\alpha_1 - \beta_1) \, - \, (\alpha_3\alpha_1)}{\alpha_1 \, - \, \beta_1}$$

$$\Pi_6 = -\frac{\alpha_3\beta_1}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.31)$$

$$(\Pi_7)$$

:

$$\Pi_7 = \alpha_1(\frac{\beta_2}{\alpha_1 - \beta_1})$$

$$\Pi_7 = \frac{\alpha_1\beta_2}{\alpha_1 - \beta_1} \quad \ldots \quad (19.2.31)$$

Despejamos para el error:

$$w_t = u_{1t} \, + \, \alpha_1(\frac{u_{2t} \, - \, u_{1t}}{\alpha_1 \, - \, \beta_1})$$

$$w_t = u_{1t} \, + \, (\frac{\alpha_1 u_{2t} \, - \, \alpha_1 u_{1t}}{\alpha_1 \, - \, \beta_1})$$

$$w_t = \frac{u_{1t}(\alpha_1 \, - \, \beta_1) \, + \, (\alpha_1 u_{2t} \, - \, \alpha_1 u_{1t})}{\alpha_1 \, - \, \beta_1}$$

$$w_t = \frac{\alpha_1.u_{2t} \, - \, \beta_1.u_{1t}}{\alpha_1 \, - \, \beta_1}$$

Entonces nuestra Cantidad de Equilibrio es:

$$Q_t \, = \, \Pi_4 \, + \, \Pi_5 I_t + \Pi_6 R_t + \Pi_7 P_{t-1} + w_t$$