#1 a) $f(x \pm \delta) \approx f(x) \pm f'(x)\delta + \frac{1}{2}f''(x)\delta^2 \pm \frac{1}{6}f'''(x)\delta^3 \ldots \pm \frac{1}{120}f^{(5)}\delta^5$

$f(x \pm 2\delta) \approx f(x) \pm 2f'(x)\delta + 2f''(x)\delta^2 \pm \frac{8}{6}f^{(3)}(x)\delta^3 \ldots \pm \frac{32}{120}f^{(5)}\delta^5$

$$
\begin{aligned}
Af(x+\delta) + Bf(x-\delta) + Cf(x+2\delta) + Df(x-2\delta) = {} & (A+B+C+D)f(x) \\
& + (A-B+2C-2D)f'(x)\delta \\
& + \left(\tfrac{A}{2}+\tfrac{B}{2}+2C+2D\right)f''(x)\delta^2 \\
& + \left(\tfrac{A}{6}-\tfrac{B}{6}+\tfrac{8C}{6}-\tfrac{8}{6}D\right)f^{(3)}(x)\delta^3 \ldots \\
& + \tfrac{1}{120}(A-B+32C-32D)f^{(5)}x\,\delta^5
\end{aligned}
$$

even terms cancel

$$
\begin{aligned}
A+B+C+D &= 0 \\
A-B+2C-2D &= 1 \\
\tfrac{A}{2}+\tfrac{B}{2}+2C+2D &= 0 \\
\tfrac{A}{6}-\tfrac{B}{6}+\tfrac{8}{6}C-\tfrac{8}{6}D &= 0
\end{aligned}
$$

$$
\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & -2 \\ \frac{1}{2} & \frac{1}{2} & 2 & 2 \\ \frac{1}{6} & -\frac{1}{6} & \frac{4}{3} & -\frac{4}{3} \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}
$$

$$
\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} & \frac{2}{3} & & \\ & -\frac{2}{3} & & \\ & -\frac{1}{12} & & \\ & \frac{1}{12} & & \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}
$$

$A = 2/3$ $\quad C = -1/12$

$B = -2/3$ $\quad D = 1/12$

$$
f' \approx \frac{1}{12}\Big(8\big(f(x+\delta) - f(x-\delta)\big) + f(x-2\delta) - f(x+2\delta)\Big) \underbrace{- \frac{f^{(5)}\delta^4}{30}}_{e_t}
$$

b) $E_{total} = e_r + e_t$

we have $f'(x) \approx \frac{8\,(f(x+\delta)-f(x-\delta)) - f(x+2\delta) + f(x-2\delta)}{12}$

we let $f(x) \to f(x)\cdot(1+g_i\epsilon)$

chose g_i s.t. the error is maximized,

then $e_r \le \frac{(8+8+1+1)\epsilon}{12\delta}|f(x)| = \frac{3}{2}\frac{\epsilon}{\delta}|f(x)|$

letting $\frac{dE_{total}}{d\delta} = \left(\frac{3}{2}\frac{\epsilon|f(x)|}{\delta} + \frac{|f^{(5)}(x)|}{30}\delta^4\right)' = 0$

$$\Rightarrow \frac{|f^{(5)}(x)|\cdot 4\delta^3}{30} = \frac{3}{2}\frac{\epsilon|f(x)|}{\delta^2}$$

$$\Rightarrow \delta = \left(\frac{45\,|f(x)|\epsilon}{4\,|f^{(5)}(x)|}\right)^{\frac{1}{5}}$$