Problem 1

let $f(x,t) = z^t \exp(ikx)$

$$\frac{f(t+dt,x) - f(t-dt,x)}{2dt} = -v\left(\frac{f(t,x+dx) - f(t,x-dx)}{2dx}\right)$$

$$\Rightarrow \frac{z^t \exp(ikx)\left(z^{dt} - z^{-dt}\right)}{2\,dt} = -v\left[\frac{z^t \exp(ikx)\left(e^{ikdx} - e^{-ikdx}\right)}{2dx}\right]$$

$$\Rightarrow z^{dt} - z^{-dt} = -v\frac{dt}{dx}\,2i\sin(kdx)$$

$$z^{2dt} + z^{dt}\,v\frac{dt}{dx}\,2i\sin(kdx) - 1 = 0$$

$$z^{dt} = \left(-2i\sin(kdx)\,\frac{v\,dt}{dx} \pm \sqrt{-4v^2\left(\frac{dt}{dx}\right)^2\sin^2(kdx) + 4}\right)/2$$

$$z^{dt} = -i\sin(kdx)\,\frac{v\,dt}{dx} \pm \sqrt{1 - v^2\left(\frac{dt}{dx}\right)^2\sin^2(kdx)}$$

$$|z^{dt}|^2 = \sin^2(kdx)\,v^2\left(\frac{dt}{dx}\right)^2 + 1 - v^2\left(\frac{dt}{dx}\right)^2\sin^2(kdx)$$

$$= 1$$

Thus there is no amplitude/Energy dissipation $\forall$ $v\,dt \le dx$