Question 1

the electric field is given by

$$E = \frac{\sigma}{4\pi\varepsilon_0}\int_0^{\pi} \frac{(z - R\cos\theta)(2\pi R\sin\theta)\,R\,d\theta}{(z^2+R^2-2zR\cos\theta)^{3/2}}$$

we let $x = \cos\theta$

s.t. $\theta = 0 \Rightarrow x = 1$

$\theta = \pi \Rightarrow x = -1$

$$dx = \sin\theta\, d\theta$$

we have

$$E = \frac{\sigma}{4\pi\varepsilon_0}\int_1^{-1} \frac{(z - Rx)\,2\pi R\,dx}{(z^2+R^2-2xzR)^{3/2}}$$

$$E = \frac{\sigma R}{2\varepsilon_0}\int_1^{-1} f(x)\,dx$$

$$f(x) = \frac{(z - Rx)}{(z^2+R^2-2xzR)^{3/2}}$$

Question 2

With the lazy integrator,
there are $\sum_{n}^{N} (2^{n+1} + 1)$ function calls.
With the efficient integrator,
There are $2^{N+1} + 1$ function calls.
Assuming $N = 50$